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Linear Programming Using ABS Method

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Abstract: ***Nowadays, we face many equations in everyday life, where many attempts have been made to find theirsolutions, and various methods have been introduced. Many complex problems often lead to the solution of systems of equations. In mathematics, linear programming problems is a technique for* optimization *of a* linear objective function *that must impose several constraints on linear inequality. Linear programming emerged as a mathematical model.In this study, we introduce the category of ABS methods to solve general linear equations. These methods have been developed by Abafi, Goin, and Speedicato, and the repetitive methods are of direct type, which implicitly includes LU decomposition, Cholesky decomposition, LX decomposition, etc. Methods are distinguished from each other by selecting parameters. First, the equations system and the methods of solving the equations system along with their application are examined. Introduction and history of linear programming and linear programming problems and their application were also discussed.***

Keywords: ABS Algorithm, Huang Method, Implicit LU, Implicit LX, Linear Programming, Linear Inequalities

1. Introduction

Nowadays, we face many equations in everyday life; where many attempts have been made to find the solutions, and various methods have been introduced. Many complex problems often lead to the solution of equations systems.

In mathematics, linear programming problems is a technique for optimization of a linear objective function that must impose several constraints on linear inequality. Informally, linear programming uses a linear mathematical model to get the best output (e.g. maximum profit, minimum work) according to given conditions (for example only 30 hours per week, illegal work not done, etc.). And more formally in a polygon or polygon on which a function with real value is defined, the goal is to find the point in these conditions where the objective function has the most or the least value. These points may not be available, but if available, searching among the vertices of a polygon will ensure that at least one of them is found. [1]

Solving the problem by linear inequality dates back to the Fourierarea. Linear programming emerged as a mathematical model and became clear during and after World War II that planning and coordination of various projects and the efficient use of scarce resources were a necessity. The US Air Force Optimized Planning Team began its serious work in June 1947. The result was the invention of the simplex method by Jorj. B. Dantzik at the end of the summer of 1947. Linear programming quickly has been interested in economists, mathematicians, statisticians, and government agencies.In the summer of 1949, a planning conference was held to plan expenditures and returns, under the auspices of the Cowles Committee for Economic Research. Papers presented at this conference were compiled shortly afterward in 1951 by T.C.Koopmans in a book entitled Production Activity Analysis and Allocation. In the same year, JanvanNeumnn developed the theory of duality, and Leonid Khashian, a Russian mathematician used simple techniques in pre-Dantesik economics, winning the 1975 Nobel Prize in Economics. [2] In this study, linear programming by ABS method was investigated.

Solving Linear Equations Systems

If F is a field, find n scalar $x_1, x_2, \ldots, x_n$ in the following Equation;

$$A_{11}x_1 + A_{12}x_2 + \ldots + A_{1n}x_n = b_1$$

$$A_{m}x_1 + A_{22}x_2 + \ldots + A_{2n}x_n = b_2$$

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$$A_{n1}x_1 + A_{n2}x_2 + \ldots + A_{mn}x_n = b_m$$

where $b \in R^m$, $A \in R^{m \times n}$, $1 \leq j \leq n$ and $1 \leq j \leq m$. The above equation is called m equation in n unknown linear. Each n of $x_1, \ldots, x_n$ elements that hold in any of the above equations is called a system solution. This equation is briefly in the form of;

$$Ax = b$$

A is matrices of equation coefficients, x unknown vectors and b isthe right value.

Tip 1. If the equation has at least one solution, it is consistent; and inconsistent if it does not have asolution at all [3]

Linear Programming

We begin our discussion by formulating a specific type of mathematical programming problem. As you can see below, any linear programming problem can be designed this way. [6]

A) Variables

Unknown values are the equation that must be decided.

B) Restrictions

The governing conditions of the problem, which are expressed as a number of mathematical equations or inequalities.

C) Objective function

It is a linear expression in which the objective of solving a problem is specified. The objective function may be Max or Min.

