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Solvency and Reinsurance Treaty of an Automobile Portfolio: Case of Sub-Saharian Countries

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Abstract: ***The objective of this article is to analyse the actuarial solutions necessary to safeguard the solvency of an automobile portfolio, particularly for sub-Saharan countries. In the majority of Sub-Saharan countries motor civil liability is compulsory, it is the most important branch: In the CIMA zone (Inter African Conference of Insurance Markets), motor and health insurance make up 60% of the turnover of all 163 insurance companies (6). If the motor industry is badly managed, this can even lead to the insolvency of the insurance company. Faced with the internal needs of insurance companies to better control the underwriting risks of affaires and to adapt to the new and more demanding regulations regarding the quantification risks (Solvency II for Europe for example, with a Solvency Capital Requirement, the coverage is 99.5% of the risks not foreseen at one year(1) , another striking example is CIMA's decision, which requires all insurance companies in French-speaking countries to multiply their minimum social capital by 5 progressively, tripling it in 3 years and quintupling it in 5 years (2) ...), the empirical methods traditionally used have been replaced by probabilistic methods, based on modelling the annual frequency of claims and their ultimate individual severity. In order for the probability of an insurer's ruin to remain below a desirable threshold, according to ((8)RIMI, 2015), the insurer mainly has 4 means at his disposal which he uses jointly: loading the pure premium, setting up a reserve allocated to the risk, calling on reinsurance, using financial products.***

Keywords: Bonus-malus system, thirst for bonus, excess of loss reinsurance, provisioning, a priori car pricing.

AMS 2000 Subject classification: 60J10, 62F03, 91B30

1. Introduction

In automobile insurance, the insured is protected against all kinds of material damage caused to the insured vehicle (property insurance), and bodily injury suffered by the driver of the vehicle.

The aim of motor insurance companies is to make each insured pay a fair premium that is proportional to the risk to be covered. The problem that arises is to be able to determine certain criteria that make it possible to differentiate between insured persons.

Depending on the type of contract taken out, motor insurance can also cover material damage or bodily injury caused by the insured vehicle to third parties, known as liability insurance.

In the majority of Sub-Saharan countries motor civil liability is compulsory, it is the most important branch: in the Democratic Republic of Congo for example, it covers more than 80% of the turnover of the National Insurance Company (Motor, Fire, Life, Maritime, ARD, etc.). (3); In Algeria, compulsory automobile insurance represents nearly 57% of the damage insurance market(5) and in the CIMA zone, automobile and health insurance represents 60% of the turnover of all the 163 insurance companies(6). If the motor industry is badly managed, this can even lead to the insolvency of the insurance company. Faced with the internal needs of insurance companies to better control the underwriting risks of affaires and to adapt to the new and more demanding regulations regarding the quantification risks (Solvency II for Europe for example, with a Solvency Capital Requirement, the coverage is 99.5% of the risks not foreseen at one year**(1)** , another striking example is the decision of the Inter African Insurance Markets Conference, CIMA, which requires all insurance companies in French-speaking countries to multiply by 5 their minimum share capital progressively, tripling it in 3 years and quintupling it in 5 years (**2) ...**), the empirical methods traditionally used have been replaced by probabilistic methods, based on the modelling of the annual frequency of claims and their ultimate individual severity. In section 2, we set out the a priori pricing, where the insurer tries to predict, as soon as a new policyholder joins, his future claims experience according to certain criteria selected at the time of subscription. By carrying out a statistical analysis of reported claims, for example in Kinshasa in 2016 , J.Lemaire((7), Manya and Malonda((7),Manya and Bukanga((7) , have shown that the Bonus Malus system can be applied to class in the Democratic Republic of Congo, because of the persistent heterogeneity of the motor portfolio. Manya and Bukanga, worked on a finite horizon, realistic because one cannot stay in the system forever, or, more precisely, it is assumed that the insured will leave the system (because he will not be able to drive a vehicle) at a certain age (say after 40 years of driving).

In some countries of the world, the bonus-malus system is imposed by the government, in which case all insurers must adopt the same system (number of classes, transition rules, etc.). For other countries, the market is completely free; each insurer builds its own system.

After having given, in section 3, some theoretical foundations on the construction of a Bonus Malus System, we build, in the following section, a Bonus System based on the two types of Bonus Malus System practiced throughout the world: Class Bonus Malus System (Belgian type) and Multiplicative Bonus Malus System (French type).

