

The Cyclic Group Used in 24-Hour Time Format Digital Clock

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Abstract: The paper presents the results of research and application of cycle group in the 24-hour time format in Digital clock. The research works we use external trustful resources and materials. Cyclic group are ordinary in our daily life. Therefore, cyclic group is widely used in many field and well worth people exploring. Cyclic group could be pattern found in nature. For example, in a urchin, a snowflake, a bell pepper and flowers. Cyclic groups can be e thought of a rotation. If we rotate an object sufficient time we will conclusively return to the initial position. Cyclic groups are used in topics such as cryptography and number Theory. In this paper we discover for the application of cyclic group in the 24-hour time format in digital clock.

Keywords: Cyclic group, Subgroup, Whole numbers($N \cup \{0\}$), 24-Hour time format, Clock Arithmetic, Addition table

1. Introduction

Let (G, o) be a group and 'c' an element of G. The Subgroup $\langle c \rangle$ generated by $\{c\}$ is denoted by $\langle c \rangle$. Thus, $\langle c \rangle = \{c^n | n \in Z\}$.

2. Definition of Cyclic group:

A group (G, o) is said to be a Cyclic group if it is generated by a single element. That is, for some c in G , $G = \{c^n | n \text{ is an element of } Z\}$ or, in addition notation, $G = \{nc | n \text{ is an element of } Z\}$. This element 'c' (which need not be unique) is called generator of G. Alternatively, we may write $G = \langle c \rangle$.

- Let G be a group and let $c \in G$. if c has infinite order, then $c^i c^j$ if and only if $i = j$. If c has finite order, say n then $\langle c \rangle = \{e, c, c^2, c^3, \dots, c^{n-1}\}$ and $c^i c^j$ if and only if n divides $i=j$.

3. Examples

- The set of integer Z under ordinary addition is cyclic. Both are 1 and -1 are generators. (Recall that, when the operation is addition, 1^n is interpreted as $(1 + 1 + 1 + \dots + 1)$ n terms, when n is positive $(\{-1\} + \{-1\} + \{-1\} + \dots + \{-1\})$ |n| terms, when n is negative)
- The $Z_n = \{0, 1, 2, \dots, n-1\}$ for $n \geq 1$ is a cyclic group under addition modulo n. Again, 1 and $-1 = n-1$ are generators. Unlike Z, which has only 2 generators, Z_n may have many generators (depending on which n we are given).

- $Z_8 = \langle 1 \rangle = \langle 3 \rangle = \langle 5 \rangle = \langle 7 \rangle$.

To verify, for instance, that $Z_8 = \langle 3 \rangle$, we note that $\langle 3 \rangle = \{3, 3+3, 3+3+3, \dots\}$ is the set $\{3, 6, 1, 4, 7, 2, 5, 0\} = Z_8$. Thus 3 is a generator of Z_8 . On other hand, 2 is not a generator, since $\langle 2 \rangle = \{0, 2, 4, 6\}$ is not equal to Z_8 .

4. Methodology

Clock arithmetic:

On a clock the numbers cycle from 1 to 24. After circulating around the clock, we do not go to 25 but restart at 1.

We have used following notations,

1 = 1.00 A.M

2 = 2.00 A.M

3 = 3.00 A.M

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12 = 12.00 P.M

13 = 1.00 P.M

14 = 2.00 P.M

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24 = 12.00 A.M or 0.00 A.M

If it was 18 o' clock what would it be in 8 hours?

$18 + 8 = 2$

Where 2 = 2.00 A.M. The set of the numbers on a clock are $C = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23\}$.