The general form of linear models is as follows:

$$\text{(MinorMax) } Z = c_1x_1 + c_2x_2 + \ldots + c_nx_n$$
$$\text{or } s.t.\ a_{11}x_1 + a_{12}x_2 + \ldots . + a_{1n}x_n \le = \ge b_1 \text{ or}$$
$$\text{or } a_{21}x_1 + a_{22}x_2 + \ldots . + a_{2n}x_n \le = \ge b_2 \text{ or}$$
$$\text{or } a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \le = \ge b_m \text{ or}$$
$$\text{unrestricted} \le \circ \text{ or } x_1, x_2, \ldots, x_n \ge \circ$$

Here, $c_1x_1 + \ldots + c_nx_n$ is objective functions (or standard functions) that need to be optimized and denoted by Z. Coefficients are $c_n, \ldots, c_1, c_1$ coefficients of the objective function (known) and $x_n, \ldots, x_2, x_1$ decision variables (variables, structural variables, or activity levels) that need to be specified. Restricted $\ge b_i$ or $=$ or $\sum_{j=1}^{n} a_{ij}x_j \le$ indicates the first constraint (implicit or functional, structural or technical), and the a_{ij} coefficients are called technical coefficients.

The column vector whose first component is b_i is called the right vector. A set of $x_n, \ldots, x_1$ variables that holds in all restricts called feasible region.

The LP problem can be summarized as follows:

$$(Max \quad Min) z = \sum_{j=1}^{n} c_j x_j$$
$$s.t. \qquad \sum_{j=1}^{n} a_{ij} x_j \le \quad = \quad b_i \qquad i = 1,000\, m \quad (1.1)$$
$$x_j \ge \circ \quad \le \circ \qquad j = 1,000\, n$$

The matrix shape (1. 1) is as follows

$$(Max \quad Min) z = c_1x_1 + c_2x_2 + \ldots + c_nx_n$$
$$\begin{aligned} s.t. \quad & a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \le \ = \ \ge b_1 \\ & a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \le \ = \ \ge b_2 \\ & . \\ & . \\ & . \\ & a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \le \ = \ \ge b_m \\ & x_1, x_2, \ldots, x_n \ge \circ \quad \le \circ \end{aligned}$$

Where, $C = (c_1, c_2, \ldots, c_n)$ is objective function coefficients, $x^t = (x_1, x_2, \ldots, x_n)^t$ the vector of variables,

$$A = \begin{bmatrix} a_{11} & a_{12} & \ldots a_{1n} \\ . & . & . \\ . & . & . \\ . & . & . \\ a_{m1} & a_{m2} & a_{mn} \end{bmatrix}$$

is restricted coefficient matrix (technical coefficients), and $b^t = (b_1, b_2, \ldots, b_m)^t$ is the requirements vector.

In economic problems, maximization and minimization problems are often seen in a special way, which we call the focal (conventional) form. The simplex method also specially solves the problem, which we call the standard form. Table (1) shows both focal and standard forms of maximization and minimization problems: [7]

Table 1: Focal Shape and Standard of Maximization and Minimization

	Maximization	Minimization
Focal (Conventional)	$Max\ Z = Cx$ $s.t.\ Ax \le b$ $x \ge \circ$	$Min\ Z = Cx$ $s.t.\ Ax \ge b$ $x \ge \circ$
Standard	$Max\ Z = Cx$ $s.t.\ Ax = b$ $x \ge \circ$	$Min\ Z = Cx$ $s.t.\ Ax = b$ $x \ge \circ$

Requirement Space

The problem of linear programming can be solved and interpreted geometrically in another space called requirement space.

Consider the following equation:

$$\begin{aligned} Min \quad & Z = Cx \\ s \cdot t \quad & Ax = b \\ & x \ge 0 \end{aligned}$$

We rewrite the problem as follows:

$$M\text{ in} \quad Z = Cx$$
$$s \cdot t \quad \sum_{j=1}^{n} a_j x_j = b$$
$$x_j \geq 0 \quad j = 1,\dots,n$$

The vectors are a_n،،a،a_2 $_1$rows A, and we want to find the negative $x_n,\dots,x_1$so that

$Z = \sum_{j=1}^{n} c_j x_j, \sum_{j=1}^{n} a_j x_j = b$ is minimized.

