Buckling Analysis of Stiffened Plate with Varying Stifenner and Various Load Position

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Abstract: ***One of the methods to study the dynamic nature of plate is to go for buckling analysis. It predicts various modes of vibration. Plate because of high strength to weight ratio are in use for many structural applications. Such structures are subjected to dynamic load many times over its life span. Strength of these structures are increased by adding stiffeners to its plate. This paper deals with the analysis of rectangular stiffened plates which forms the basis of structures. In order to continue this analysis various research papers were studied to understand the previous task. The aim of this study is to highlight the effectiveness of stiffeners in the plate. The results presented herein are for deformation and stresses which can be useful for fixing the geometry of stiffener in the plate. The cost effectiveness of the stiffened plate may be studied further for achieving the economy in the construction of real life structures having stiffened plates.***

Keywords: Buckling Analysis, Plates, Biaxial Forces, Boundary Condition

Notation: a, b- dimensions of the plate, E- modulus of elasticity , h- thickness of the plate ,w- out of plane displacement, q- load intensity,μ -Poisson's ratio ,ζ -non-dimensional element coordinate ,λ -buckling load parameter,α-aspect ratio a/b, m and n -no of half waves in x and y direction, $\mathbf{N_x}$ and $\mathbf{N_y}$ -critical buckling load in x and y direction,D- rigidity factor of plate,Ii- second moment of inertia of the stiffener cross section,Ai -area of the stiffener cross section

1. Introduction

Man has always been inspired from the nature be it art or engineering. Perhaps one of the derivatives of such inspiration is stiffened engineering structures. Sea shells, leaves, trees, vegetables - all of these are in fact stiffened structures. Observations of structures created by nature indicate that in most cases strength and rigidity depend not only on the material but also upon its form. This fact was probably noticed long ago by some shrewd observers and resulted in the creation of artificial structural elements having high bearing capacity mainly due to their form such as girders, arches and shells. The buckling and vibration characteristics of stiffened plates subjected to uniform and non-uniform in-plane edge loadings are of considerable importance to aerospace, naval, mechanical and structural engineers. Aircraft wing skin panels, which are made of thin sheets, are usually subjected to non-uniform in-plane stresses caused by concentrated or partial edge loading at the edges, and due to panel stiffener support conditions.

Goals

In order to estimate flat plates critical buckling load, aeronautical industry uses Finite Element Method software, as well as some equations and graphs for quick calculations. The main goal of this project is to compare both methods, determining and quantifying the difference between them.

For that purpose, previously the following goals need to be achieved through the process.

- Understand the theoretical solutions available and the theory behind them.
- Understand Finite Element method.
- Model plates accurately in FEM for different loadings and boundary conditions.
- Validate FEM results.
- Compare FEM and theoretical results for different loadings and boundary conditions. The achievement of the main and secondary goals is shown and proven along the work.

2. Methodology

There are some different theoretical approaches to solve combined loading buckling problems. However, there is no general theory developed applicable to all cases of loading and boundary conditions. This section will present the analytical results for plate buckling problems and the theory behind it. Additionally, the state of the art of plate buckling phenomena and its solutions will be presented along the section.

Basic Theory

Equilibrium method is one of the most common method to do the analysis of plate structure. The standard differential equation comes to be :

$$D\left(\frac{d^4(w)}{dx^4}\right) + 2D\left(\frac{d^4(w)}{dx^2 * dy^2}\right) + D\left(\frac{d^4(w)}{dy^4}\right) = N_x\left(D\left(\frac{d^2(w)}{dx^2}\right)\right) \ldots(1)$$

Where Nx is the internal force acting on the middle plane of the plate.
D is the plate rigidity ratio and w is the deflection of plate along the thickness of the plate.

In order to have a trivial solution for such problem the main consideration is to have the initial, flat configuration of equilibrium. However the coefficient of governing equation depends on the magnitudes of the stress resultants, which are in turn, connected with the applied in plane external forces, and we can find values of these loads for which nontrivial solution is possible. The smallest value of these loads will correspond to critical load. A more general formulation of the equilibrium method transforms the stability problem into an eigenvalue problem. For this purpose we multiply a reference value of the stress resultant (Nx) by load parameter λ, ie.

$$Nx = \lambda Nx \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

By substituting equation-2 in equation -1 we get:

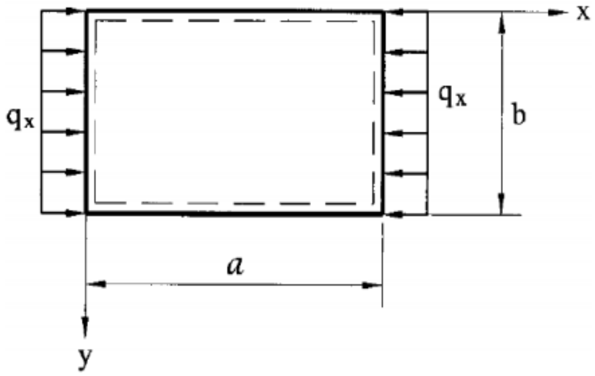
$$\Delta^4(w) + \boldsymbol{\lambda}\frac{Nx}{D}\left(D\left(\frac{d^2(w)}{dx^2}\right)\right) = 0 \ldots\ldots(3)$$

Following assumptions will be applicable throughout the discussion:

a) Prior to loading plate is ideally flat and all the externally applied load is acting on the mid plane.
b) States of stress described by equations of linear plane elasticity. Any change in the plate dimensions prior to buckling will be ignored.
c) The plate bending equations are described by Kirchhoff's plate bending theory.

Simply supported rectangular plate without stiffener subjected to uniaxial loading

Simply supported plate with load acting along X axis a shown below:



Critical load for this case is given as:

$$N_x = \frac{D\pi^2}{b^2}\left(\frac{mb}{a} + \frac{n^2 a}{mb}\right)^2 \ldots\ldots\ldots\ldots\ldots(4)$$

Where ,

$$D = \frac{Eh^3}{12(1-\mu^2)} \ldots\ldots\ldots\ldots\ldots\ldots(5)$$

h= Plate thickness

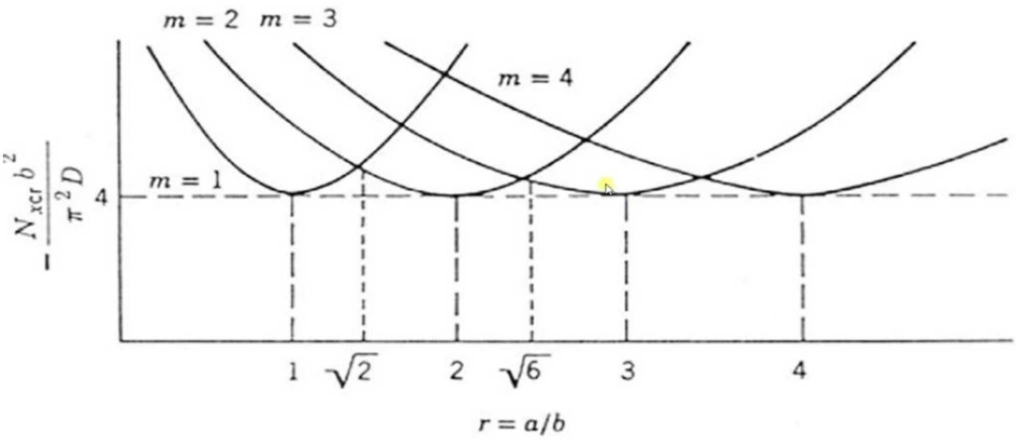


Figure 2: Buckling load as a function of aspect ratio of simply supported isotropic beam

Simply supported rectangular plate without stiffener subjected to biaxial loading

Consider a simply supported flat isotropic plate undergoing buckling under the action of biaxial in-plane loads as shown in Figure 1. Let us assume that the thickness of the plate in the z- direction, is far smaller than the length and width of the plate in the x- and y-directions respectively.

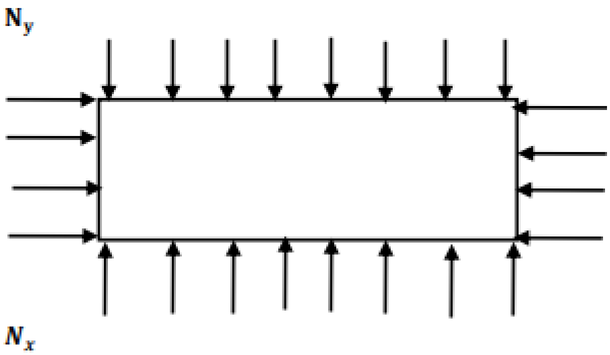


Figure 3: Simply supported isotropic flat plate under biaxial in-plane loads

The critical buckling equation for plates undergoing biaxial buckling is as given by Equation

$$\mathbf{N_x}\left(\frac{\partial^2 w}{\partial x^2}\right) - \mathbf{N_y}\left(\frac{\partial^2 w}{\partial y^2}\right) = D\left[\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2\,\partial y^2} + \frac{\partial^4 w}{\partial y^4}\right] \ldots\ldots(6)$$

Above equation , is an equation of the biaxial forces acting on the plate. These forces (both internal and Equation (is an equation of the biaxial forces acting on the plate. These forces (both internal and external), act to deform the plate. If the average deformation on the plate by the forces is "w", then the work done by the forces while acting on the plate, shall be obtained by multiplying Equation by "w". This yields the work equation as Equation;

$$-\mathbf{N_x}\left(\frac{\partial^2 w}{\partial x^2}\right).\mathbf{w} - \mathbf{N_y}\left(\frac{\partial^2 w}{\partial y^2}\right).w = D\left[\frac{\partial^4 w}{\partial x^4}.w + 2w.\frac{\partial^4 w}{\partial x^2\,\partial y^2} + \frac{\partial^4 w}{\partial y^4}.w\right] \ldots\ldots\ldots(7)$$

This equation is the Galerkin's expression for the buckling analysis of plates subjected to biaxial forces.