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((7) J. Lemaire1975) has shown that the thirst for the Bonus, which encourages policyholders to bear the costs resulting from small claims themselves, means that they can make savings at the expense of the insurance company of up to 36% of the total sum paid by an insured who is unaware of dynamic programming and its applications. In the case of the Democratic Republic of Congo, these savings can amount to around 38%.

The Insurer will realise that the insured only report large claims and will once again be exposed to ruin. We close this paper by studying, as a third approach, the possibility for the insurance company to resort to reinsurance in order to safeguard its solvency.

The nature of the peak exposures accepted by reinsurance companies, and sometimes the limited statistical information available, are a challenge for actuaries in achieving their pricing approach. They are therefore faced with major difficulties in choosing and parameterising the probability distributions to be used in risk modelling.

The most appropriate form of reinsurance for insurance companies in sub-Saharan countries will be "excess of full" **or XP (*surplus share*) because** it reduces the risks taken by the ceding company, as the insurer knows in advance what the maximum amount it has to pay in the event of a claim, and premiums and claims are shared according to a pre-defined ratio. The reinsurer will only intervene on policies that exceed a certain guarantee amount, known as the retention amount or *line. Other means can be used to keep the probability of an insurer's ruin below a desirable threshold, including the use of financial products* ((8)RIMI, 2015) *We will not deal with this case in this article because few insurance companies in sub-Saharan countries are listed on the stock markets.*

2. A Priori Pricing of a Car Portfolio

As soon as a new policyholder joins the company, the insurer tries to predict his future claims record according to certain criteria that are set out at the time of subscription.

In motor insurance, pricing is based on the segmentation of the insurance portfolio into homogeneous classes: the aim is to classify policyholders according to their potential risk. The aim is therefore to select pricing criteria that are relevant and commercially usable. After segmentation, the insurance portfolio is divided into homogeneous classes, where policyholders belonging to the same class pay the same premium. This method consists in predicting the expected number of claims based on the characteristics observed a priori among the insured such as the use of the vehicle, the age of the vehicle (seniority), the sex, the age of the insured, the power of the vehicle, etc. The observable characteristics of the insured are called classification variables or *a priori* variables.

The pricing a priori therefore depends on the specific characteristics of the insured good (Vehicle) as well as the characteristics linked to the insured (the driver profile). These observable characteristics are called *classification variables or a priori variables or exogenous variables.*

It is difficult for statistical and practical reasons to take into account all the characteristics, so each company selects a few that it considers the most significant. In the Democratic Republic of Congo, for example, at the Société National d'Assurances (SONAS), only four criteria are taken into account for a priori pricing:

- The power of the vehicle: horsepower
- Its use: commercial use, taxi, rental or tourism
- Its engine: diesel or petrol
- The age of the vehicle.

Table 1: A priori pricing applied to SONAS/DRC

Class	Vehicle power	Annual bonus in relation to the duration of the vehicle	
		≥ 6 *ans*	< 5 *ans*
1	1 to 5 H.P.	173 $	163 $
2	6 to 9 H.V.	217 $	201 $
3	10 to 13 CVs	285 $	262 $
4	17 resumes ≥ 18	375 $	343$
5		508 $	466$

By collecting data relating to claims in Kinshasa, from an observed sample of 6,475 vehicles for the year 2016, drawn using the Simple Random Sampling method, Bukanga and Manya(6) have shown that pricing is not fair, good drivers are overcharged and bad drivers are undercharged. this is likely to discourage some policyholders and even contribute to their refusal to pay for insurance. This would jeopardise the solvency of the car portfolio.
A system of ex-post pricing must therefore be introduced.

3. A Posteriori Pricing of an Automobile Portfolio

- ***Model 1: Poisson Model (Homogeneous Portfolio)***: as a first approximation, we assume that all policyholders are equal in terms of risk, i.e. the probability of having an accident is the same for all policyholders. In this case, the occurrence of claims is then a random event and there is no need to penalise the insureds responsible for the claims. If, in addition, we make the following intuitive assumptions :

a) The probability of having 1 claim during a time interval $]t, t+\Delta t[$ is proportional to the length of this interval and does not depend on the number of claims at time t.
b) The probability of having more than one accident during the time interval$]t, t+\Delta t[$ is negligible.
c) The number of claims relating to two non-encroaching time intervals are independent.
d) Under assumptions (a), (b), (c), the distribution of the number of claims in the portfolio is a fish distribution of parameter$\lambda = 0.126$, for SONAS/DRC.

