Analysis of the Grace Data Processing on the Commonly used Filtering Methods Spatial Filtering

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Abstract: ***According to the different GRACE data sources, the inversion methods of surface quality migration are divided into two categories. The first type is directly inversion based on the GRACE Level-2 Earth time-varying gravity field model, also known as the "bit theory method". This method was first proposed by Wahretal and is one of the most commonly used inversion methods. It has been widely used in research and development; the second type is inversion by Level-1B observation data, such as mass tumor (Mascon) method. This is a theoretical method. The method is briefly introduced below.***

1. Methodology

The shape of the geoid reflects the structure and distribution of the material inside the Earth. When studying the Earth's gravity field, it can be replaced by the geoid. Combined with the gravity field of the Earth. The formula and the Bronx formula can obtain a spherical harmonic expansion with a high geoid.

$$N(\theta,\lambda)=a\sum_{l=0}^{\infty}\sum_{m=0}^{l}\overline{P}_{lm}(\cos\theta)(C_{lm}\cos(m\lambda)+S_{lm}\sin(m\lambda))$$

Where a represents the average radius of the Earth, θ,λ represent the left and right latitudes of the Earth and the longitude of the Earth, C_{lm} and S_{lm} are regular spherical harmonic coefficients, and l and m represent the degree and the order of the spherical harmonic coefficients. $\overline{P}_{lm}(\cos\theta)$ is a fully normalized Legendre function.

Many institutions in the world provide a full order of spherical harmonic coefficients C_{lm} and S_{lm}. The change in mass will cause a corresponding change in the geoid, assuming that the geoid N changes with time, that is, the geoid at a certain moment relative to The difference between the geoid at a later time, or the difference between the geoid at a certain time and the average geoid at a certain time. Introducing the variation of the spherical harmonic coefficient ΔC_{lm}, ΔS_{lm} to represent ΔN:

$$\Delta N(\theta,\lambda)=a\sum_{l=0}^{\infty}\sum_{m=0}^{l}\overline{P}_{lm}(\cos\theta)(\Delta C_{lm}\cos(m\lambda)+\Delta S_{lm}\sin(m\lambda))$$

Introducing the density change $\Delta\rho(r,\theta,\lambda)$ that causes the change of the geoid to obtain the relationship between the changes in the spherical harmonic coefficients ΔC_{lm}, ΔS_{lm} and $\Delta\rho(r,\theta,\lambda)$.

$$\begin{Bmatrix}\Delta C_{lm}\\ \Delta S_{lm}\end{Bmatrix}=\frac{3}{4\pi a\rho_{ave}(2l+1)}\int\Delta\rho(r,\theta,\lambda)\overline{P}_{lm}(\cos\theta)(\frac{r}{a})^{l+2}\begin{Bmatrix}\cos(m\lambda)\\ \sin(m\lambda)\end{Bmatrix}\sin\theta d\theta d\lambda dr$$

Where ρ_{ave} represents the average density of the Earth and $\Delta\rho(r,\theta,\lambda)$ is the density of a film with a surface thickness of H. The film should contain changes in the mass of the atmosphere, ocean, ice sheet and groundwater, usually in the range of 10-15 km. Define $\Delta\sigma(\theta,\lambda)$ as the change in areal density. It is the change in mass per unit area, then the relationship between $\Delta\sigma(\theta,\lambda)$ and $\Delta\rho(r,\theta,\lambda)$ is:

$$\Delta\sigma(\theta,\lambda)=\int\Delta\rho(r,\theta,\lambda)dr$$

If the thickness H of the thin layer is small enough, it satisfies:

$$(l+2)H/2 = 1$$

$$(r/a)^{l+2} \approx 1$$

The change caused by the surface mass migration of the geoid can be expressed as:

$$\begin{Bmatrix} \Delta C_{lm} \\ \Delta S_{lm} \end{Bmatrix}_{surf\_mass} = \frac{3}{4\pi a \rho_{ave}(2l+1)} \int \Delta\sigma(\theta,\lambda)\overline{P}_{lm}(\cos\theta) \begin{Bmatrix} \cos(m\lambda) \\ \sin(m\lambda) \end{Bmatrix} \sin\theta d\theta d\lambda$$

Since the earth is a hysteresis, the change of its surface mass load will also cause the earth to change. This deformation will also cause the geoid to change. Introducing the load love number k_l can be obtained:

$$\begin{Bmatrix} \Delta C_{lm} \\ \Delta S_{lm} \end{Bmatrix}_{solid\_Earth} = \frac{3k_l}{4\pi a \rho_{ave}(2l+1)} \int \Delta\sigma(\theta,\lambda)\overline{P}_{lm}(\cos\theta) \begin{Bmatrix} \cos(m\lambda) \\ \sin(m\lambda) \end{Bmatrix} \sin\theta d\theta d\lambda$$

Therefore, the expression for the change in the spherical harmonic coefficient caused by the total mass change is:

$$\begin{Bmatrix} \Delta \hat{C}_{lm} \\ \Delta \hat{S}_{lm} \end{Bmatrix} = \begin{Bmatrix} \Delta C_{lm} \\ \Delta S_{lm} \end{Bmatrix}_{surf\_mass} + \begin{Bmatrix} \Delta C_{lm} \\ \Delta S_{lm} \end{Bmatrix}_{solid\_Earth}$$

Using the spherical harmonic coefficient to develop the areal density $\Delta\sigma(\theta, \lambda)$:

$$\Delta\sigma(\theta, \lambda) = a\rho_w \sum_{l=0}^{\infty}\sum_{m=0}^{l} (\Delta\hat{C}_{lm}\cos(m\lambda) + \Delta\hat{S}_{lm}\sin(m\lambda)\overline{P}_{lm}(\cos\theta)$$

Where ρ_w is the water density, and $\Delta\hat{C}_{lm}$ and $\Delta\hat{S}_{lm}$ are the corresponding spherical harmonic coefficients, which can be expressed as:

$$\begin{Bmatrix} \Delta \hat{C}_{lm} \\ \Delta \hat{S}_{lm} \end{Bmatrix} = \frac{1}{4\pi a \rho_w} \int_0^{2\pi} d\lambda \int_0^{\pi} \Delta\sigma(\theta,\lambda)\overline{P}_{lm}(\cos\theta) \begin{Bmatrix} \cos(m\lambda) \\ \sin(m\lambda) \end{Bmatrix} \sin\theta d\theta d\lambda$$

$$\begin{Bmatrix} \Delta \hat{C}_{lm} \\ \Delta \hat{S}_{lm} \end{Bmatrix} = \frac{\rho_{ave}}{3\rho_w} \frac{2l+1}{1+k_l} \begin{Bmatrix} \Delta C_{lm} \\ \Delta S_{lm} \end{Bmatrix}$$

Combined with the above formula we can get:

$$\Delta\sigma(\theta, \lambda) = \frac{a\rho_{ave}}{3} \sum_{l=0}^{\infty} \frac{2l+1}{1+k_l} \sum_{m=0}^{l} (\Delta C_{lm}\cos(m\lambda) + \Delta S_{lm}\sin(m\lambda)\overline{P}_{lm}(\cos\theta)$$

Express the change in the surface quality of the earth as the equivalent water height $EWH(\theta, \lambda)$

$$EWH(\theta, \lambda) = \frac{a\rho_{ave}}{3\rho_w} \sum_{l=0}^{\infty}\sum_{m=0}^{l} \frac{2l+1}{1+k_l} (\Delta C_{lm}\cos(m\lambda) + \Delta S_{lm}\sin(m\lambda)\overline{P}_{lm}(\cos\theta) \qquad (2\text{-}11)$$

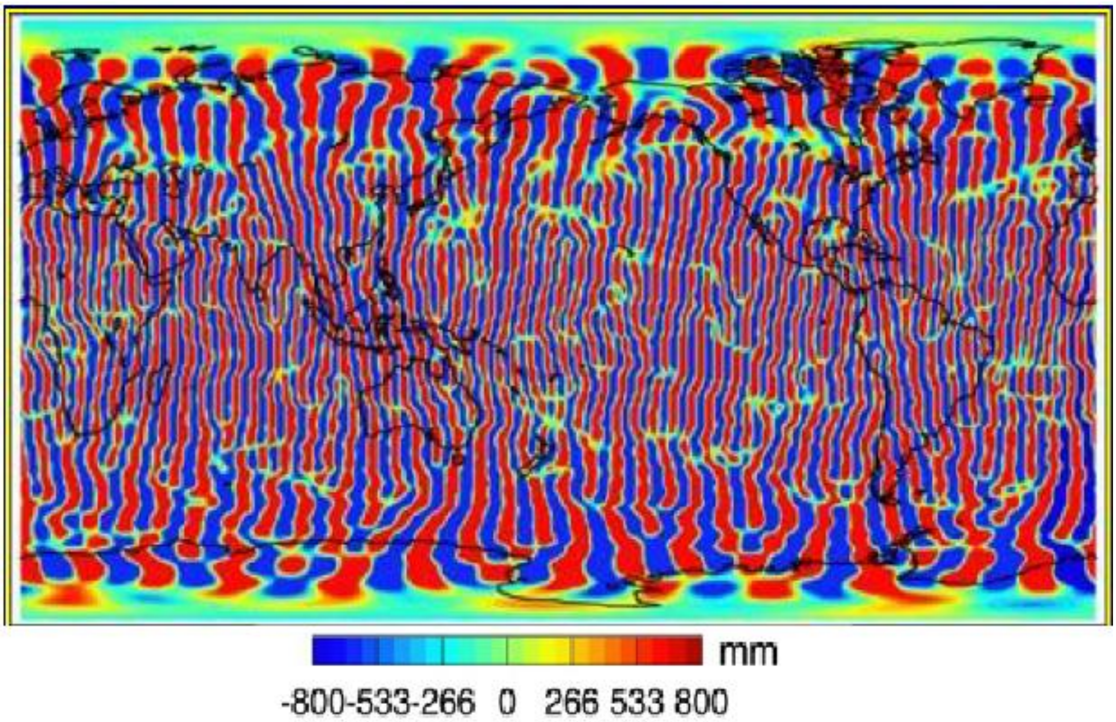
The values of load love number are shown in the table below.