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The general equation used for determining the critical buckling loads, of a biaxially compressed thin rectangular isotropic plate as Equation

$$N_x = -\frac{\left(\frac{D}{\alpha^2}\right)\iint \left[\left(\frac{\partial^4 H}{\partial R^4}\right)H + \frac{2H}{\alpha^2}\left(\frac{\partial^2 H}{\partial R^2 \partial Q^2}\right) + \frac{H}{\alpha^4}\left(\frac{\partial^4 H}{\partial Q^4}\right)\right]\partial R \partial Q}{\iint \left[\left(\frac{\partial H}{\partial R}\right)^2 + \frac{K}{\alpha^2}\left(\frac{\partial H}{\partial Q}\right)^2\right]\partial R \partial Q} \quad \ldots\ldots\ldots\ldots(8)$$

Where

$H = (R-2R^3 + R^4)(Q-2Q^3+Q^4)$

$$\iint_0^1 \left(\frac{\partial H}{\partial R}\right)^2 \partial R \partial Q = 0.0239$$

$$\iint_0^1 \left(\frac{\partial H}{\partial Q}\right)^2 \partial R \partial Q = 0.0239$$

$$\iint_0^1 \left(\frac{\partial^4 H}{\partial R^4}\right)^2 \partial R \partial Q = 0.2362$$

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$$\iint_0^1 \left(\frac{\partial^4 H}{\partial R^4 \partial Q\wedge 2}\right)^H \partial R \partial Q = 0.2359$$

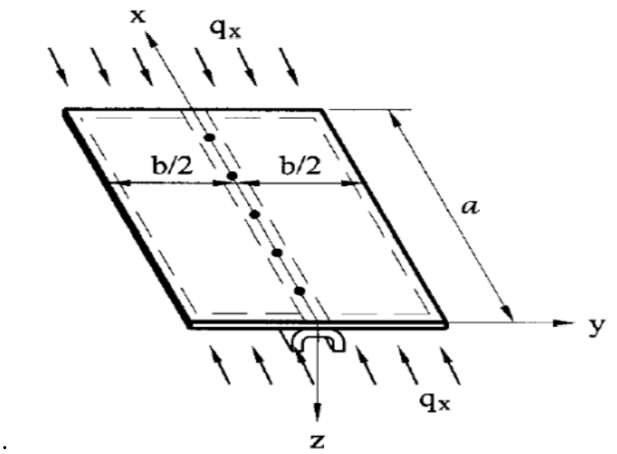
Hence the critical buckling load equation for an all-round simply supported rectangular isotropic plate as Equation

$$N_x = -\frac{\left(\frac{D}{\alpha^2}\right)\left[0.2362 + \frac{2}{\alpha^2}(0.2359) + \frac{1}{\alpha^4}(0.2362)\right]}{\left[0.0239 + \frac{K}{\alpha^2}0.0239\right]} \ldots\ldots\ldots\ldots(9)$$

Simply supported rectangular plate with stiffener at mid along applied uniaxially loaded

The plate is subjected to in plane compressive forces, uniformly distributed along the edges x=0, x=a.

In this analysis we will consider the buckling analysis of the two halves and stiffener separately. In order to get the desired solution we will apply the equation 2 to the one half of the plate with the standard solution.



$$W(x, y) = F_Y \sin\left(\frac{m\pi x}{a}\right) \ldots\ldots\ldots(10)$$

Putting the above equation in equation 1, we get:

$$D\left(\frac{d^4 FY}{dy^4}\right) - 2\left(\frac{m\pi}{a}\right)^2 \frac{d^2 FY}{dy^2}\left(\frac{m\pi}{a}\right)^2 \left[\left(\frac{m\pi}{a}\right)^2 - \frac{qx}{D}\right] \ldots\ldots..(11)$$

The solution comes in the form

$$F(Y) = A\cosh\alpha y + B\sinh\alpha y + C\cos\beta y + D\sin\beta y \ldots\ldots.(12)$$

Where:

$$A = \left[\mu\left(\mu + \left(\frac{qx}{D}\right)^{\wedge 0.5}\right]^{0.5}, \quad \beta = \left[\mu\left(\left(\frac{qx}{D}\right)^{\wedge 0.5} - \mu\right)\right]^{\wedge 0.5}, \quad \mu = \frac{m\pi}{a} \quad (13)$$

