Basics of Schrödinger's Wave Equation

Nikhil Singh

Abstract: ***Erwin Schrödinger is considered one of the founding father of quantum mechanics. Schrodinger's equation is fundamental of understanding Quantum Mechanics. Laws of Newtonian system don't work in quantum system. In this Research paper I have discussed brief overview on Schrodinger's wave equation.***

Keywords: Quantum mechanics, Schrodinger's equation, Schrödinger, wave function

1. Introduction

What is wave? Wave is a dynamic disturbance which carries a definite amount of energy. To understand waves we first have to be familiar with some terms.

Amplitude of wave which is denoted by A. it is the distance between highest point or crest and centre of the wave.

Wavelength which is denoted by λ (Lambda). It is the distance between two consecutive crest or trough.

Frequency is number waves passed through in unit time. It is denoted by v.

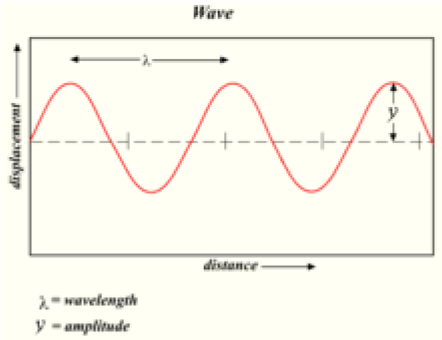

Image 1

In classical mechanics, wave equation is used to study behaviour of waves in space-time

The Wave equation used to understand motion of waves in classical mechanics can be represented as

$$\mathbf{Y(x,t) = A\sin(kx - \omega t)} \text{ or } \boldsymbol{\Psi(x,t) = A\, e^{i(kx-\omega t)}}$$

A is amplitude	k is $\frac{2\pi}{\lambda}$
x is position in x-direction	ω is angular frequency $2\pi v$
t is time.	e is Euler's number≈ 2.71828

So why do we need quantum wave equation? When scientists were arguing if light is wave or particle a physicist named Louis de Broglie developed a revolutionary idea.

De Broglie idea of matter wave

Louis de Broglie postulated in his Ph.D.'s thesis that electron also have dual characteristics i.e. acts like both wave and particle, just like light. Wavelength of De Broglie waves is inversely proportional to its mass. So in short we all have a wavelength but it is negligible because our mass is pretty huge.

He gave relationship between wavelengths and mass. It is as follows-

$$\boldsymbol{\lambda = \frac{h}{p} = \frac{h}{mv}} \qquad (1)$$

De Broglie idea of matter waves needed a wave equation to determine wave nature of quantum particle.

Schrödinger's wave equation

Erwin Schrödinger was an Austrian-Irish physicist he is considered one of the founding father of quantum mechanics. He developed the wave equation which is fundamental of quantum mechanics. His equation used the principle of conservation of energy from classical mechanics and applied it to quantum mechanical system

Time-dependent Equation is

$$\boldsymbol{i\bar{h}\frac{\partial\Psi(x,t)}{\partial t} = -\frac{\bar{h}^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} + V(x)\Psi} \qquad (3)$$

$$i = \sqrt{-1}, \bar{h} = \frac{h}{2\pi}$$

Derivation of Schrödinger's equation (2)

$$\boldsymbol{\Psi(x,t) = A\, e^{i(kx-\omega t)}} \qquad |1|$$

$$\boldsymbol{\Psi(x,t) = A\, e^{i(\frac{P}{\bar{h}}x - \frac{E}{\bar{h}}t)}}$$

($E = \bar{h}\omega$, $P = \bar{h}k$(Einstein-de Broglie relation))

Differentiating equation with respect to x

$$\boldsymbol{\frac{\partial\Psi(x,t)}{\partial x} = A\, e^{\frac{i}{\bar{h}}(Px-Et)}\,\frac{i}{\bar{h}}\,P}$$

Differentiating again

$$\boldsymbol{\frac{\partial^2\Psi(x,t)}{\partial x^2} = A\, e^{\frac{i}{\bar{h}}(Px-Et)}\,(\frac{i}{\bar{h}}P)\,(\frac{i}{\bar{h}}P)}$$

$$\boldsymbol{\frac{\partial^2\Psi(x,t)}{\partial x^2} = \frac{-P^2}{\bar{h}^2}\,\Psi(x,t)} \qquad |2|$$

(Putting value of Ψ from [1])