Table 1.1: The values of load love number

i	k_i	i	k_i
0	0	10	-0.069
1	0.027	12	-0.064
2	-0.303	15	-0.058
3	-0.194	20	-0.051
4	-0.132	30	-0.04
5	-0.104	40	-0.033
6	-0.089	50	-0.027
7	-0.081	70	-0.02
8	-0.076	100	-0.014
9	-0.072	150	-0.01

1.2 Spatial filtering

The potential coefficients of GRACE gravity field model are influenced by satellite orbit error and model error, which makes the high-order coefficients of the published gravity field model data contain more noise, and the influence of noise increases with the increase of order. At the same time, the coefficients are correlated and the strip error is produced. The influence on the calculation results can not be neglected. Fig. 2.1 is the equivalent water height calculated directly by formula (2-11) without any filtering. We can see that there are obvious effects of North-South band noise and other noises in the calculated results. When using time-varying gravity field model to calculate mass change, W can not directly use the formula (2-11) deduced above. In order to suppress the influence of noise, it is necessary to adopt a certain spatial filtering method to weaken the noise in the data post-processing. The paper further shows that generalized least squares will produce consistent estimates of those parameters that are not time varying.



2. Analysis of commonly used filtering methods Spatial filtering

2.1. Gaussians filtering

Gaussian filtering is an isotropic filtering method. In isotropic filtering, the smoothing function W($\varphi,\lambda,\varphi',\lambda'$) is only a function of the two-point $(\varphi,\lambda),(\varphi',\lambda')$ angular distance on the spherical surface. The relationship between the two is W($\varphi,\lambda,\varphi',\lambda'$)=W(γ),

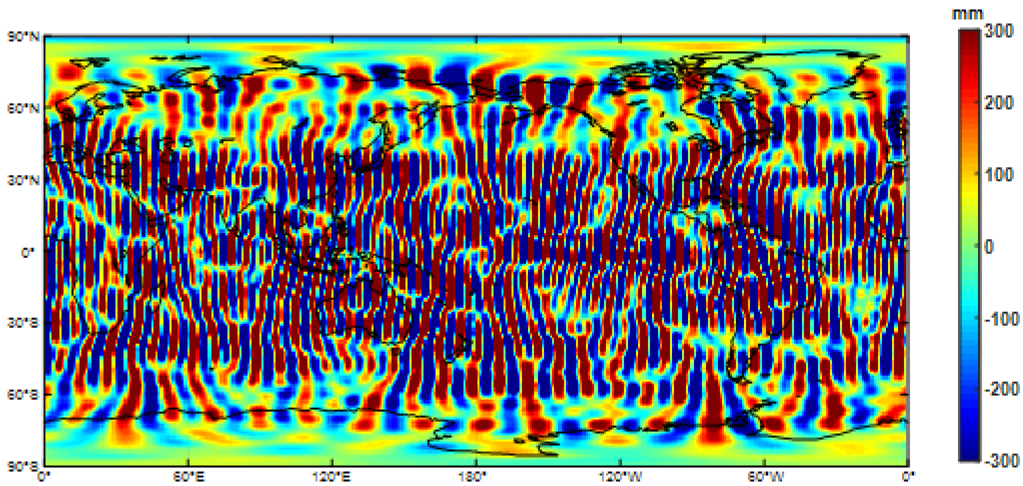


Figure 2.1: Analysis of commonly used filtering methods Spatial filtering

where $\cos\gamma = \cos\varphi\cos\varphi' + \sin\varphi\sin\varphi'\cos(\lambda-\lambda')$,

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available:

$$\Delta\sigma = \frac{a\rho_{ave}}{3}\sum_{n=0}^{\infty}\sum_{m=0}^{n}\frac{2n+1}{1+k_n}W_{nm}\overline{P}_{nm}(\sin\varphi)[\Delta\overline{C}_{nm}\cos(m\lambda)+\Delta\overline{S}_{nm}\sin(m\lambda)]$$

Where: $W_n = \int_0^{\pi} W(\varphi)\overline{P_n}(\cos\gamma)\sin\gamma d\gamma$ is the Gaussian filter weight coefficient only related to the order, $\overline{P_n}$ is the Legendre polynomial, $\overline{P_n} = \frac{P_{nm}=0}{\sqrt{2n+1}}$

From the data of GRACE RL05 which has been published, the highest degree is 60. The data is not infinite, but truncated to 60th degree. This result will inevitably produce truncation error and eliminate high-order ball harmonics. The influence of coefficient error on the calculation of areal density requires the introduction of a smoothing factor Wn to weaken the effects of truncation error and high-order error. Gaussian filtering is based on the principle of using the Gaussian filter coefficient proposed by Jekeli. It is related to the degree of the spherical harmonic coefficient, and has nothing to do with order. The Jekdi Gaussian filter coefficients are as follows:

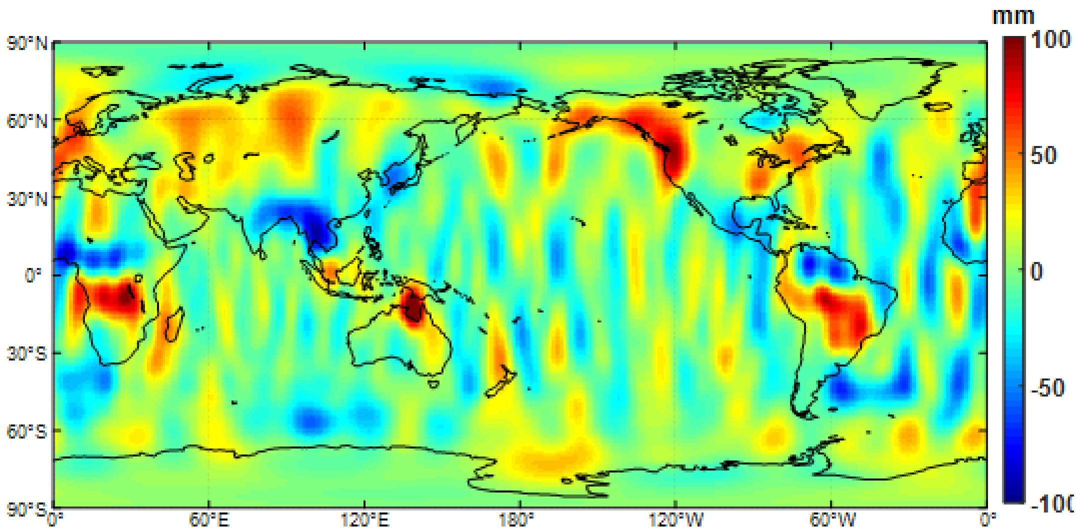
$$W(\varphi,\lambda,\varphi',\lambda') = W(\gamma) = \frac{b}{2\pi}\frac{\exp(-b(1-\cos\gamma))}{1-\exp(-2b)}$$

Among them, $b = \frac{\ln 2}{1-\cos(r/a)}$, a is the radius of the Earth's equator, r is the spherical distance between two points when $\gamma=0$, the smoothing kernel function value is reduced to 1/2, which is the filtering radius. At the same time, Gaussian filter weight coefficients of different degrees can be recursively solved by the following three formulas:

$$W_0 = 1$$

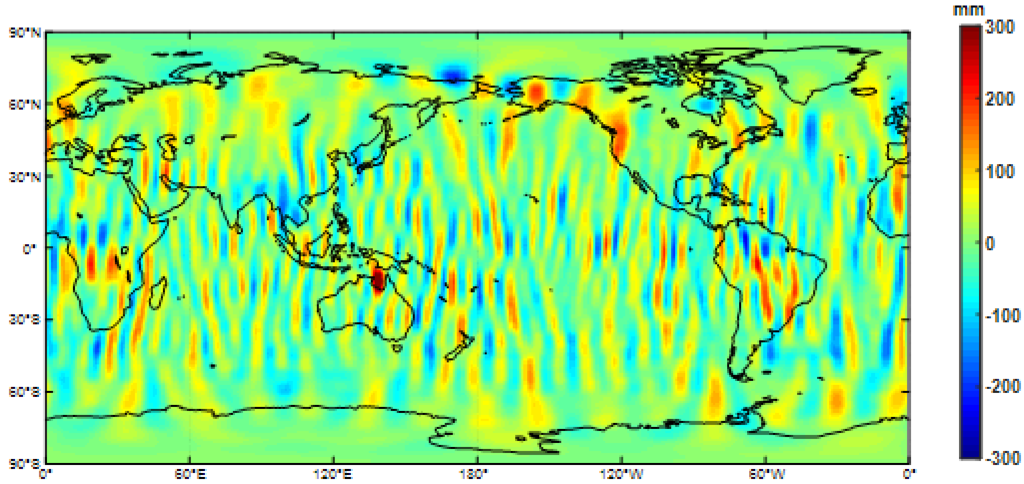
$$W_1 = \frac{1+e^{-2b}}{1-e^{-2b}} - \frac{1}{b}$$

$$W_n = -\frac{2n-1}{b}W_{n-1} + W_{n-2}$$



R=300km

R=500km



R=800km

The paper further shows that generalized least squares will produce consistent estimates of those parameters that are not time varying.