The boundary conditions on the edge y=0.5b are:

$$W = 0, \frac{d^2 w}{dy^2} = 0 \text{ at } y = 0.5 \ldots..(14)$$

Here it will be assumed that the plate is buckled together with the stiffener, then the bent surface of plate must be symmetric about the line y=0. This results in the below mentioned condition:

$$\frac{dw}{dy} = 0, \text{ at } y = 0 \ldots.(15)$$

The difference in the reaction forces from the two strip of the plate will be represented as below:

$$R_Y = -D\left[\frac{d^3 w}{dy^3} + (2-\mu)\frac{d^3 w}{dyd\,x^2}\right] \ldots\ldots..(16)$$

This reaction will be transmitted to the stiffener. Since the plate and stiffener both are of same material the first part will be of importance only while the second part will get cancelled out. If we assume that stiffener along with the plate is subjected to the compressive force then the governing equation will be as below;

$$EI_i \frac{d^4(w)}{dx^4}) + \sigma_x A_i \frac{d^2(w)}{dx^2} + 2D\frac{d^3(w)}{dy^3} = 0, \text{at } y = 0 \ldots\ldots\ldots(17)$$

Where Ii and Ai are the second moment of inertia and area of the stiffener cross section, respectively. Few parameters are required to be introduced here as below:

$$\gamma = EI_i \qquad \zeta = \frac{a}{b} \qquad \delta = A_i/bh \ldots\ldots\ldots..(18)$$

Introducing the equation -7 into the conditions of equation 14,15& 18, we obtain a system of linear homogeneous equations for A, B, C, D. Equating the determinant of this system to zero, give the below equation:

$$\left(\frac{1}{b\alpha}\tanh\left(\frac{b\alpha}{2}\right) - \frac{1}{b\beta}\tanh\left(\frac{b\beta}{2}\right)\right)\left(\frac{\gamma m^2}{\alpha^2} - K\delta\right)\frac{m^2\pi^2}{\alpha^2} - \frac{4m\sqrt{k}}{\alpha} = 0 \ldots\ldots..(19)$$

Where;

$$K = \frac{\sigma x, cr}{\sigma x, E} \qquad \sigma x, cr = \frac{qx, cr}{h} \qquad \sigma x, E = \frac{4D\pi^2}{b^2 h} \ldots\ldots(20)$$

For m=1 & δ >2, its solution using the equation 13 the expression for critical stress is achieved as:

$$N_x = \frac{D\pi^2}{b^2}\frac{(1+\zeta^2)^2 + 2\gamma}{\zeta^2(1+2\delta)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots(21)$$

Finite Element Modelling

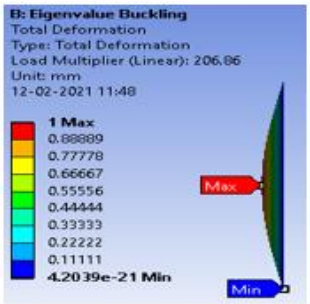
The modelling of any finite element problem includes generally five steps;

a) Defining the material properties of the model,
b) Creating the geometry of the model,
c) Discretizing the model into number of finite elements (i.e. meshing of the geometry),
d) Applying boundary and loading conditions,
e) Solving the problem for its subsequent results.

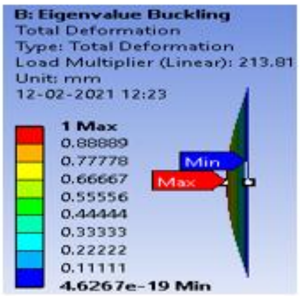
Parameters:

Thickness of plate(h)	2mm
E(MPa)	70GPa
Poisson's ratio	0.33
Load on edge	10N/m

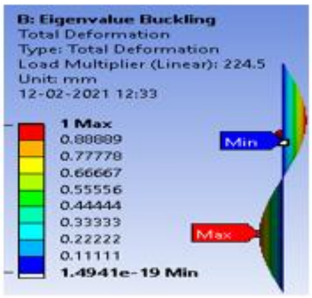
Case 1: (a) Simply supported rectangular plate without stiffener subjected uniaxialloading