Table: 24 -Hour time format of digital clock addition

Z	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
0	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	0
2	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	0	1
3	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	0	1	2
4	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	0	1	2	3
5	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	0	1	2	3	4
6	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	0	1	2	3	4	5
7	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	0	1	2	3	4	5	6
8	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	0	1	2	3	4	5	6	7
9	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	0	1	2	3	4	5	6	7	8
10	10	11	12	13	14	15	16	17	18	19	20	21	22	23	0	1	2	3	4	5	6	7	8	9
11	11	12	13	14	15	16	17	18	19	20	21	22	23	0	1	2	3	4	5	6	7	8	9	10
12	12	13	14	15	16	17	18	19	20	21	22	23	0	1	2	3	4	5	6	7	8	9	10	11
13	13	14	15	16	17	18	19	20	21	22	23	0	1	2	3	4	5	6	7	8	9	10	11	12
14	14	15	16	17	18	19	20	21	22	23	0	1	2	3	4	5	6	7	8	9	10	11	12	13
15	15	16	17	18	19	20	21	22	23	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
16	16	17	18	19	20	21	22	23	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
17	17	18	19	20	21	22	23	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
18	18	19	20	21	22	23	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
19	19	20	21	22	23	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
20	20	21	22	23	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
21	21	22	23	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
22	22	23	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21
23	23	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22

This set of numbers is a group. The Identity element is 0 what we will think of as 24. If we add 24 hours to anywhere on the clock, we will end up in the same position.

Since there are infinitely many whole numbers, It is not possible to write to complete table of addition facts for that set. Such a table, to show the sum every possible pair of whole numbers, would have an infinite number of rows and columns, making it infeasible to construct. On the other hand, the 24 hours clock system use only the whole numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22 and 23.

A table of possible sums for this system requires only 24 rows and 24 columns. The 24 - hourtime format of digital clock **addition table** is shown in below table. Since the 24 - hour system is built upon a finite set, it is called a **finite mathematical system**.

5. Results and Discussion

- (1) Stuart arrives at his office every day at 9.00 am and works there for about 8 hours every day then comes straight home from there. But if one day his work is done in 7 hours, what time would Stuart have come to his house that day???

Here we will use the table shown above to know what time Stuart must have come to his house so that it can be easily understood. Time to reach Stuart's office = 9.00 AM

Total working hours in the office = $9 + 7 = 16 = 4.00$ PM

[modulo integer z under addition in the above table (9th column and 7th row) = 16 or (7th column and 9th row) = 16]

Therefore, Stuart reached his house at 4.00 pm that day.

- (2) The time of arrival of a train at Ahmedabad station is 4 pm and for some reason the train is running 7 hours

later than its scheduled time, so what time will the train arrive at Ahmedabad station now?

Arrival time at Ahmedabad station here = 4.00 P.M = 16

Hear, given the train is already showing that it is running 7 hours late, so

$$16 + 7 = 23 = 11.00 \text{ P.M}$$

Therefore, train will reach Ahmedabad station at 11 P.M

Thus, the cyclic group in the 24-hour time format Digital clock is used for a lot of importance in practical life.

6. Benefits

It should encourage students to discover more about this topic. It is a new relation which creates some new questions related to the usage cyclic group in everyday actions around us.

7. Conclusion

Human minds are designed for pattern recognition and we can find algebraic structures in common objects and things around us. Maths is unavoidable. It is a deeply fundamental thing. It is a main matter of the real-life that can cover all aspects of real world. Cyclic groups are the simplest groups that have an object that can generate the whole set. The object can generate the set by addition, multiplication, or rotations. Cyclic groups are not only common in pure mathematics, but also in patterns, shapes, music, and chaos. Also, important of star polygons in art and culture. Cyclic groups are an imperative part of number theory used with the Chinese remainder theorem and Fermat's theorem. Knowing if a group is cyclic could help determine if there can be a way to write a group as a simple circuit. This circuit could simplify the process of generation to discover the most efficient way to generate the object for use of future

applications in mathematics and elsewhere. As we know the human life indirect and direct have close relationship with mathematics, here we mentioned the application of cyclic group in the clock which is match and connected in our valuable life.

24 - hour time format in Digital clock elements set

$C = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23\}$, has the algebraic structure of the group and its cyclic group.

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