3. Fan filtering

Unlike Gaussian filtering, fan filtering is a non-isotropic filtering method that is developed based on isotropic filtering. Above equation uses Gaussian filtering only for the degree, based on the original filtering of the sector filtering, in which Gaussian filtering is introduced in the formula. In simple terms, the fan filtering is the same as the spherical harmonic Gaussian filtering is performed on the degree and the order of the spherical coefficient, and the formula for calculating the mass variation is:

$$\Delta\sigma=\frac{a\rho_{ave}}{3}\sum_{n=0}^{\infty}W_n\sum_{m=0}^{n}\frac{2n+1}{1+k_n}W_{nm}\bar{P}_{nm}(\sin\varphi)[\Delta\bar{C}_{nm}\cos(m\lambda)+\Delta\bar{S}_{nm}\sin(m\lambda)]$$

Among them, Wn、Wm is the Gaussian filter weight coefficient of the order and degree and the second correlation with the GRACE ball harmonic coefficient. From equation, it can be seen that when calculating the mass change of a certain place, for a certain order of the ball The harmonic coefficient, the weight value of the fan filter is smaller than the Gaussian filter, especially for the high-order term, the spherical coefficient weight obtained by the sector filtering is very small.

Non filtering

Non-filtering is also based on Gaussian filtering. The smoothing kernel function is:

$$W_{nm}=W_n(r_{1/2}(m)),r_{1/2}(m)=\frac{r_1-r_0}{m_1}m+r_0$$

where W_n is the classical Gaussian smoothing kernel function, r_1, r_0 is the smoothing radius, m1is the selected order, n,m is the degree and order of harmonics of the GRACE gravity field ball. r_0 determines the smooth radius along the longitude direction, r1,m1 determines the smooth radius along the twist direction. The Non filter is also related to the order of the spherical harmonic coefficients, and the higher the number of the same order, the smaller the value of the calculated smooth kernel function, and the smaller the weight of the corresponding spherical coefficient. The formula for solving the mass change based on Fan filter is:

$$\Delta\sigma(\varphi,\ \lambda)=\frac{a\rho_{ave}}{3}\sum_{n=0}^{\infty}\sum_{m=0}^{n}\frac{2n+1}{1+k_n}W_{nm}\bar{P}_{nm}(\sin\varphi)[\Delta C_{nm}\cos(m\lambda)+\Delta S_{nm}\sin(m\lambda)]$$

4. De-correlation filtering

The equivalent water height change map mainly shows the north-south band error. Swenson and Wahr pointed out that the appearance of the north-south band indicates that the GRACE error has a high spatial correlation. By comparing the spectral domain variation characteristics of the median coefficient of the time-varying gravity model, it is found that there is a correlation between the GRACE time-varying gravity model spherical coefficients. When the value of the second m is constant, there is a correlation between the odd terms in the order n of the coefficient, and the correlation also exists between the even terms. Taking m=15 as an example, the common coefficient C15,15、C16,15、C17,15…C58,15、C59,15、C60,15 ，where the odd term of n is C15,15、C17,15、C19,15…C55,15、C57,15、C59,15 has a correlation, and the even term is also between C16,15、 C18,15、C20,15…C56,15、C58,15、C60,15 Correlation. According to this, Swenson proposed a sliding window decorrelation method to reduce the correlation between the spherical coefficients to weaken the influence of the north-south band.

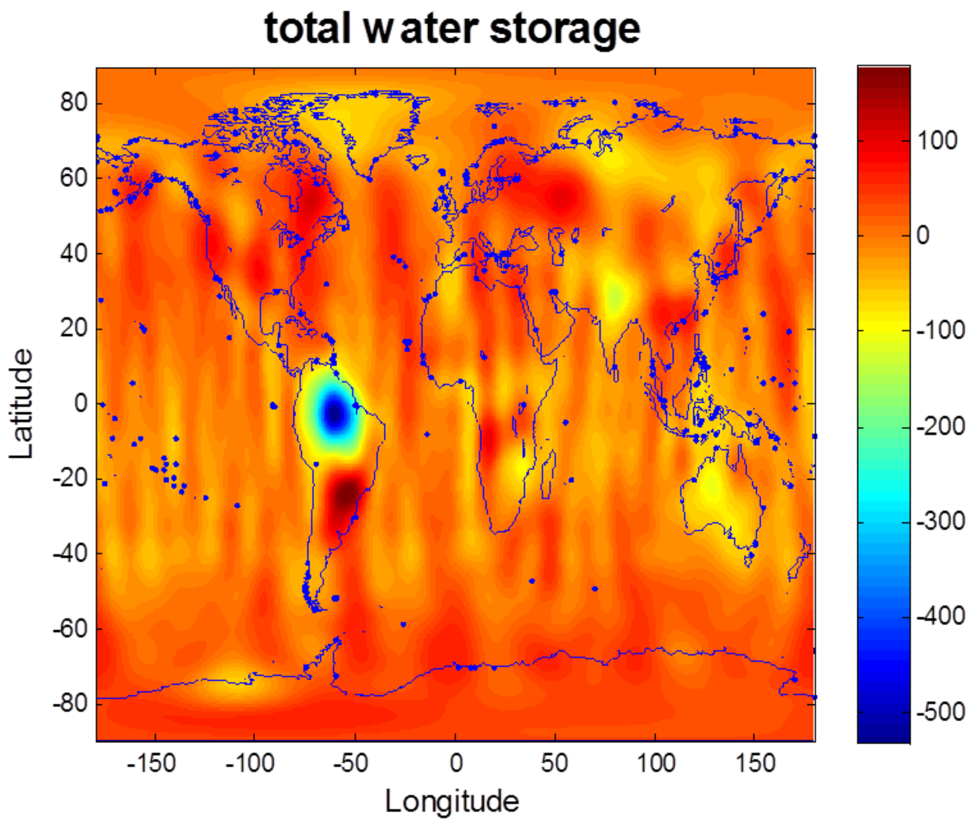


Figure 4.3: (a) Total water storage

The principle of de-correlation filtering is: the order-m of the fixed-ball harmonic coefficient Cnm, using a sliding window of size w, with the degree n as an independent variable, and the even-numbered and odd-numbered pairs of the alignment coefficient are fitted by polynomial, the formula as follows:

$$C_{nm}^{ce} = \sum_{i=0}^{p} Q_{nm}^{i} n^{i}$$

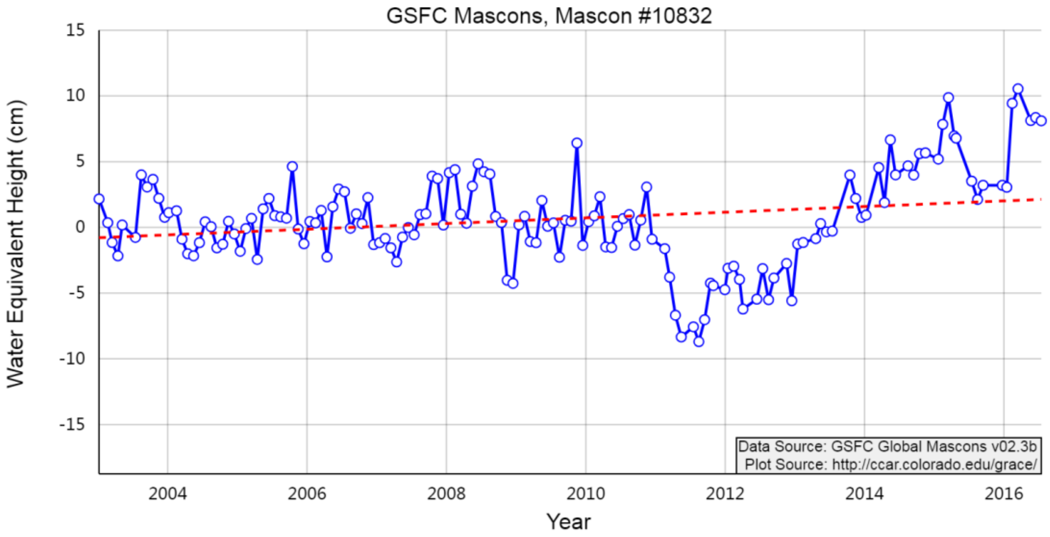
Where C_{nm}^{ce} is the polynomial fitting value of Cnm , the fitting value is used as the correction value of Cnm, Q_{nm}^{i} is the fitting polynomial coefficient, p is the number of polynomials, and the principle is the same when calculating the fitting value of Snm And thus the coefficient can be calculated as:

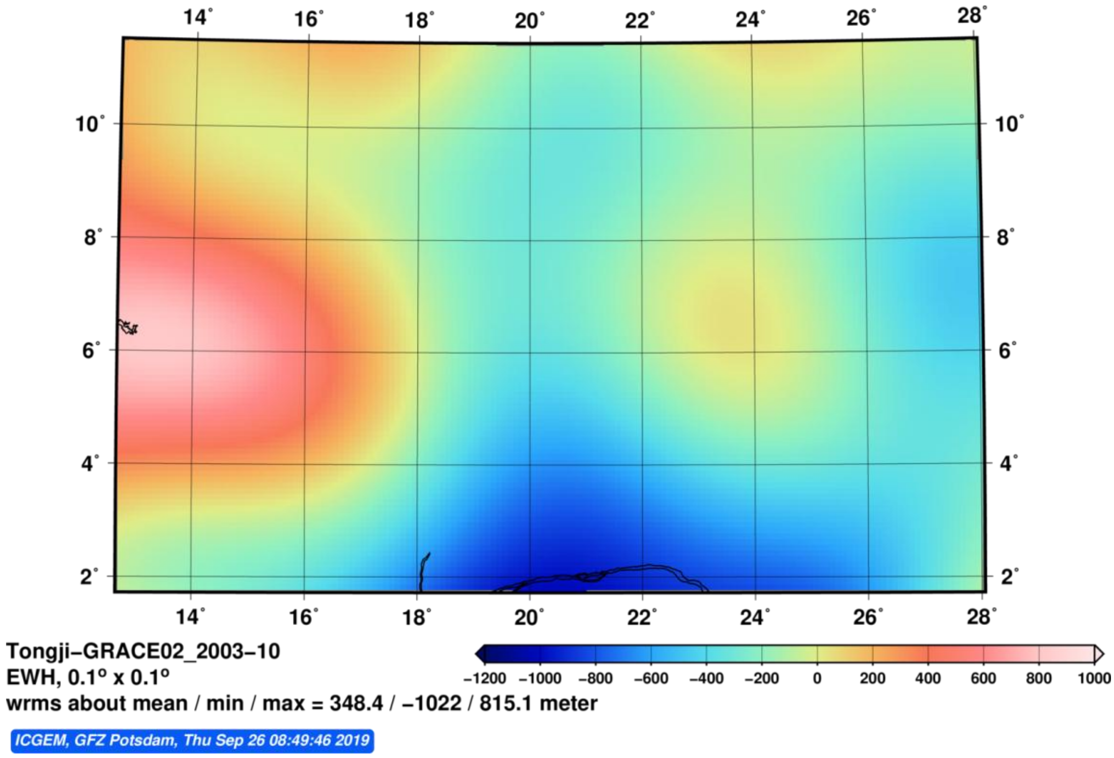
$$Q_{nm}^{i} = \sum_{i=0}^{p} \sum_{l=n-w/2}^{n+w/2} L_{ij}^{-1} l^{j} C_{lm}$$

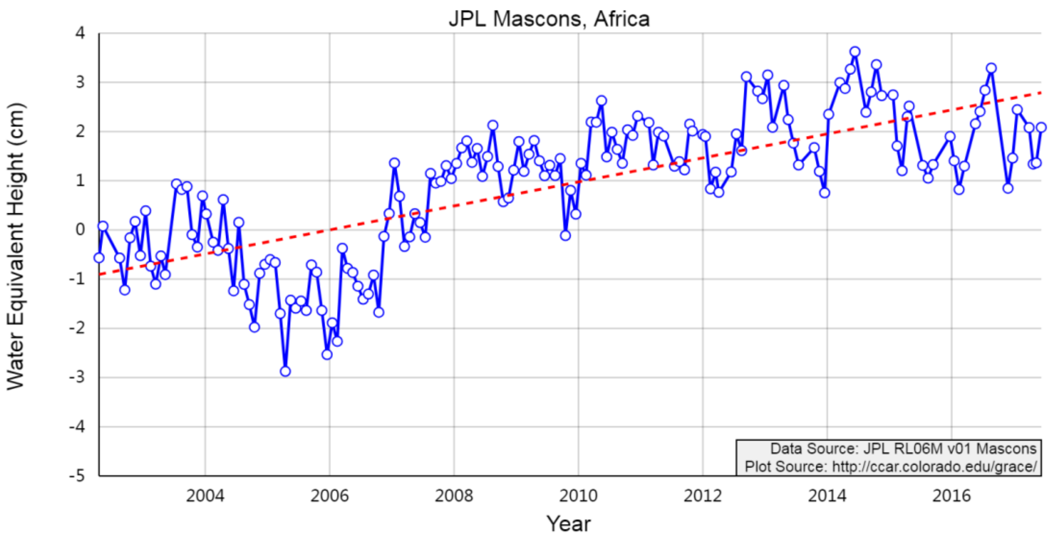
$$L_{ij} = \sum_{l=n-w/2}^{n+w/2} l^{i} l^{j}$$

l refers to the degree of participation in the fitting of the clamp coefficient. Since there is only a correlation between the even term and the even term, the odd term and the odd term, when n is an even term, the value of l is even. The value range is [n-w/2,n+w/2] and l≥w. Similarly, when n is an odd number, the degree values of the participating fitting coefficients should also be odd. Therefore, after decorrelation filtering, the mass density variation formula can be converted into:

$$\Delta\sigma(\varphi, \lambda) = \frac{a\rho_{ave}}{3} \sum_{n=0}^{\infty} \sum_{m=0}^{n} \frac{2n+1}{1+k_n} W_{nm} \bar{P}_{nm}(\sin\varphi)[\Delta\bar{C}_{nm}\cos(m\lambda) + \Delta\bar{S}_{nm}\sin(m\lambda)]$$


GSFC Mascons, Mascon #10832
Water Equivalent Height (cm)
Year
Data Source: GSFC Global Mascons v02.3b
Plot Source: http://ccar.colorado.edu/grace/