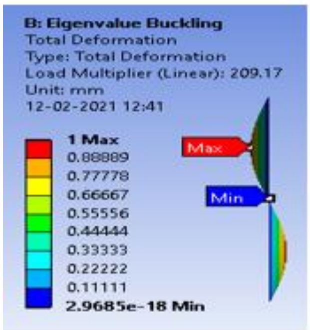
Aspect ratio 1



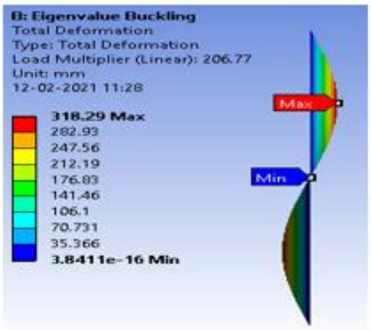
aspect ratio 1.2



aspect ratio 1.5



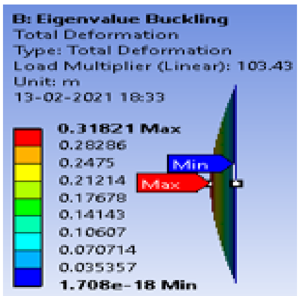
Aspect ratio 1.8



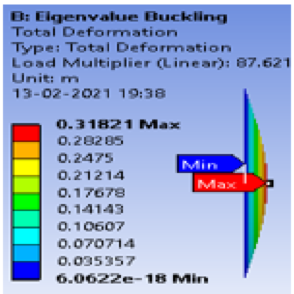
aspect ratio 2

Aspect ratio (a/b)	APPLIED LOAD (P)	CRITICAL LOAD (Pcr)
1	10N/m	206.86
1.2	10N/m	213.81
1.5	10N/m	224.5
1.8	10N/m	209.17
2	10N/m	206.77

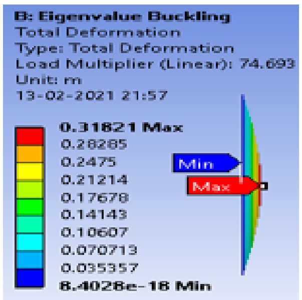
b) Biaxial loading



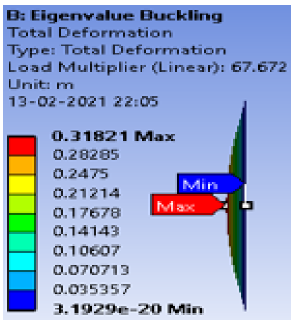
Aspect ratio 1



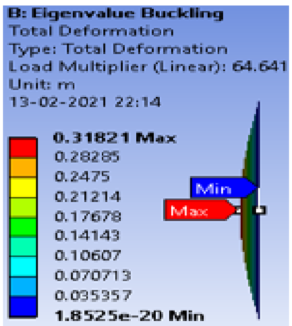
aspect ratio 1.2



aspect ratio 1.5



Aspect ratio 1.8



aspect ratio 2

Aspect Ratio(a/b)	Applied Load (q)	Critical Load
1	10N/m	103.43
1.2	10N/m	87.62
1.5	10N/m	74.69
1.8	10N/m	67.67
2	10N/m	64.64

Case 2: (a) Simply supported rectangular plate with stiffener at mid subjected to uniaxially loaded

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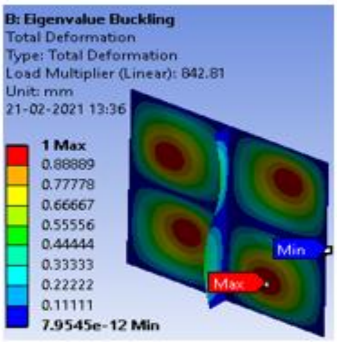
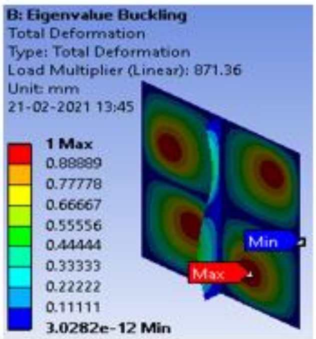
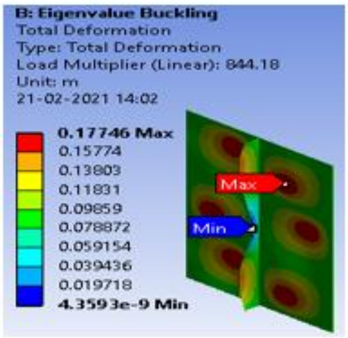