In a statistical analysis of the homogeneous portfolio model, bukanga and manya (6) showed, using K.Pearson's fit test, that the homogeneity assumption, that all insureds are equal in terms of risk, is rejected. We are therefore led to reject model 1

Model 2: Binomial Negative Model (Heterogeneous Portfolio)*:* Here we assume that not all insureds are equal with respect to risk, i.e. each insured has his or her own claims distribution. The qualities of a driver are therefore

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entirely summarised by the value of his or her claims frequencyλ. The portfolio is therefore made up of good and bad insureds.

Assuming once again assumptions a), b), c) of the previous model, the distribution of the number of claims of each insured is a fish distribution of parameter λ, parameter which varies from policy to policy and whose distribution function (structure function) is U(λ).

If the random variable □is distributed according to a gamma distribution (Γ) frequency function $dU(\lambda) = \frac{\tau^a \; e^{-\lambda\tau} \,.\, \lambda^{a-1}}{\Gamma(a)}$ (a, τ>0), then the distribution of the number of claims in the portfolio is a Negative Binomial, for the demonstration see [3]. Its probability distribution will be given by $p_k = \binom{k+a-1}{a-1}\left(\frac{\tau}{1+\tau}\right)^a\left(\frac{1}{1+\tau}\right)^k$, k=0,1,2,… of average $m = \frac{a}{\tau}$ and variance $\sigma^2 = \frac{a}{\tau^2}(1+\tau)$

Also doing a statistical analysis of this heterogeneous portfolio, bukanga and manya (6) showed, using K. Pearson's fit test, that the fit is good. The second model comes closest to reality, demonstrating the heterogeneity of the portfolio.

4. Building an Optimal Bonus-Malus System For Sub-Saharan African Countries

4.1 Model Assumptions

An insurance company uses a bonus-malus system when:

a) The set of fonts in a given group can be partitioned into a finite number of fonts. $s$classes$c(i = 1, \dots , s)$ in such a way that the annual premium depends only on the class.
b) The class at a given point in time is univocally determined by the class of the previous period and the number of claims reported during the period.
c) There are two final classes, one in which all policies are found after a sufficiently large number of years without claims and the other in which all policies with a sufficiently large number of accidents are found.

Such a system is determined by the following three factors:

a) The number of classes (noted s)
b) The bonus scale b_i $(i = 1, \dots , s)$ such as those insured in the i pay the premium b_i and $\forall i$, $i = 1, \dots ,s$ we have :$b_i \leq b_{i+1}$ iii.Transition rules, i.e. the laws governing the transition from one class to another when the number of claims is known.

These transition rules can be presented in the form of transformations T_k such as $(i) = j$ which means that the policy is transferred from the class c_i to the class $c_j$if $k$claims have been reported.

These transformations can also be presented in the form of a matrix$\left(t_{ij}^{(k)}\right)$.

The probability of an insured person moving from one class to another in the SBM depends on the transition rules pre-determined in the system.

Assuming that k accidents have been reported by the insured, the transition rules allowing the insured to transfer from one class to another are defined as follows:

$$t_{ij}^{(k)} = \begin{cases} 1 & \text{si} T_k(i) = j \\ \\ 0 & \text{sinon} \end{cases}$$

These are λ the average annual frequency of claims within the portfolio, and N_t the annual number of claims caused by an insured.

Consider an insurance company using a bonus-malus system. Each insured occupies a class in the bonus-malus scale that counts $(s + 1)$ classes (numbered from 0 to s).

Degree 0 gives the right to a maximum bonus and the relative bonus increases with the level to reach its maximum by s.

Note that:
L_t the class occupied by the insured between the moments t and $+ 1$.
$\{L_t,\ t\in\ \mathbb{N}\}$ the discrete-time stochastic process that represents the trajectory of the insured.

The system is such that an insured person's degree for a given period of insurance is determined by the degree of the previous period and the number of claims relating to that period.