Note that the set of vectors $\sum_{j=1}^{n} a_j x_j$ in which there are $x_1, x_2,\dots,x_n \geq \circ$ cones created by the $a_n,\dots a_2, a_1$vectors is therefore problematic if the vector b is placed in this cone.

ABS solution to a specific linear programming problem

We now consider the problem of linear programming, which $rankA = m \ ,m \leq n \ ,b \in R^m \ ,x \in R^n$

$$\max c^T x : Ax \leq b \quad (1)$$

And using the above problem is equivalent to:

$$\min(c^T M\gamma + C^T H^T q): \quad \gamma \in R^m, q \in R^n \quad (2)$$

We have,

$$\bar{c} = M^T c$$
$$I^+ = \left\{ i \mid \bar{c}_i > \circ \right\},$$
$$I^\circ = \left\{ i \mid \bar{c}_i = \circ \right\},$$
$$I^- \left\{ i \mid \bar{c}_i < \circ \right\}$$

Theorem

Suppose we use the ABS algorithm for A as in the proposition of 308, then we have $x^* = A_{W_m}^{-T} b$:

(a) If $Hc \neq 0$ then problem (1) is infinite and has no solution .

(b) If $Hc = 0$ and $I^- \neq \phi$ then problem (1) is infinite and has no solution .

(C) If $Hc = 0$ and $I^- = \phi$ then an infinite number of optimal solutions for (12. 3) form

$$x = x^* - H^T q - \sum_{j \in I^\circ} \gamma_j M e_j$$

We have,

$$j \in I^O \text{،} \gamma_j \geq \circ \text{ ؛ } q \in R^n$$

Are arbitrary and ej is the unit vector of j in R^m.

Proof:

(a) Given (2), Hc vectors are not zero, q can be chosen appropriately to obtain the desired small value for the objective function.

We now assume that $Hc = 0$ using the above symbol, such as Problem (1), can be written as follows:

$$\min z = \sum_{j \in I^-} \bar{c}_j \gamma_j + \sum_{j \in I^+} \bar{c}_j \gamma_j + \sum_{j \in I^\circ} o\gamma_j : \gamma_j \geq \circ \ \forall_j$$

(b) Since $I^- \neq \phi$ after $k \in I^-$ is available. By placing $\gamma = tek$ then we have; $Z \to -\infty$ when $t \to \infty$ is therefore infinite and has no solution .

(c) In this case $z = \sum_{j \in I^+} \bar{c}_j \gamma_j + \sum_{j \in I^\circ} \circ\, \gamma_j$, where the z-minimizers are written as $\gamma = \sum_{j = I^\circ} \gamma_j e_j$, in which $\gamma_j \geq \circ, j \in I^\circ$ are arbitrary, so the optimal solutions in (1) are $x = x^* - H^T q - \sum_{j \in I^\circ} \gamma_i M e_j$, in which $q \in R^n$, $r_j \geq 0$, and $j \in I^\circ$ are arbitrary.

Note 11. 3: According to the characteristics of ABS, $HA^T = 0$ and hence H nullity=A^T is the condition for $Hc = \circ$. Regarding Koohan Taker condition $c = A^T u$, for u, since A^T has a full column rank, then u is unique. The $\frac{1}{d}\bar{c}$ vector, on the other hand, holds $A^T u = c$ because lines $A_{W_m}^{-1}$ are linearly independent, and the solution $A^T u = c$ is equivalent to the solution $A_{w_m}^{-1} A^T u = A_{w_m}^{-1} c$.

Since $A_{W_m}^{-1} A^T = I$ then $u = A_{W_m}^{-1} c = \frac{1}{d}\bar{c}$. So when w_i are selected, so that $\bar{c}, d^m > 0$ has the same symbol. Just as multiplications (Lagrange) are components. If $\bar{c} \geq 0$ then the problem is infinite and has no solution, then we find that for $i \in I^-$, $u_i < 0$ and for every $i \notin I^-$, $u_i \geq 0$. If $\bar{c} \geq \circ$ then there are an infinite number of optimal solutions to the problem.

Here, as expected for optimization, for each i, $u_i \geq 0$. Using the above results, the following algorithm is proposed to solve the linear programming problem (1):

1) We use the ABS algorithm with matrix A.