Tongji–GRACE02_2003–10
EWH, 0.1° x 0.1°
wrms about mean / min / max = 348.4 / –1022 / 815.1 meter
ICGEM, GFZ Potsdam, Thu Sep 26 08:49:46 2019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Water Equivalent Height (cm)
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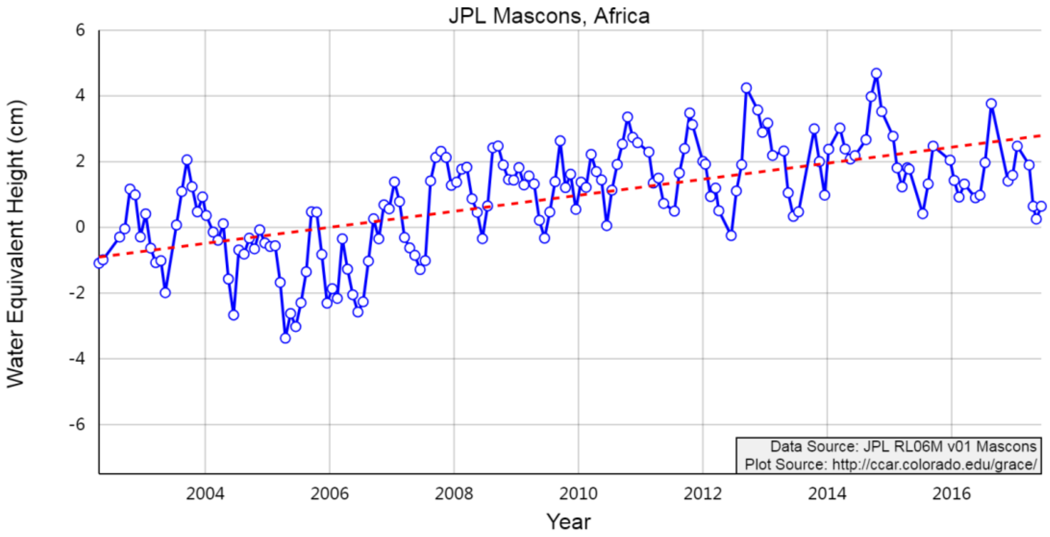
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Water Equivalent Height (cm)
Year
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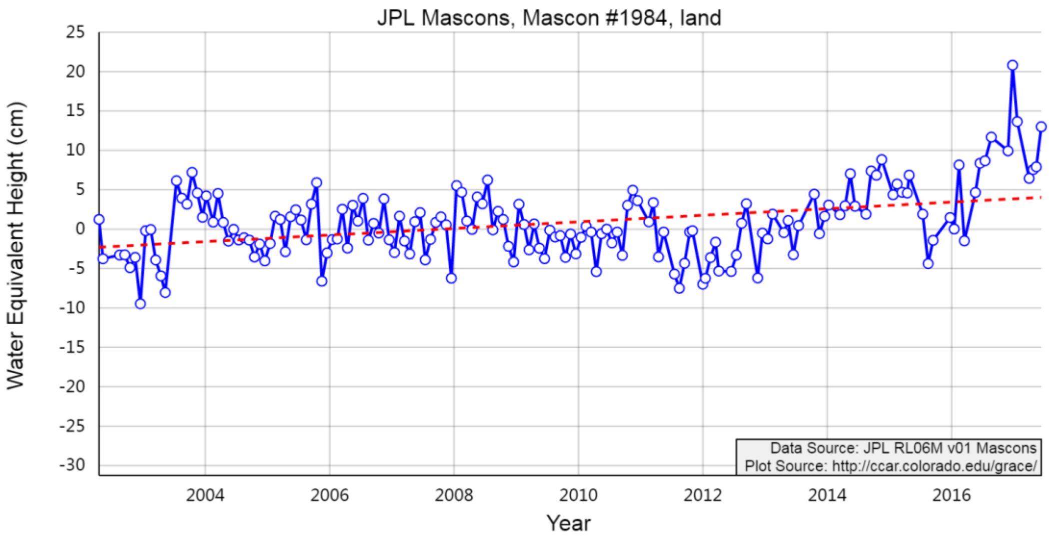
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Water Equivalent Height (cm)
Year
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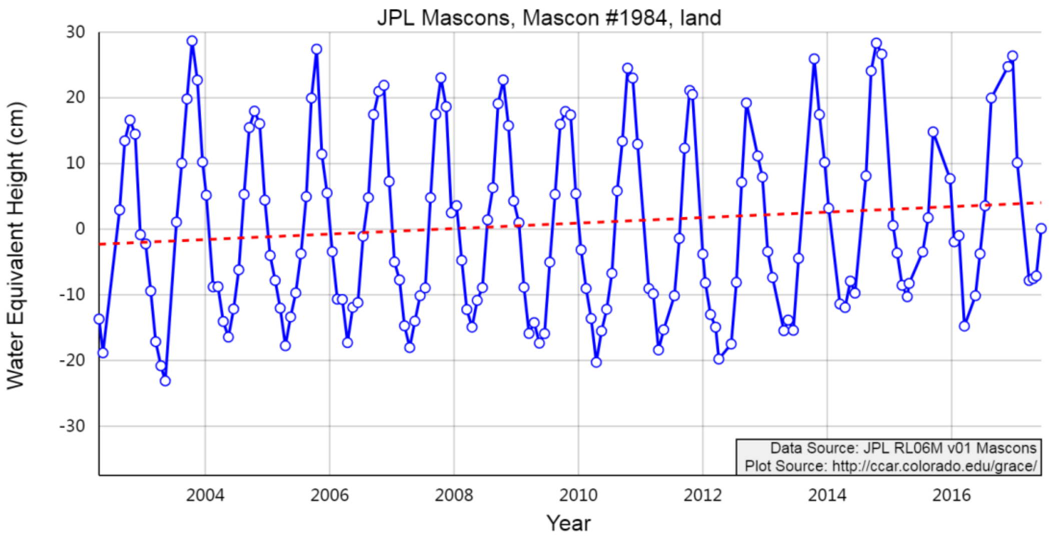
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Water Equivalent Height (cm)
Year
Data Source: JPL RL06M v01 Mascons
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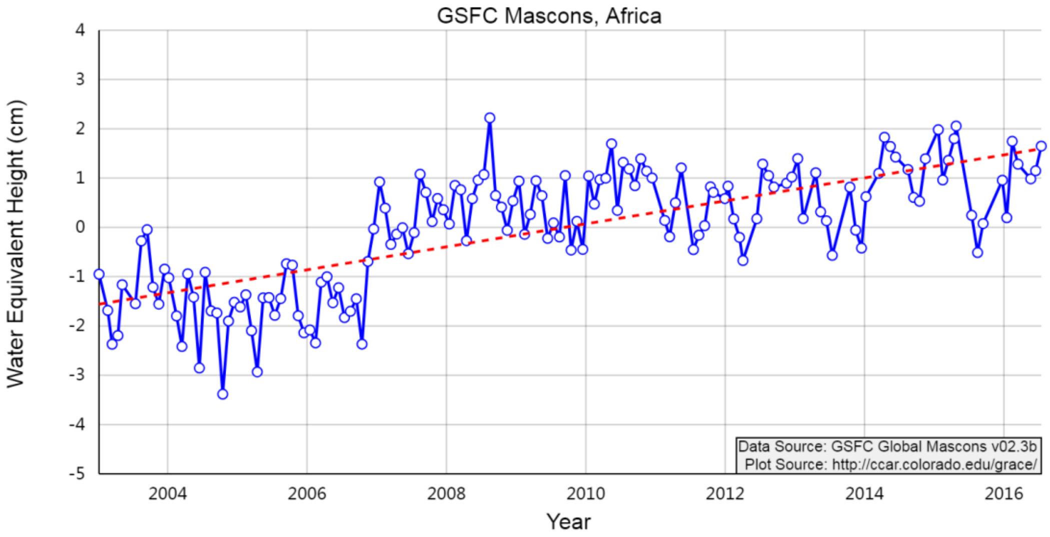
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Water Equivalent Height (cm)
Year
Data Source: GSFC Global Mascons v02.3b
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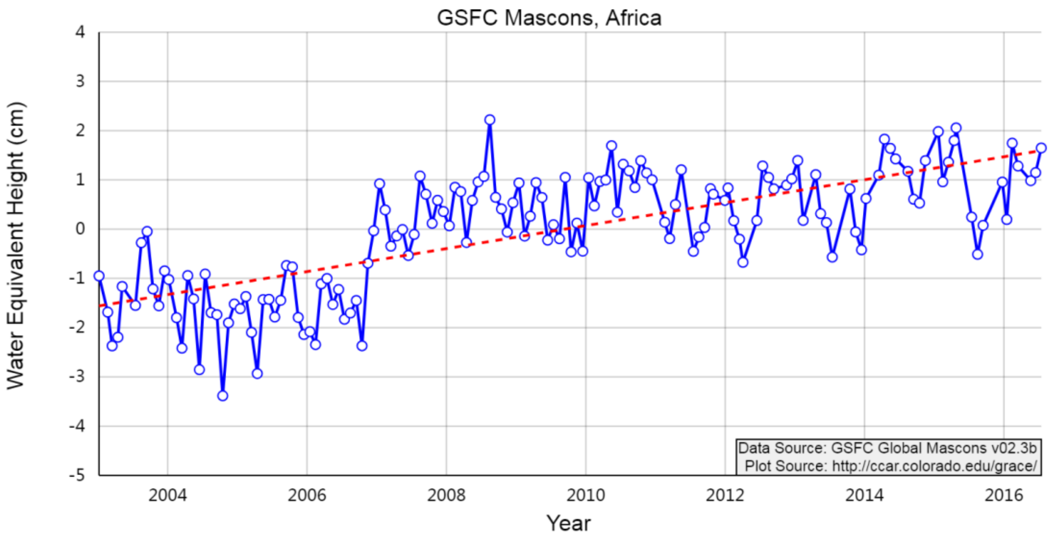
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Water Equivalent Height (cm)
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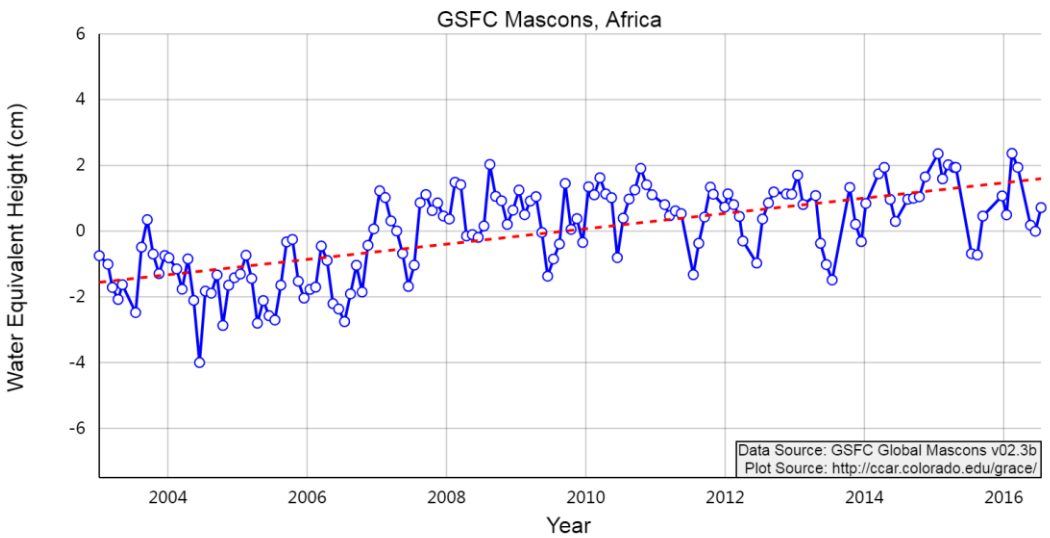
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Water Equivalent Height (cm)
Year
Data Source: GSFC Global Mascons v02.3b
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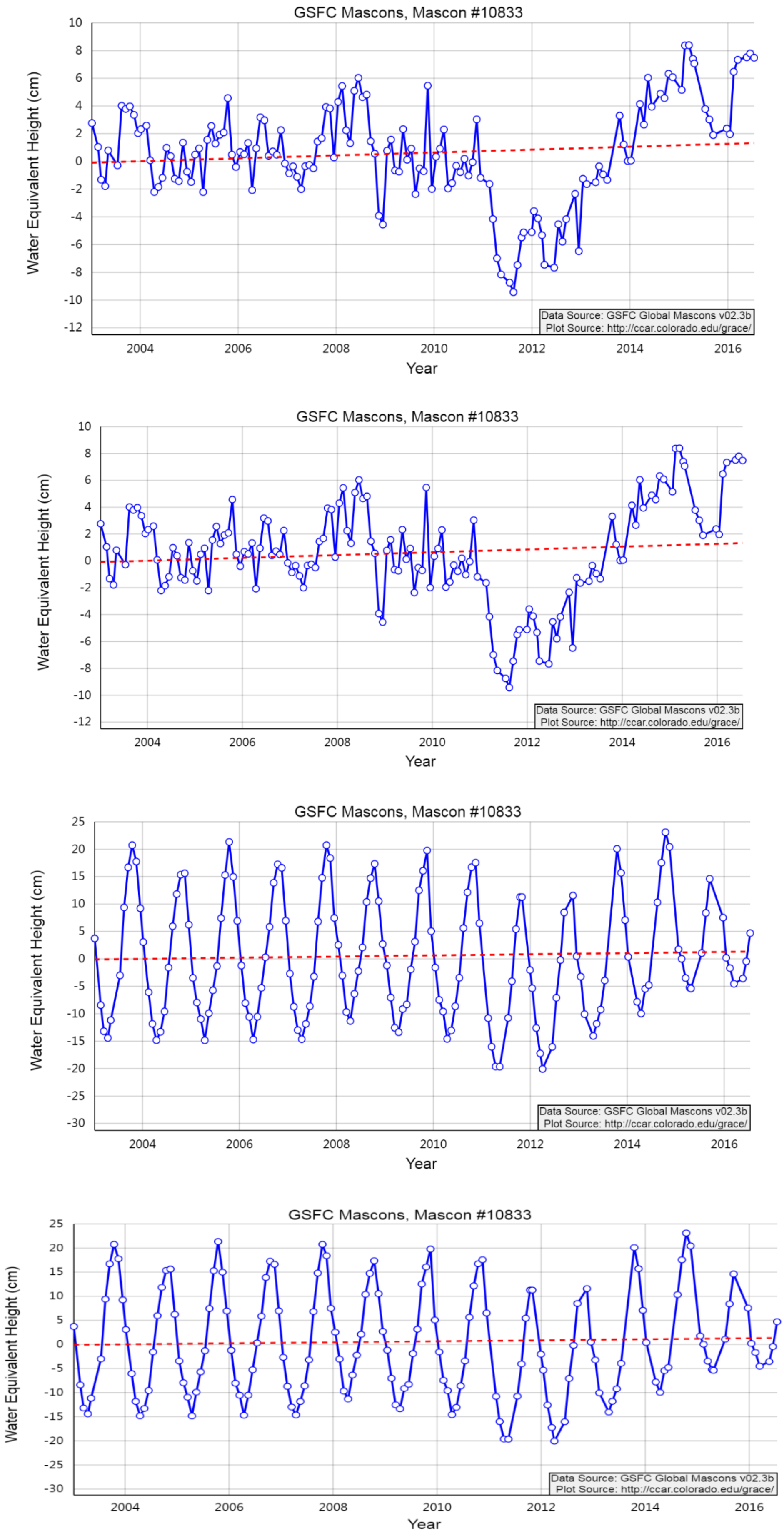

