International Journal of Science and Research (IJSR)
ISSN: 2319-7064
SJIF (2020): 7.803

Spinning Light Clocks Reveal Time Contraction in Spinning Noninertial Frames

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Abstract: ***Spinning light clocks are invented and their behaviors are investigated. For a spinning light clock, the two mirrors of the clock rotate around a point between the two mirrors that are mathematically abstracted as two points. The trajectory of the light ray in the spinning frame is plotted, and it is curved; light bending appears. The formula to calculate the time interval for the light ray to travel from one mirror to the other mirror in the spinning frame is given. It is found that time contraction happens in the spinning frames.***

Keywords: spinning light clock; light bending; time contraction

1. Introduction

General relativity [1-2] is an extension of special relativity, and it copes with non-inertial frames; the relative velocity is not necessarily a constant. In special relativity, the relative velocity is a constant, and light clock is a conceptual clock [1-2] to study the time-dilation in inertial frames. In stationary frame, the light ray travels up and down with the speed c ($c = 3\times 10^8 \mathrm{m/s}$) along the straight line connecting the top and bottom mirrors. In uniform moving frame, the light ray still travels up and down along the straight line connecting the top and bottom mirrors from the perspective of the observer moving with the clock, but the light ray speed is reduced to $c\sqrt{1-\frac{V^2}{c^2}}$, so time dilation happens.

Mathematically, the top and bottom mirrors of a light clock can be abstracted as two points, and a light ray travels between the two points. In inertial frames, the mirrors ‘see’ that the light ray travels along a straight-line segment with the two points (representing the two mirrors) as the endpoints. In non-inertial frames, the mirrors may ‘see’ that the light ray travels along curved line segments between the two mirrors.

In this paper, two kinds of spinning light clocks are investigated. One is the spinning such that the spinning center is the midpoint of the line segments connecting the two points representing the two mirrors; the other is the spinning such that the spinning center is not at the center of this line segment.

2. Spinning Light Clock (A-type)

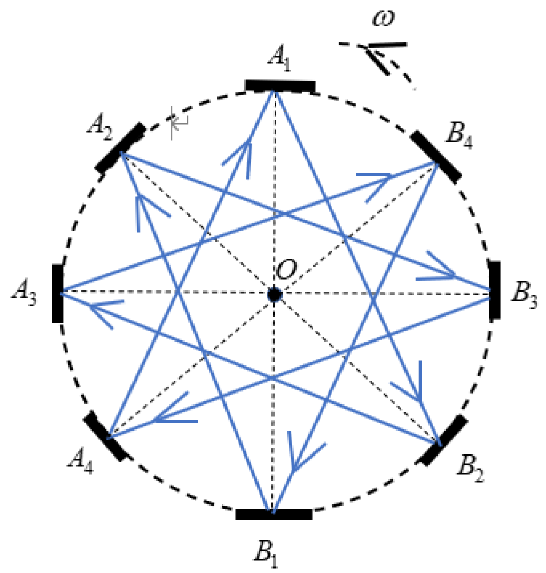

Figure 1: A spinning light clock (A-type)

In figure 2, it shows a spinning light clock (A type) in which both the top and the bottom mirrors rotate around the point O , and the distances between the spinning center O and the two mirrors are same. The radius of the circle is denoted as R .

$$2R = c\cdot\Delta t_1 \tag{1}$$

where Δt_1 is the time interval in the stationary frame, c is light speed.

Point A represents the top mirror and point B the bottom. The spin is counterclockwise and the angular velocity ω is a constant. In the beginning, the top mirror is at the point A_1

and the bottom mirror is at B_1, a light ray starts from B_1 and then hits A_2, and then hits B_3, and so on. In the triangle $\Delta B_1A_2A_1$, we know $|B_1A_2| = c\cdot\Delta t_2$, $\angle A_2OA_1 = \omega\cdot\Delta t_2$, and

$\angle A_2OB_1 = \pi - \angle A_2OA_1 = \pi - \omega\cdot\Delta t_2$.

where Δt_2 is the time interval for the light ray to travel from one mirror to the other mirror in the spinning frame.