If the insured descends unconditionally one class per year down the ladder and each claim is penalized by a rise of ω degree, the class L_{t+1} where the insured will be positioned at the moment $t + 1$ is given by:

$$L_{t+1} = \max\{\min\{L_t + \omega N_{t+1} - 1,\ s\},\ 0\}$$

Generally speaking, $L_{t+1} = \Psi(L_t; N_{t+1})$ where $\Psi(. , .)$ is a growing function in both of its arguments.

Conditional on the quality of the risk

$P[L_{t+1} = l_{t+1}|L_t = l_t, \dots , L_0 = l_0, \theta] = P[L_{t+1} = l_{t+1}|L_t = l_t, \theta]$
(III. 1.)

As long as the trajectory $_0, \dots , l_t$ is possible, i.e.

$$P[L_t = l_t, \dots , L_0 = l_0] > 0.$$

The relationship (III. 1) expresses the fact that the state currently occupied by the insured in the scale summarizes all the information useful to know its future evolution.

What this means is that the forecast of future developments is improved only by having occupied levels at times 1, 2,...

It is this property that allows the evolution of an insured person to be modelled using Markov processes. Indeed, a Markov chain is a stochastic process in which future development depends solely on the present state and not on the history of the process or the way in which the present state has been reached. It is a process *without memory* such

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that the different states of the chain represent the different levels of the bonus-malus system.

Knowledge of the level occupied at the present time and the number of claims caused by the insured during the year are sufficient to determine the level he will occupy the following year. It is therefore not necessary to know how the insured has reached the level he or she currently occupies.

Manya and Bukanga (6) showed that the proposed class system for the DRC has 23 classes.

The class system, the scale of bonuses and the rules for transitions are shown in Table VIII below:

Table 12: The premium scale and transition rules in relation to the number of claims reported by the policyholder

Class	Bonus level	T_0	T_1	T_2	T_3	T_4	$T_k(k \geq 5)$
22	508	21	22	22	22	22	22
21	482	20	22	22	22	22	22
20	459	19	22	22	22	22	22
19	437	18	22	22	22	22	22
18	416	17	22	22	22	22	22
17	397	16	21	22	22	22	22
16	378	15	20	22	22	22	22
15	360	14	19	22	22	22	22
14	343	13	18	22	22	22	22
13	326	12	17	22	22	22	22
12	311	11	16	21	22	22	22
11	296	10	15	20	22	22	22
10	282	9	14	19	22	22	22
9	268	8	13	18	22	22	22
8	256	7	12	17	22	22	22
7	243	6	11	16	21	22	22
6	232	5	10	15	20	22	22
5	221	4	9	14	19	22	22
4	210	3	8	13	18	22	22
3	200	2	7	12	17	22	22
2	191	1	6	11	16	21	22
1	182	0	5	10	15	20	22
0	173	0	4	9	14	19	22

The bonus level is in US dollars ($). An insured will pay $296 if he/she is in class 11. People whose vehicles have between 1 and 9 h.p. access the system in class 9 and others in class 14.

The rules for transition from one class to another are as follows:

Downgrading by one grade per year with no claims per year involving one or more claims; raising by 4 classes for the first claim declared and raising by 5 classes for subsequent claims.$T_1(11)$=15, i.e. after declaring a claim, any insured in class 11 will be transferred to class 15

The restriction to this system is that: Whatever the number of accidents caused, the insured will not exceed classes 0 and 22.

5. Thirst for the Bonus and Loss of the Insurance Company on a Finite Horizon

It is much more realistic to consider that the insured cannot stay in the system forever, or, more precisely, it is assumed that the insured will leave the system (because he or she will not be able to drive a vehicle at a certain age (say after a maximum of 40 years of driving). We therefore assume that the maximum duration of the insured is N periods. Let Wn: The probability that the risk is insured for the nth period given that it was insured during the (n-1)th period. It goes without saying that W1=1 and WN+1=0.

The approach of a rational policyholder will be to minimize the discounted expectation of future payments.

Manya and Malonda (6) proposed the following algorithm while working in Retrospective Analysis on a finite horizon. At each period, calculate the optimal policy $\bar{x}$ (n) and the corresponding updated expectation $\bar{v^n}$. The existence of the optimal policy is guaranteed by R. BELLMAN's optimality theorem.