Figure 4.3: (b) Differents models variatins of water storage over the years

4.1 DDK filtering

DDK filtering is a decorrelated smoothing filter, which is similar to fan-shaped filtering. It is also a non-isotropic filtering function. The smoothing function of non-isotropic filtering is related to the degree of spherical coefficients and the order of spherical coefficients. Related. The smooth kernel function is: □

$$W_a = (E^{-1} + aS^{-1})^{-1} E^{-1}$$

where E is the error covariance matrix and S is the signal covariance matrix.

The spherical coefficient after DDK filtering can be expressed as:

$$\begin{cases} C_{nm}^{filt} = \sum\limits_{n'=n_{\min},\, n'\in parity(n)}^{n_{\max}} w(n,n',m,a)\, C_{n'm} \\ S_{nm}^{filt} = \sum\limits_{n'=n_{\min},\, n'\in parity(n)}^{n_{\max}} w(n,n',m,a)\, S_{n'm} \end{cases}$$

Where $n' \in parity(n)$ means that n' and n have the same parity, and the left side of the equation is the ball harmonic coefficient after ddk filtering.

The existing ddk filters mainly include ddkl, ddk2, ddk3, ddk4, and ddk5. The latter two have been used in the new version of the RL05 data. The agencies have released the coefficients after DDK filtering, which can be downloaded from the relevant website ICGEM.

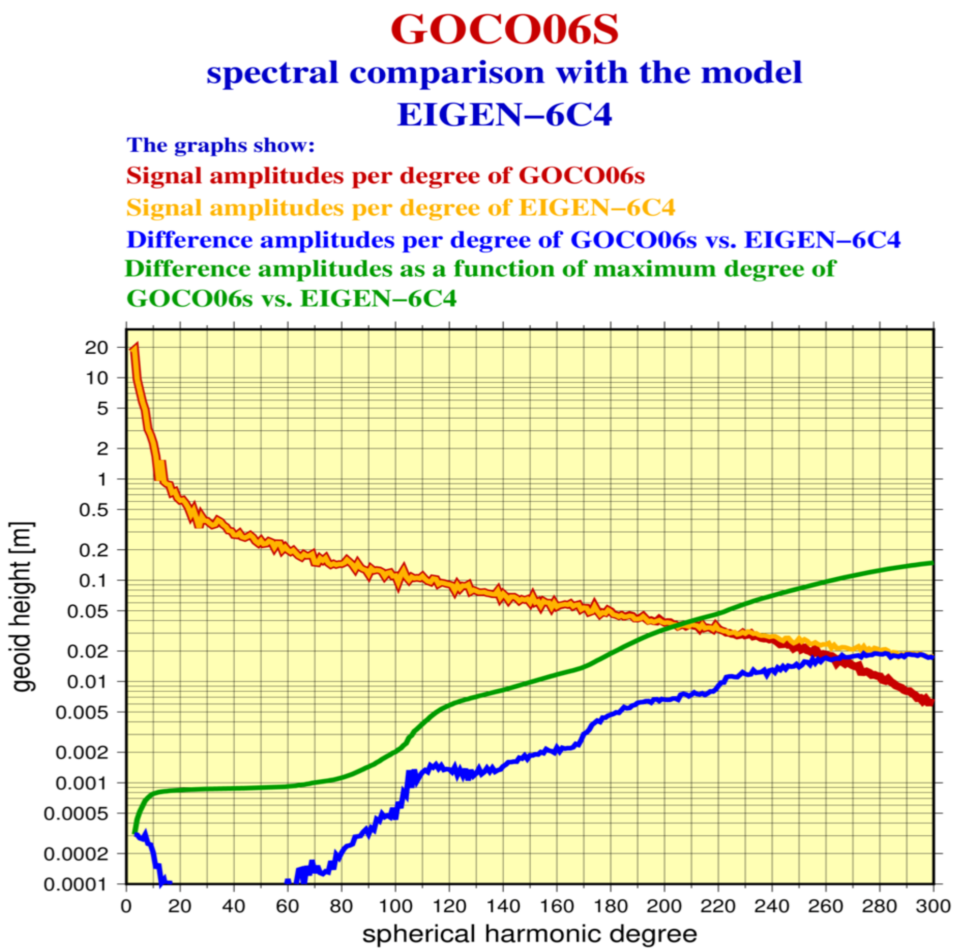

Figure 4.1: Spectral comparison with the model

4.2 Fan filtering

Unlike Gaussian filtering, fan filtering is a non-isotropic filtering method that is developed based on isotropic filtering. Above equation uses Gaussian filtering only for the degree, based on the original filtering of the sector filtering, in which Gaussian filtering is introduced in the formula. In simple terms, the fan filtering is the same as the spherical harmonic Gaussian filtering is performed on the degree and the order of the spherical coefficient, and the formula for calculating the mass variation is:

$$\Delta\sigma = \frac{a\rho_{ave}}{3}\sum_{n=0}^{\infty} W_n \sum_{m=0}^{n} \frac{2n+1}{1+k_n} W_{nm} \bar{P}_{nm}(\sin\varphi)[\Delta\bar{C}_{nm}\cos(m\lambda) + \Delta\bar{S}_{nm}\sin(m\lambda)]$$

Among them, Wn、Wm is the Gaussian filter weight coefficient of the order and degree and the second correlation with the GRACE ball harmonic coefficient. From equation, it can be seen that when calculating the mass change of a certain place, for a certain order of the ball The harmonic coefficient, the weight value of the fan filter is smaller than the Gaussian filter, especially for the high-order term, the spherical coefficient weight obtained by the sector filtering is very small.