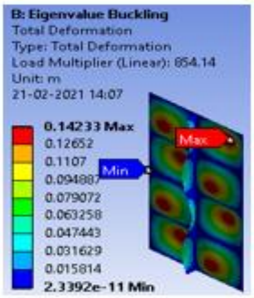
Aspect ratio 1



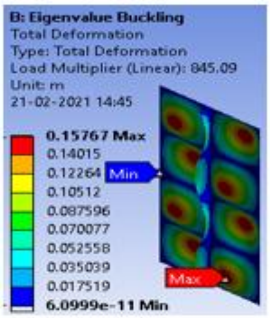
aspect ratio 1.2



aspect ratio 1.5



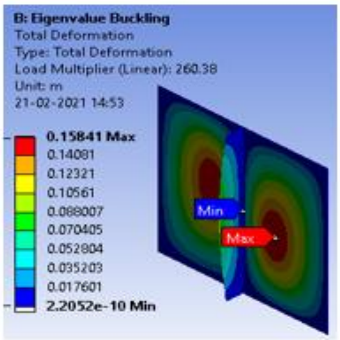
Aspect ratio 1.8



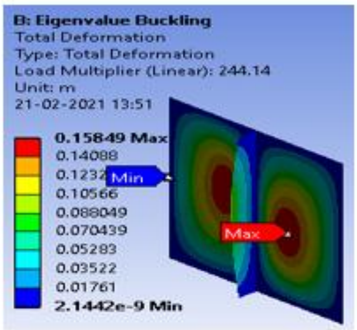
aspect ratio 2

Aspect Ratio(a/b)	Applied Load (q)	Critical Load
1	10N/m	842.81
1.2	10N/m	871.36
1.5	10N/m	844.18
1.8	10N/m	854.14
2	10N/m	845.09

(b) biaxial loading



Aspect ratio 1



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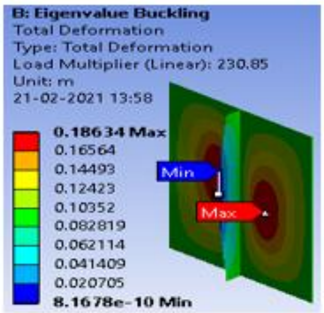

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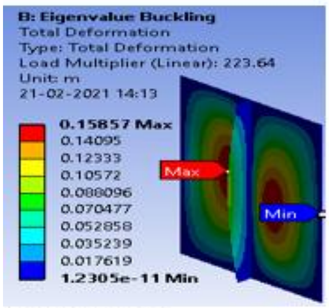

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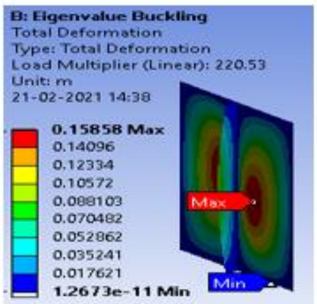

aspect ratio 2

Aspect Ratio(a/b)	Applied Load (q)	Critical Load
1	10N/m	260.38
1.2	10N/m	244.14
1.5	10N/m	230.85
1.8	10N/m	223.64
2	10N/m	220.53

Case 3: (a) Simply supported rectangular plate with varying stiffener length subjected to uniaxial loading

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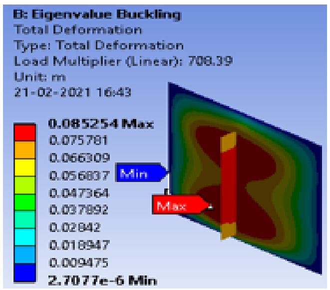
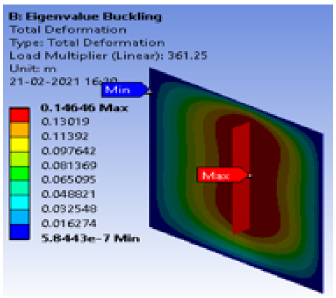
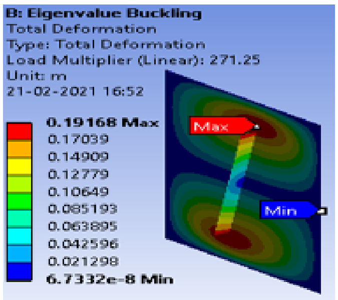