Algorithm

Given a policy $\bar{x}_n = (x_1(n), \ldots, x_s(n))$;

Start

n = N

For i from 1 to s

$(N) = 0$

$N \ (N)$

$vi \ = b_i$

End For

For n from N - 1 to 1 For i from 1 to s

$$x_i(n) = W_{n+1} \cdot \beta^{1-t} \cdot \sum_{k=0}^{L} \bar{p}_k^i[\lambda(1-t);n] \cdot \left[v^{n+1}_{T_{k+m+1(i)}} - \bar{v}^{n+1}_{T_{k+m(i)}} \right]$$

$$v_i^n = E[x_i(n)] + W_{n+1} \cdot \beta \cdot \sum_{k=0}^{L} \bar{p}_k^i[\lambda, n] \cdot v^{n+1}_{T_{k\,(i)}}$$

End For

End For

End

Where v_i^n is the discounted expectation of all future payments for an insured person at the beginning of the nth period in class c_i.

$[(n)]$ is the expected cost (premium + personally compensated claims) in the nth period. xi(n): retention limit of an insured who, during his nth class period, is in a class c_i

t∊[0,1] is the time in the year when a claim occurs.

β< 1 is the discount rate

$\bar{p}_k^i[\lambda, n]$is the probability that an insured with a claims frequency of k claims in λ his nth period will report k claims if he is in class i.

It is then clear that the retention limit $\bar{x}$ (n) of the insured person depends on two parameters for each period:

λ = frequency of claims

β = discount rate

which are generally not perfectly known by the insured. It is therefore interesting to study the variation of $\bar{x}$ (n) according to these parameters.

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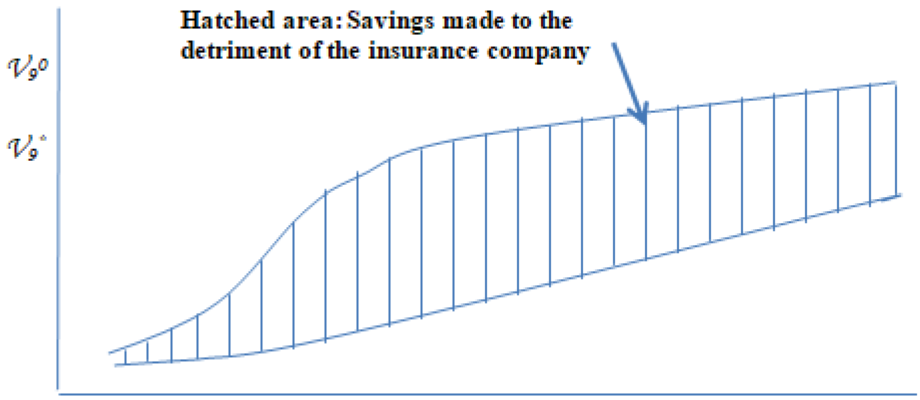
Applying the algorithm 4.2 to the Bonus Malus system in Table 12 calculate $\bar{x}(n)$ by first varying λ and then β

For a constant interest rate (4%), we have for all calculated $\bar{x}$ usual values of λ. Figure 1 shows the optimum retention limit for the most characteristic classes.

The highest limits are obtained in classes 16, 17 and 15 for the usual values of λ (i.e. for λ ≤ 0, 8). This means that policyholders are required to pay very large claims.

The absolute maximum is reached at the point λ= 0.2 in class 16. It is therefore in the interest of the insured in this class to indemnify himself for any claim under US$254, which amounts to covering 71% of the claims.



The savings that policyholders can realize at the expense of the insurance company by applying the optimal policy can be considerable. The hatched area can reach 40% of the total sum paid by an insured ignoring the dynamic programming and its applications; the insurance company will resort to reinsurance to safeguard its solvency.

6. Optimal Reinsurance Treaty for Insurance Companies in Subsaharan Countries: "Full Surplus" Or XP (Surplus Share)

In 2015, the global reinsurance volume is estimated at 230 billion dollars, with the following breakdown: 28% in life and 72% in non-life. Here again, we see the large share of non-life reinsurance.

The sums insured for certain risks are enormous. In aviation, for example, the sums insured can be in the order of US$200 million and US$500 million for passengers and cargo. A subSaharan insurance company cannot bear such a risk alone without putting itself in danger of bankruptcy in the event of a claim.