Non filtering

Non-filtering is also based on Gaussian filtering. The smoothing kernel function is:

$$W_{nm} = W_n(r_{1/2}(m)), r_{1/2}(m) = \frac{r_1 - r_0}{m} m + r_0$$

where W_n is the classical Gaussian smoothing kernel function, r_1, r_0 is the smoothing radius, m1is the selected order, n,m is the degree and order of harmonics of the GRACE gravity field ball. r_0 determines the smooth radius along the longitude direction, r1,m1 determines the smooth radius along the twist direction. The Non filter is also related to the order of the spherical harmonic coefficients, and the higher the number of the same order, the smaller the value of the calculated smooth kernel function, and the smaller the weight of the corresponding spherical coefficient. The formula for solving the mass change based on Fan filter is:

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4.3 De-correlation filtering

The equivalent water height change map mainly shows the north-south band error. Swenson and Wahr pointed out that the appearance of the north-south band indicates that the GRACE error has a high spatial correlation. By comparing the spectral domain variation characteristics of the median coefficient of the time-varying gravity model, it is found that there is a correlation between the GRACE time-varying gravity model spherical coefficients. When the value of the second m is constant, there is a correlation between the odd terms in the order n of the coefficient, and the correlation also exists between the even terms. Taking m=15 as an example, the common coefficient C15,15、C16,15、C17,15…C58,15、C59,15、C60,15 ，where the odd term of n is C15,15、C17,15、C19,15…C55,15、C57,15、C59,15 has a correlation, and the even term is also between C16,15、C18,15、C20,15…C56,15、C58,15、C60,15 Correlation. According to this, Swenson proposed a sliding window decorrelation method to reduce the correlation between the spherical coefficients to weaken the influence of the north-south band.

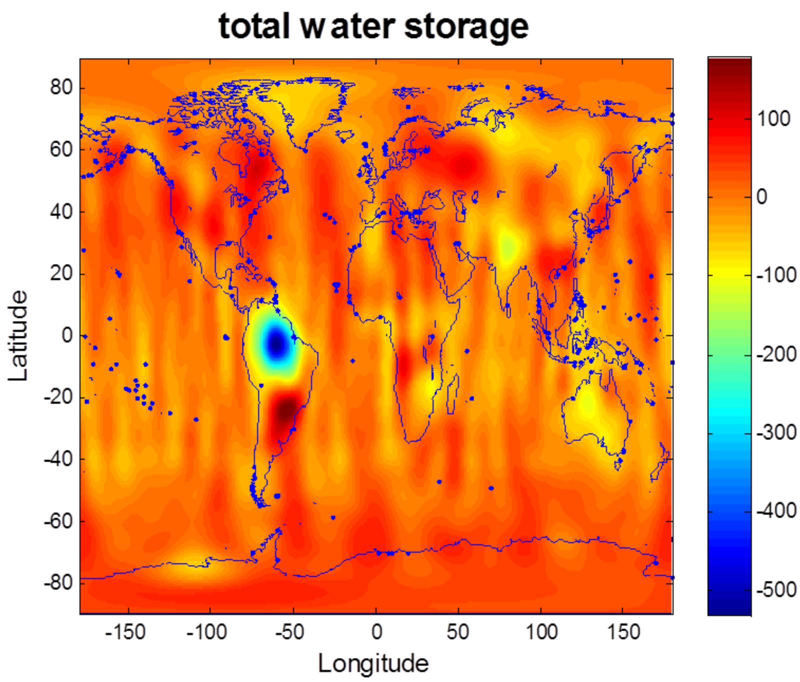


Figure 4.3: (a) Total water storage

The principle of de-correlation filtering is: the order-m of the fixed-ball harmonic coefficient Cnm, using a sliding window of size w, with the degree n as an independent variable, and the even-numbered and odd-numbered pairs of the alignment coefficient are fitted by polynomial, the formula as follows:

$$C_{nm}^{ce} = \sum_{i=0}^{p} Q_{nm}^{i} n^{i}$$

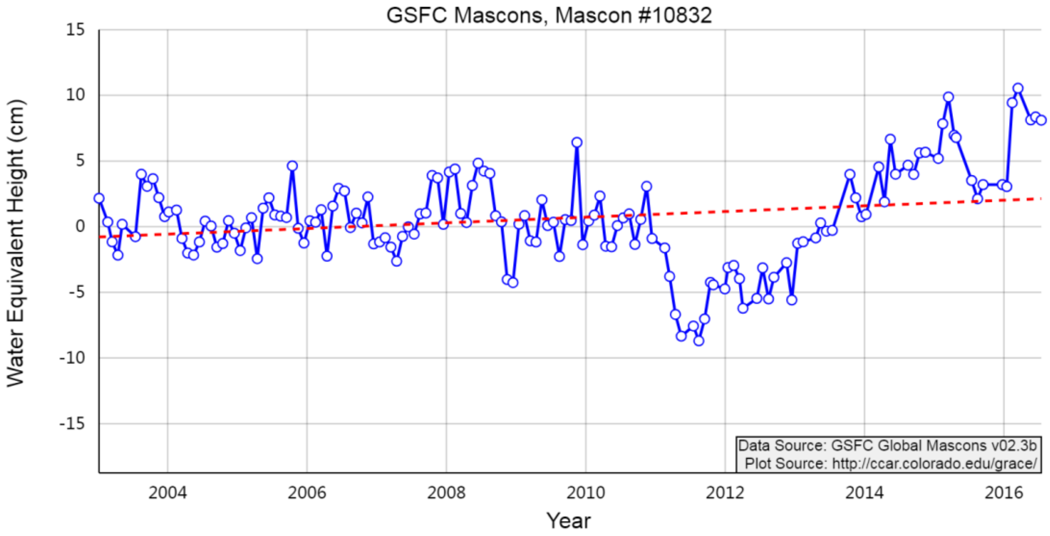
Where C_{nm}^{ce} is the polynomial fitting value of Cnm , the fitting value is used as the correction value of Cnm, Q_{nm}^{i} is the fitting polynomial coefficient, p is the number of polynomials, and the principle is the same when calculating the fitting value of Snm And thus the coefficient can be calculated as:

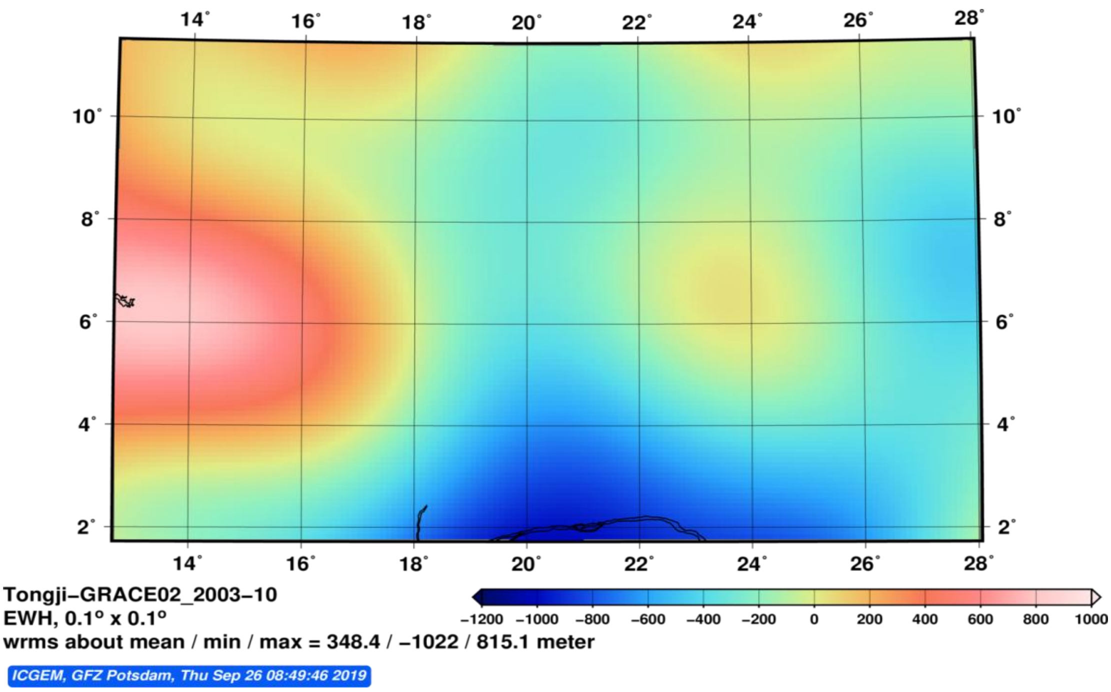
$$Q_{nm}^{i} = \sum_{i=0}^{p} \sum_{l=n-w/2}^{n+w/2} L_{ij}^{-1} l^{j} C_{lm}$$

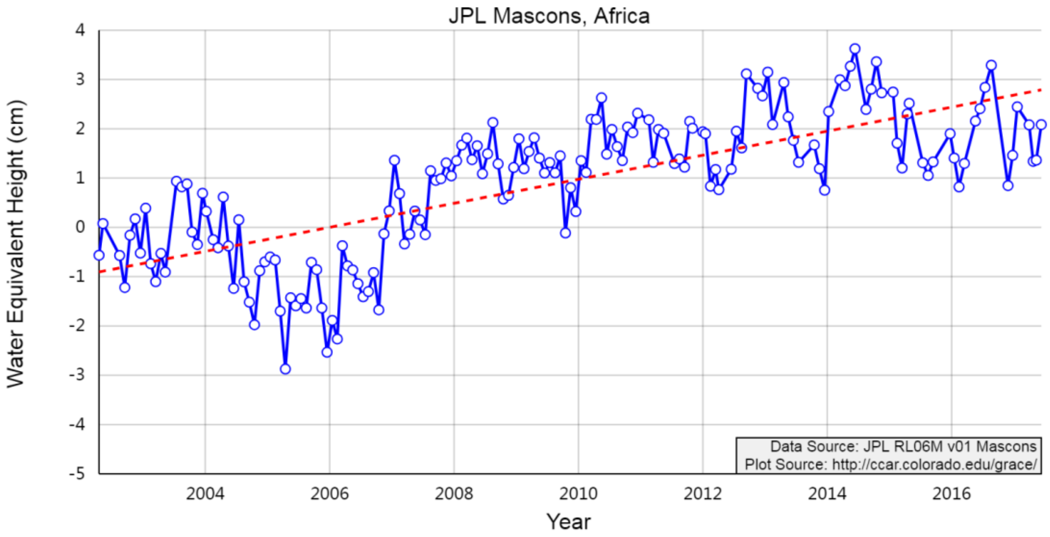
$$L_{ij} = \sum_{l=n-w/2}^{n+w/2} l^{i} l^{j}$$