$A_{W_m}^{-T}$ and H؛

$$d = \left|\det(w_m^T H_1 A^T)\right|$$
$$M = dA_{W_m}^{-T}$$

Calculate.

2) If $Hc \neq C$ then stop (the problem is infinite so there is no solution).

3) Hypothesis $\bar{c} = M^T c$ and its form are in the form of the following sets:

$$I^{+} = \{ i \mid \bar{c}_i > \circ \}$$

$$I^{-} = \{ i \mid \bar{c}_i < \circ \}$$

$$I^{\circ} = \{ i \mid \bar{c}_i = \circ \}$$

4) If $I^{-} \neq \phi$ then stop (it is an infinite problem and has no solution).

5) The equation

$$x^{*} = A_{w_m}^{-T} b \,\text{،}(I^{-} = \phi)$$

Which are the optimal solutions

$$x = x^{*} - H^T q - \sum_{j \in I^{\circ}} \gamma_j M e_j$$

Where,

$$q \in R^n, \gamma_j \geq 0, j \in I^O$$

And are arbitrary numbers, and ej denotes the unit vector j in R^m. Stop.

We have used ABS algorithms to solve real linear equations for full-rank linear inequalities and linear programming problems in which the number of inequalities is less than or equal to the number of variables, and the optimal and infinity conditions in the algorithm are obtained.

2. Conclusion

In this study, the ABS method is proposed to solve various equations and linear inequalities. This algorithm is a well-defined algorithm when the rank of the matrix A (coefficient matrix) equal to m. In this case, the method is more efficient and the number of calculations is less.

These methods while having the ability to produce suitable solutions for inactive equations. They are also effective in solving large equations. They are more efficient at solving large equations (large m or n) than conventional direct methods. The ABS method is also used to solve linear programming problems. This method is equivalent to the Symplex method with the Bland rule in linear programming to find a feasible region that can be found from $Ax \geq b$ linear inequality equations.

$$A \in R^{m \times n}, b \in R^m, x \in R^n, m \leq n$$

Which is used in a finite number of steps and the results obtained for ABS methods are not only theoretically important but also remarkable from the point of view of numerical calculations.

3. Research Suggestions

Here are some of the works that can be done in future studies:

1) Applying the ABS method to solve the system of nonlinear equations with the incomplete rank
2) Applying the ABS method to solve the system of nonlinear inequalities with the incomplete rank
3) Applying the ABS method to solve the system of nonlinear equations with full rank
4) Applying the ABS method to solve integer mixed planning problems
5) Applying the ABS method to solve integer planning problems
6) Applying the ABS method to solve quadratic planning problems

References

[1] Numerical analysis (2), Ahmadi Mohammad Baqer (2011), Payame Noor University Press.
[2] Numerical methods in linear algebra, Tofiq Allah Viranloo,(2008) Published: Tehran: Islamic Azad University.
[3] Operations Research, Authors: Zahedi SereshtMaziar, KhosraviShahrzad (2009), NegahDanesh Publications.
[4] Explanation of market linear planning Problems: Authors: Abbasi. Sh. And Zahedi Seresht .M, 2011, Published by: University book.
[5] Operations Research, Jahanshahloo. Gh, (2014), Published by: Payame Noor University.
[6] Hoffmann Linear Algebra, translated by Jamshid Farshidi, University Publishing Center, Tehran (1993).
[7] E. Esmaeili, N. Mahdavi-Amiri and E. Spedicato, ABS solutions of a class oflinear inequalities and integer LP problems, Optim. Methods Softw. 16(2001), 179-192.
[8] E. Spedicato, B. Bodon, A. Del Popolo and N. Mahdavi-Amiri, ABSinethods andABSPACK for linear systems and optimization: A review, 40R.Quarterly Journal of the Belgian,French and Italian Operations ResearchSocieties 1 (2003), 51-66.
[9] E. Spedicato and M. Zhu, Reformulation of the ABS algorithm via full rankAbaffian, Proceedings EAMA Conference, Seville, (1997), 390-403.
[10] L. Zhang, X. Xia and E. Feng, Introduction to ABS Methods forOptimization (in Chinese), Dalian Technology Univeersity Press, Dalian, 1998.