The most suitable form of reinsurance for these insurance companies will be the "excess of full" or "excess of sum" or "surplus share" or XP (surplus share) because it reduces the risks taken by the ceding company, as the insurer knows in advance what the maximum amount it has to pay in the event of a claim, the premiums and the claims are shared according to a ratio defined in advance. The reinsurer will only intervene on policies that exceed a certain guarantee amount, known as the retention amount or line. It is therefore a proportional and individual contract.

For each risk j in the portfolio, we define the retention rate or retention coefficient aj, ($0<a_j<1$), with j = 1, 2, ..., n.

Here, the cession rate is calculated policy by policy, and for policy i, the ceded claims (and premium) rate is ai

The form ''Excess of full'' is defined by the following table:

	Total risk	Retained risk conservé	Risk ceded
Claims	$S = \sum_{j=1}^{n} S_j$	$\sum_{j=1}^{n} a_j S_j$	$\sum_{j=1}^{n} (1 - a_j).S_j$
Prime	$P = \sum_{j=1}^{n} P_j$	$\sum_{j=1}^{n} a_j P_j$	$\sum_{j=1}^{n} (1 - a_j).P_j$

S: Total amount of claims to be paid in one year for this insurance portfolio
P: Total annual premium (loaded) charged for this insurance portfolio.
The random variables X and P characterize the original portfolio (before reinsurance).
Sj is the annual amount for risk j (j =1, 2, ..., n) or the annual risk for contract j.
Pj is the annual premium received for the risk j (j = 1, 2, ..., n) in the portfolio under consideration. In practice, for each policy, the reinsurer only pays for the
the portion of the risk exceeding a level of capital, called the full retention. The effective disposal rate is in fact

$$\theta = \frac{(min\{plein\ de\ souscription - capitaux\ assurés\} - plein\ de\ souscription)_+}{min\{plein\ de\ souscription - capitaux\ assurés\}}$$

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It should be noted that the portfolio held by the insurer is capped.

7. Discussions

a) This form of reinsurance is proportional: for each risk j, the proportion of the claim to be borne by the reinsurer is known in advance;
b) Reinsurance is determined on a risk-by-risk basis (individual reinsurance), the total amount of the claim borne by the reinsurer depends on each of the V.A. Sj ;
c) This form of reinsurance is appropriate when the portfolio is heterogeneous, which is in line with an insurance company's motor portfolio;
d) This method is a bit more benefic for the insurer, however, the insurer still has to face the risk of accumulation of claims (large number of claims per year) ;
e) The full surplus optimizes the retention because at a given level of reserves (or equity capital), proportional reinsurance increases the safety coefficient and thus reduces the probability of ruin. The major disadvantage of this method is that it requires a precise tariff grid to be defined and communicated to reinsurers.

8. The Provisioning of an Automobile Portfolio Line-by-Line Provisioning in Liability Insurance

Technical provisions in non-life insurance are mainly provisions for claims payable (PSAP), provisions for unearned premiums (PPNA), provisions for risks in progress (PPRE) and equalization provisions (PPE).

The PSAP is the largest share; it represents on average 85% of the reserves of non-life insurance companies (an individual). In terms of solvency, the company should build up as many provisions as possible, but in terms of performance and profitability vis-à-vis shareholders, it wishes to build up as little as possible. The difficulty lies in predicting future benefits as accurately as possible. A good estimate of this is therefore a major challenge for the company.

Third Party Liability is a non-life insurance branch that is considered to be long term in the sense that the insurer is still required to pay compensation for claims that have occurred several years before. This is due to the waiting time between the occurrence of the claim and the final court decision and to the fact that many claims are only reported a few years after the year in which they occurred. It is clear, in this case, how important it is to set aside a provision for claims at the time of the inventory in order to meet subsequent payments for claims relating to the current or previous financial years.

For sub-Saharan countries, the most appropriate method in motor insurance is developed in (an individual): it is a method of calculating provisions on a claim-by-claim basis (also known as a line by-line provisioning model), for claims that have already been reported to the insurer. To do this, each claim is considered individually and is characterized by: a date of occurrence, a settlement process and a status process (closed or in the process of being settled).