According to the law of cosines, the following holds:

$$|B_1A_2|^2 = |OA_2|^2 + |OB_1|^2 - 2\cdot|OA_2|\cdot|OB_1|\cdot\cos(\angle A_2OB_1) \tag{2}$$

Due to $|B_1A_2| = c\cdot\Delta t_2$, $|OA_2| = |OB_1| = R = \frac{1}{2}c\cdot\Delta t_1$, we have the following:

$$(\Delta t_2)^2 = \frac{1}{2}\cdot(\Delta t_1)^2 + \frac{1}{2}\cdot(\Delta t_1)^2\cdot\cos(\omega\cdot\Delta t_2) \tag{3}$$

Eq(3) can be written as the following:

$$\frac{(\Delta t_2)^2}{1+\cos(\omega\cdot\Delta t_2)} = \frac{1}{2}(\Delta t_1)^2 \tag{4}$$

The above equation (4) establishes the relationship between the time interval in the stationary frame (Δt_1) and the time interval in the spinning frame (Δt_2).

From (4) we know:

$$\frac{\Delta t_2}{\Delta t_1} = \sqrt{\frac{1+\cos(\omega\cdot\Delta t_2)}{2}} \le 1 \tag{5}$$

The equal sign in (5) corresponds to the situation of $\cos(\omega\cdot\Delta t_2) = 1$, which requires

$$\omega\cdot\Delta t_2 = 2n\pi \quad (n = 0, \pm1, \pm2\cdots). \tag{6}$$

The following will prove that the number n in eq (6) can only take the value of zero, that corresponds to the situation of stationary light clock.

Assume eq(6) is valid for an integer whose absolute value is greater than zero $|n| > 0$, then we know the following based on (5) and (6):

$$\frac{\Delta t_2}{\Delta t_1} = 1 \tag{7}$$

From (1) we know that

$$\Delta t_1 = \frac{2R}{c} \tag{8}$$

Combining (7) and (8) we know the following:

$$\Delta t_2 = \frac{2R}{c} \tag{9}$$

Plug(9) into (6), then we have:

$$\omega\cdot\frac{2R}{c} = 2n\pi \tag{10}$$

where $|n| > 0$.

From (10) we know the following:

$$\omega\cdot R = n\pi c \tag{11}$$

where $|n| > 0$.

So we know $|\omega R| = |n\pi c| > c$. ($|\omega R|$ is the tangent translational speed of the mirror). This means that the speed is over light speed. This is impossible to happen.

The time contraction that appears here can also be understood by comparing the lengths of the paths the light ray travels along in stationary and spinning frames. In fig.1, we see that the light ray travels along the segment (such as B_1A_2) that does not pass the center of the circle in the spinning frame. While the segment (such as B_1A_1) passes the center of the circle in the stationary frame. The length of the segment B_1A_2 correspond to the time interval of the half-tick of the light clock in the spinning frame, and the length of the segment B_1A_1 correspond to the time interval of the half-tick of the light clock in the stationary frame. Since $|B_1A_2| < |B_1A_1|$, it gives rise to the time contraction.

For the case corresponding to fig.2, $\omega \cdot \Delta t_2 = \dfrac{\pi}{4}$, so we know $\dfrac{\Delta t_2}{\Delta t_1} = 0.9239$ according to eq (5).

3. Spinning Light Clock (B-type)

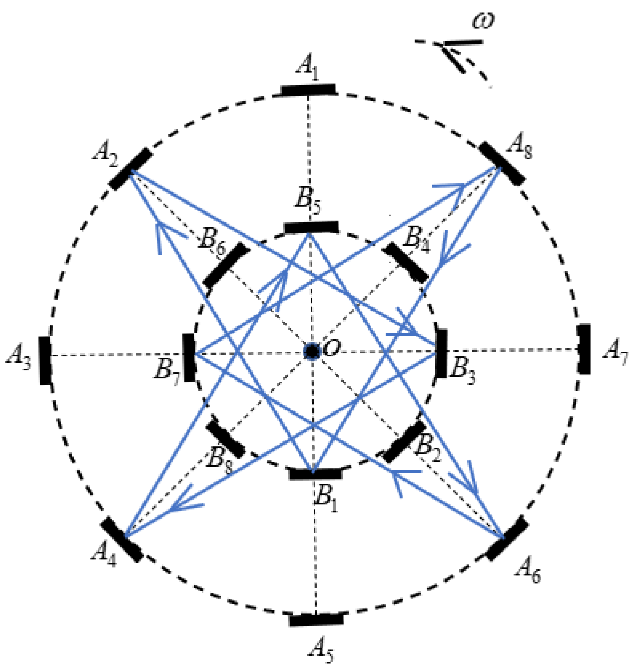

Figure 2: A Spinning light clock (B-type)

In figure 2, it shows a spinning light clock (B-type) in which both the top and the bottom mirrors rotate around the point O, and the distances between the spinning center O and the two mirrors are not same. The distance between the spinning center and the top mirror is denoted as R_1 ($|A_1O| = R_1$), and the distance between the spinning center and the top mirror is denoted as R_2 ($|B_1O| = R_2$). The ratio between the two distances is denoted as k and it is :

$$\frac{R_1}{R_2} = k \tag{12}$$

where $k \neq 1$. If $k = 1$, it becomes type A.
The time interval in the stationary frame for the light ray to travel from point B_1 to the point A_1 is denoted as Δt_1.