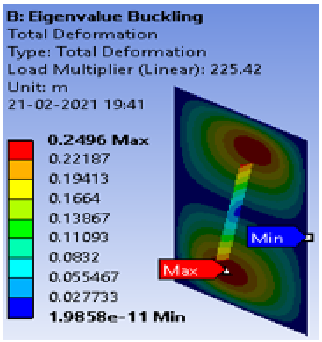
Aspect ratio 1



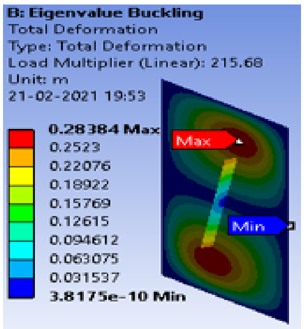
aspect ratio 1.2



aspect ratio 1.5



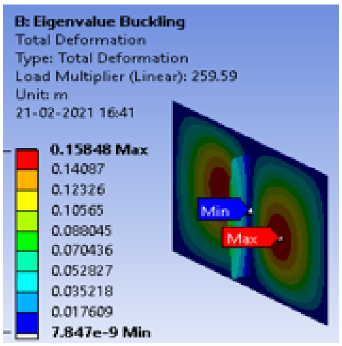
Aspect ratio 1.8



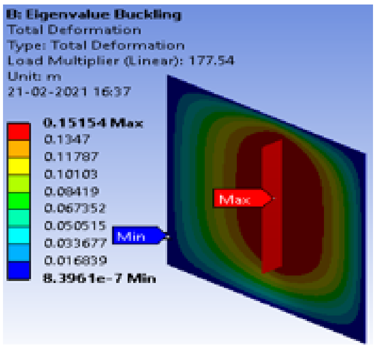
aspect ratio 2

Aspect Ratio(a/b)	Applied Load (q)	Critical Load
1	10N/m	708.39
1.2	10N/m	361.25
1.5	10N/m	271.25
1.8	10N/m	225.42
2	10N/m	215.68

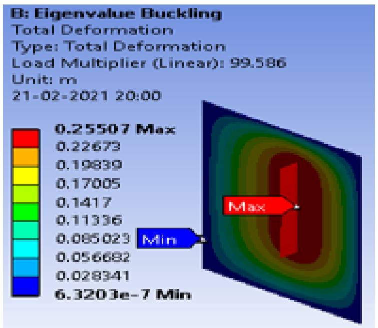
(b) biaxial loading



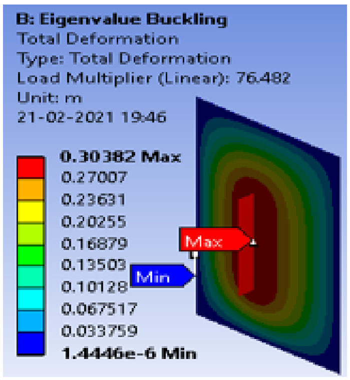
Aspect ratio 1



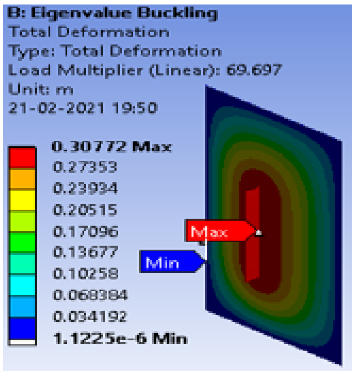
aspect ratio 1.2



aspect ratio 1.5



Aspect ratio 1.8



aspect ratio 2

Aspect Ratio(a/b)	Applied Load (q)	Critical Load
1	10N/m	259.59
1.2	10N/m	177.54
1.5	10N/m	99.586
1.8	10N/m	76.482
2	10N/m	69.697

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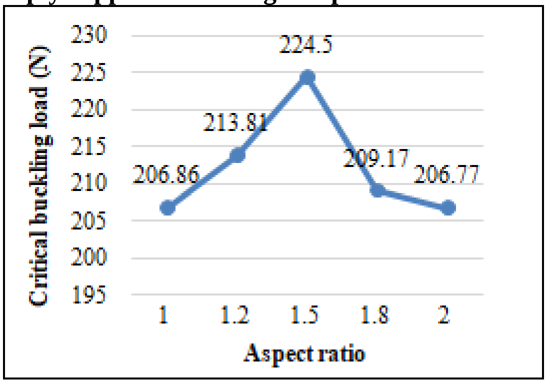
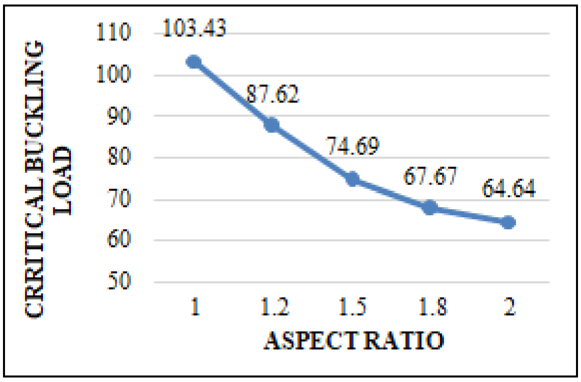
3. Result and Discussion