Now differentiating the equation with respect to time

$$\boldsymbol{\frac{\partial\Psi(x,t)}{\partial t} = A\, e^{\frac{i}{\bar{h}}(Px-Et)}(\frac{-i}{\bar{h}}E)}$$

$$\boldsymbol{\frac{\partial\Psi(x,t)}{\partial t} = \frac{-i}{\bar{h}}E\,\Psi(x,t)}$$

$$\boldsymbol{E\,\Psi(x,t) = i\bar{h}\,\frac{\partial\Psi(x,t)}{\partial t}} \qquad |3|$$

We know

$$\boldsymbol{E = \frac{p^2}{2m}}$$

Now

$$\boldsymbol{E\Psi(x,t) = \frac{p^2}{2m}\Psi(x,t)}$$

$$\boldsymbol{E\Psi(x,t) = \frac{-\bar{h}^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2}} \qquad |4|$$

Equating |3| and |4|

$$i\hbar \frac{\partial \Psi(x,t)}{\partial t} = \frac{-\hbar^2}{2m}\frac{\partial^2 \Psi(x,t)}{\partial x^2}$$

Adding potential energy to the equation we get

$$i\hbar \frac{\partial \Psi(x,t)}{\partial t} = \frac{-\hbar^2}{2m}\frac{\partial^2 \Psi(x,t)}{\partial x^2} + V(x)\Psi(x,t)$$

This is the **Schrödinger's wave equation**

Remember it is in only in one dimension for sake of our convenience.

So what does the Schrödinger's equation really determine?
Schrödinger's equation plays a role logically analogous to Newton's Second law: Given suitable initial conditions (typically, Ψ(x,0)), the Schrödinger's equation determines Ψ(x,t) for all future time, just as in classical mechanics, Newton's law determines x(t) for all future times(4)

What is wave function?
Wave function is everything we know about a quantum system. It is the mathematically description of the quantum state of an isolated system.

What is use of wave function?
Max born gave his statistical interpretation of the wave function, which says $|\psi(x)|^2$*determines the probability of finding the particle at point x, at time t*

$$\int_a^b |\Psi(x,t)(x,t)|^2 dx \quad = \{p(N)\}$$

$p(N)$ *= Probability of finding the particle between a and b, at time t*
Probability is the area under the graph of $|\Psi(x,t)|^2$ (5)

Imagine we put an electron in box? Where would be the electron? We know Probability of finding electron outside or on the box is zero. Probability density graph would look like something like this

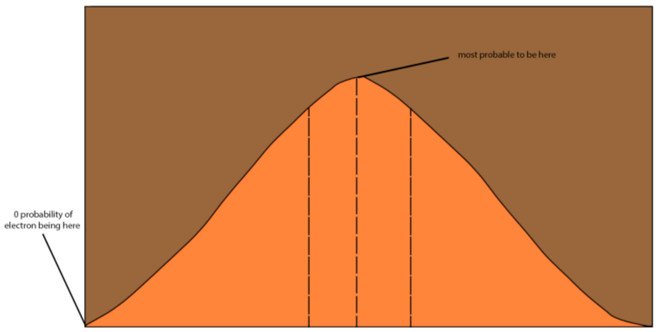

Image 2

We can see that there is zero probability of finding electron at the ends of box or outside the box and is most likely to be at point where the graph peaks. And when someone measures where electron is in the box the wave function collapses.

2. Acknowledgment

I would like to thank my Parents, Dr Jyoti Singh and Dr Raju Singh who encouraged me to write my first scientific paper. I am also grateful of my physics teacher Mr Hirendra Akolia teacher in Vasishtha Genesis School for this project.

References

[1] 3.1, Chapter-3, Quantum Physics by H.C Verma.
[2] Eo-10 Getting to Schrodinger s Equation by hc Verma Quantum Mechanics by H C Verma course; https://www.youtube.com/watch?v=q92BHRb1HRk&t=262s
[3] 1.1, Chapter-1, Introduction to Quantum Mechanics by David j Griffiths.
[4] Pg 2, Chapter-1, Introduction to Quantum Mechanics by David j Griffiths.
[5] Pg 2, Chapter-1, Introduction to Quantum Mechanics by David j Griffiths.

Author Profile

Nikhil Singh born in Kolkata on 1st of August, 2005. I am currently in High school in Surat, Gujarat, India. Contact details-niksud200@gmail.com