Then we have the following:

$$R_1 + R_2 = c \cdot \Delta t_1 \tag{13}$$

where c is light speed.

Combining (12) and (13), we can get R_1 and R_2:

$$R_1 = \frac{k}{k+1} \cdot c \cdot \Delta t_1 \tag{14}$$

$$R_2 = \frac{1}{k+1} \cdot c \cdot \Delta t_1 \tag{15}$$

In the triangle ΔB_1A_2O, we have the following according to the law of cosines:

$$|B_1A_2|^2 = |B_1O|^2 + |A_2O|^2 - 2 \cdot |B_1O| \cdot |A_2O| \cdot \cos(\angle A_2OB_1) \tag{16}$$

The time interval in the spinning frame for the light ray to travel from one mirror to the other mirror is denoted as Δt_2.

In the triangle ΔA_2B_1O, we know

$$|B_1A_2| = c \cdot \Delta t_2 \tag{17}$$

$$\angle A_2OB_1 = \pi - \angle A_1OA_2 = \pi - \omega \cdot \Delta t_2 \tag{18}$$

$$|B_1O| = R_2 = \frac{1}{k+1} \cdot c \cdot \Delta t_1 \tag{19}$$

$$|A_2O| = R_1 = \frac{k}{k+1} \cdot c \cdot \Delta t_1 \tag{20}$$

Plug (17) –(20) into (16), then we have the following:

$$\frac{(\Delta t_2)^2}{\left(\dfrac{k^2 + 1 + 2k\cos(\omega \cdot \Delta t_2)}{(k+1)^2}\right)} = (\Delta t_1)^2 \tag{21}$$

Eq (21) gives the relationship between the time intervals in the stationary and the spinning frames.

From (21) we know

$$\frac{(\Delta t_2)^2}{(\Delta t_1)^2} = \frac{k^2 + 1 + 2k\cos(\omega \cdot \Delta t_2)}{(k+1)^2} \leq 1 \tag{22}$$

The equal sign in (22) takes only when $\cos(\omega \cdot \Delta t_2) = 0$, this means $\omega \cdot \Delta t_2 = 2n\pi \ (n = 0, \pm 1, \pm 2 \cdots)$. By the proof method in the previous section (based on (6)-(11)), we know that n can only be equal to zero, which corresponds to situation of stationary light clock. Otherwise, the speeds of both the top and the bottom mirrors would be larger than the speed of light.

4. Light Bending

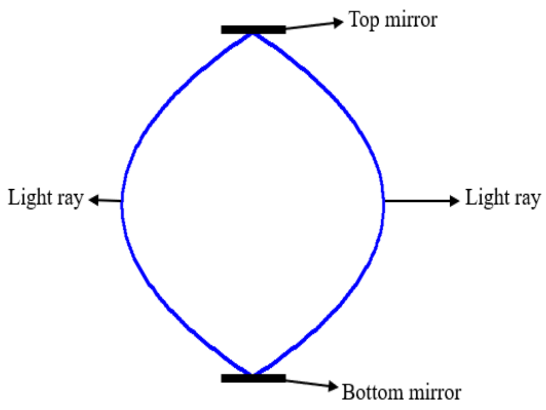

Figure 3: The bending of the light ray ‘seen’ by the mirrors of spinning light clock

In fig.3 it shows the light bending phenomenon; the light trajectory ‘seen’ by the mirrors in the spinning frame is curved. The trajectory is obtained through the numerical simulations in which the light speed is set as 1, and the distance between the two mirrors set as 1.

5. Conclusions

The behaviors of two types of spinning light clocks are investigated. Time contraction and light bending are found in the spinning frames. The explanation for the time contraction is given, and the qualitative relationship between the time intervals for the light ray to travel from one mirror to the other mirror in both the stationary and spinning frames is established.

References

[1] G. F. R. Ellis, R. M. Williams, Flat and Curved Space Times, Oxford University Press, 2000.
[2] R. Ferraro, Einstein’s Space-Time: An Introduction to Special and General Relativity, Springer Science, 2007.