1) Simply supported rectangular plate without stiffener

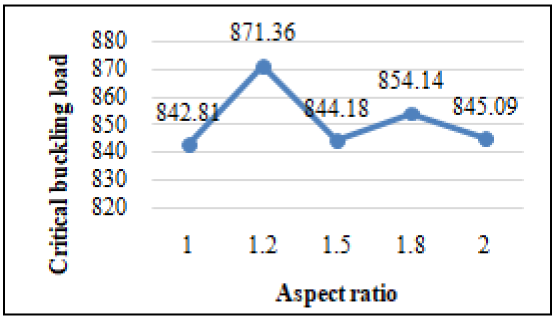


Uniaxially loaded



Biaxially loaded

2) Simply supported rectangular plate with stiffener at mid



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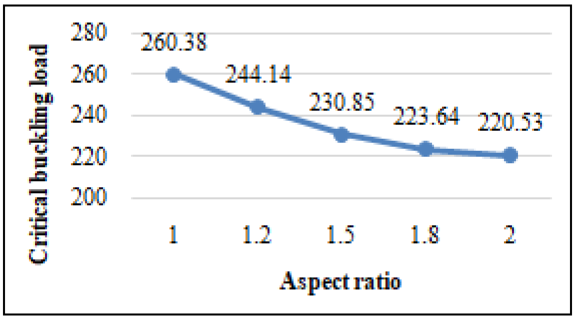
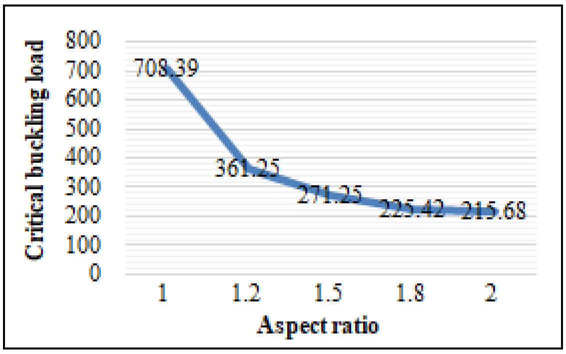


Biaxially loaded

3) Simply supported rectangular plate with varying stiffener length



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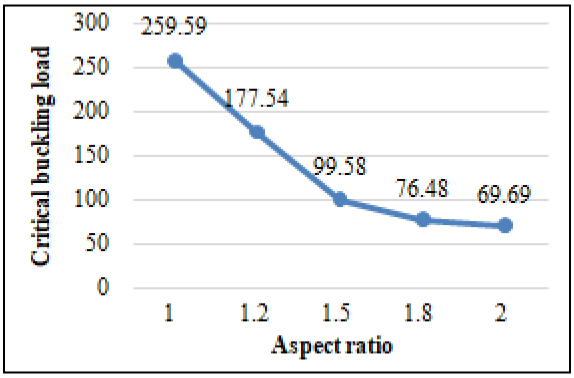

Biaxially loaded

4. Conclusion and Future Scope

The results from the study of the compressive buckling of a stiffened plate subjected to uniform in-plane stress distribution can be summarized as follows

a) Analytical calculation and FEA results are in good agreement to each other
b) Buckling analysis shows almost similar pattern for both analytical calculation and FEA analysis. Simply supported rectangular plate with stiffener at full length of plate subjected to uniaxially loaded gets maximum buckling load.
c) .It is found that Eigen value is decreasing with the reducing the length of stiffener.
d) It is also observed that the maximum stress decreases with the increase in aspect ratio in each cases.

The aim of this study is to highlight the effectiveness of stiffeners in the plate. The results presented herein are for deformation and stresses which can be useful for fixing the geometry of stiffener in the plate. The cost effectiveness of the stiffened plate may be studied further for achieving the economy in the construction of real life structures having stiffened plates.

There is further analysis scope to analyze the same case with different stiffeners geometry.

References

[1] A.K.L. Srivastava et al. / International Journal of Mechanical Sciences 45 (2003)

[2] IOSR Journal of Mechanical and Civil Engineering (IOSR-JMCE) e-ISSN: 2278-1684,p-ISSN: 2320-334X

[3] JafarpourHamedani, Shahed, and Ahmad Rahbar Ranji. "Buckling analysis of stiffened plates subjected to non-uniform biaxial compressive loads using conventional and super finite elements", Thin-Walled Structures (2013)

[4] International Research Journal of Engineering and Technology (IRJET) VOL 3, e-ISSN: 2395 -0056 p-ISSN: 2395-0072

[5] D. K. Singh, S. K. Duggal and P. Pal, "Analysis of stiffened plates using FEM – a parametric study," International research journal of engineering and technology, Vol. 02, July 2015

[6] D. Venugopal Rao, A. H. Sheikh and M. Mukhopadhyay, "A finite element large displacement analysis of stiffened plates," Computers & Structures, Vol. 47, No.6, 1993, pp. 987-993

[7] Timoshenko S.P. and Gere J.M. (1961): Theory of Elastic Stability. – New York: McGraw-Hill.

[8] Ali Reza Pouladkhan, Jalil Emadi, Majid Safemehr, The vibration of thin plates using modal analysis. World academy of science,Engineering and Technology,2011

[9] A.K.L.Shrivastava, P.K.Datta,A.H.Seikh, Transverse vibration of stiffened plates with cutouts subjected to in-plane uniform edge loading at the plate boundary, IOS Press,2002.

[10] Nildem T.I. (2010): Determination of thickness and stiffener locations for optimization of critical buckling load of stiffened plates. – Scientific Research and Essay, vol.5, pp.897-910