The use of stochastic methods to determine the level of provisioning, although it does not always make it possible to reduce the provisioning charge, nevertheless provides information on the risk inherent in the level of provisioning used. The use of these techniques reveals in particular that the calculation of provisions using the Chain Ladder method, which is an estimate based on the average, sometimes leads to an allocation that is not very prudent. If a distribution can be associated with claims settlements, a determination of the provision using quartiles is more appropriate. For this reason, the stochastic approach is at the heart of discussions on the overhaul of solvency indicators for insurance companies.

9. The Solvency Analysis

The solvency analysis criterion based on the approach by the probability of "ruin" and simulation are treated in (Lise HE ENSAE 2003-2004)

Using a probability of ruin approach, the aim is to determine the minimum level of equity capital so that the probability of ruin is negligible.

Economically speaking, the assets of an insurance company are characterised by a set of positive flows which correspond to the income generated by the assets in the investment portfolio and the liabilities by a set of negative flows which correspond to future benefits.

In order for the insurance undertaking to be solvent, it must have sufficient resources to pay future benefits, which means that the asset/liability margin defined as the difference between the asset/liability between the NAV of the assets and the NAV of the liabilities can be covered by the initial equity. We seek to determine the level of initial *FP* equity capital that must be held by the company to remain solvent in *1-x% of* cases. This is equivalent to looking for the level of initial equity capital such that the probability of insolvency is below a threshold *x%; and* given:

Proba (VAP [Assets] - VAP [Liabilities] < FP) < x

This definition of solvency is similar to the definition of VaR (Value at Risk), i.e. the amount of potential loss that will not be exceeded in x% of cases.

Proba (VAP [Liabilities] - VAP [Assets] > FP) x□

The simulation of the trajectories of the Asset-Liability margin (difference between the NAV (Assets) and the NAV (Liabilities)) of Monte Carlo type, can be built by the computer tool, under VBA of Excel or MATLAB.

10. Conclusion

In this paper, we have analysed the actuarial solutions necessary to safeguard the solvency of an automobile portfolio, particularly for sub-Saharan countries.

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In the majority of Sub-Saharan countries motor third party liability is compulsory, it is the most important branch: In the CIMA zone (Inter African Conference of Insurance Markets), motor and health insurance make up 60% of the turnover of all 163 insurance companies(6).

If the motor industry is badly managed, this can even lead to the insolvency of the insurance company. Faced with the internal needs of insurance companies to better control the underwriting risks of affaires and to adapt to the new and more demanding regulations regarding the quantification risks (Solvency II for Europe for example, with a Solvency Capital Requirement, the coverage is 99.5% of the risks not foreseen at one year**(1),** another striking example is CIMA's decision, which requires all insurance companies in French-speaking countries to multiply their minimum social capital by 5 progressively, tripling it in 3 years and quintupling it in 5 years (**2) ...**), the empirical methods traditionally used have been replaced by probabilistic methods, based on modelling the annual frequency of claims and their ultimate individual severity.

In order for the probability of an insurer's ruin to remain below a desirable threshold, according to ((8)RIMI, 2015), the insurer mainly has 4 means at his disposal which he uses jointly: loading the pure premium, setting up a reserve allocated to the risk, calling on reinsurance, using financial products. We have shown that in spite of the loading of the pure premium, by practising a priori or a posteriori pricing, the solvency of the automobile portfolio is not guaranteed (moral Alea, thirst for Bonus, insufficient provisions...), so we have proposed other alternatives: the recourse to reinsurance and the constitution of provisions for claims to be paid.

- As the pricing model for the automobile portfolio is individual (each insured pays according to the danger or risk he or she poses to the community), we have chosen the following two adapted models: Excess of full reinsurance or XP (surplus share) determined on a risk-by-risk basis (individual reinsurance), is appropriate when the portfolio is heterogeneous, which is in line with an insurance company's motor portfolio; it reduces the risks taken by the ceding company. In addition, the proportionality of this model increases the safety coefficient and therefore reduces the probability of ruin.
- The provisioning model using the method for calculating provisions on a claim-byclaim basis (also known as the line-by-line provisioning model).

Finally, we used a solvency analysis model based on the "ruin probability" approach. The simulation of the trajectories of the Asset-Liability margin (difference between the NAV (Assets) and the NAV (Liabilities)) of Monte Carlo type, can be built by the computer tool, under Excel VBA or on MATLAB.

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