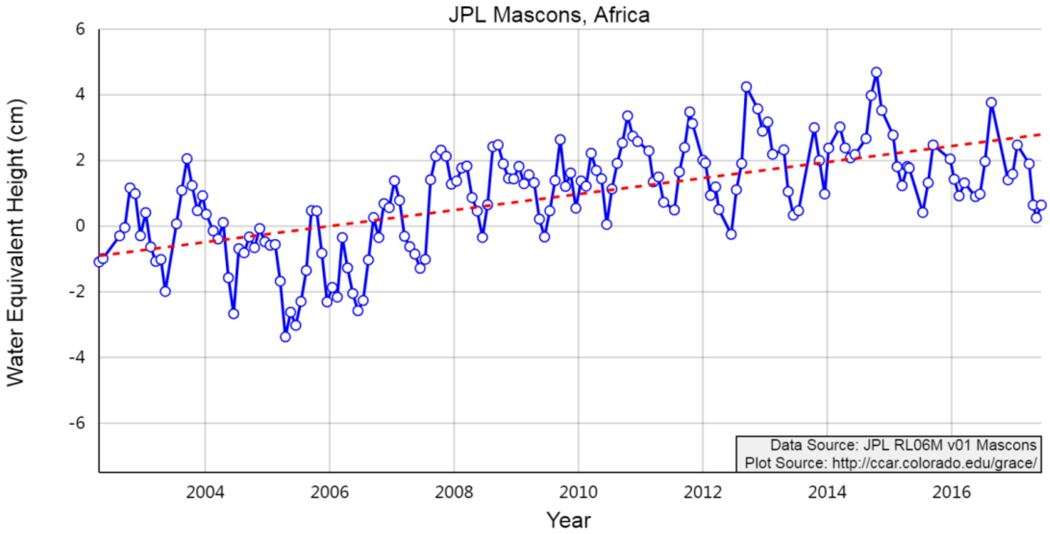
l refers to the degree of participation in the fitting of the clamp coefficient. Since there is only a correlation between the even term and the even term, the odd term and the odd term, when n is an even term, the value of l is even. The value range is [n-w/2,n+w/2] and l≥w. Similarly, when n is an odd number, the degree values of the participating fitting coefficients should also be odd. Therefore, after decorrelation filtering, the mass density variation formula can be converted into:

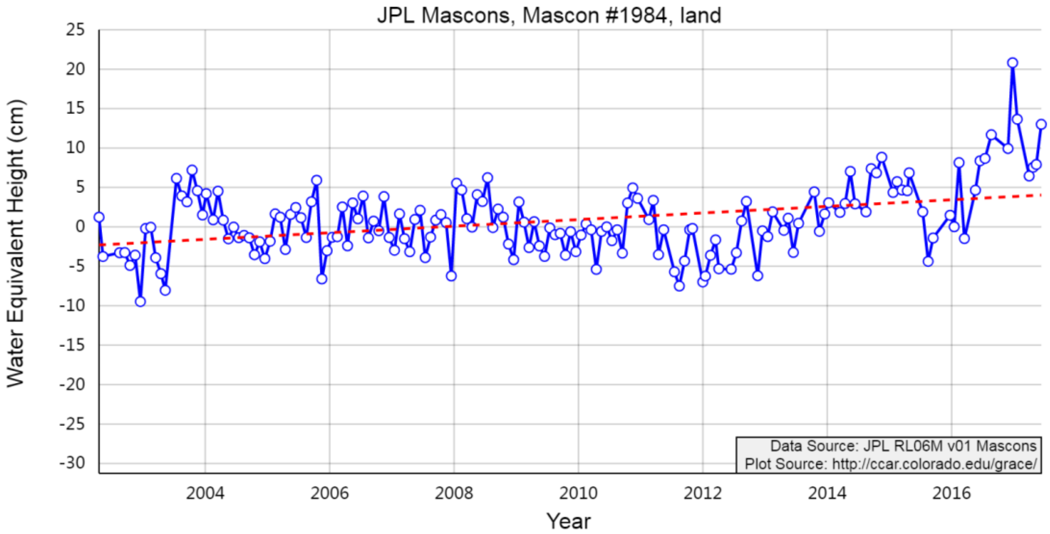
$$\Delta\sigma(\varphi, \lambda) = \frac{a\rho_{ave}}{3} \sum_{n=0}^{\infty} \sum_{m=0}^{n} \frac{2n+1}{1+k_n} W_{nm} \bar{P}_{nm}(\sin\varphi)[\Delta\bar{C}_{nm}\cos(m\lambda) + \Delta\bar{S}_{nm}\sin(m\lambda)]$$











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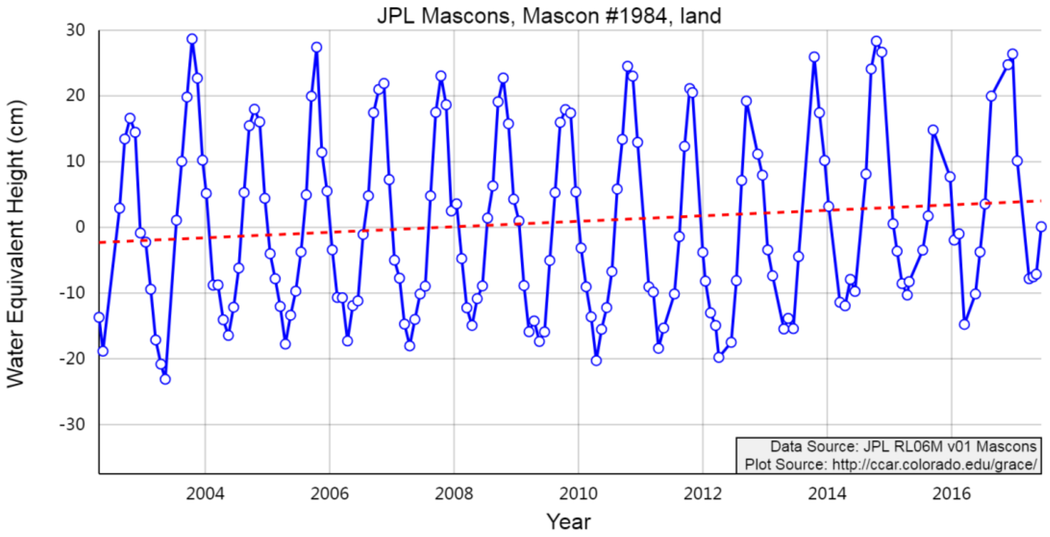
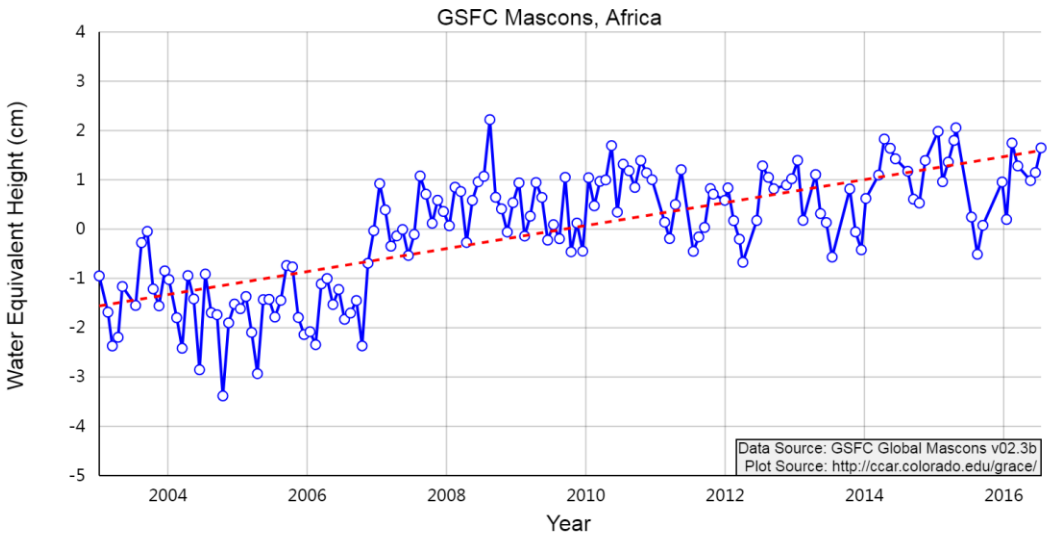
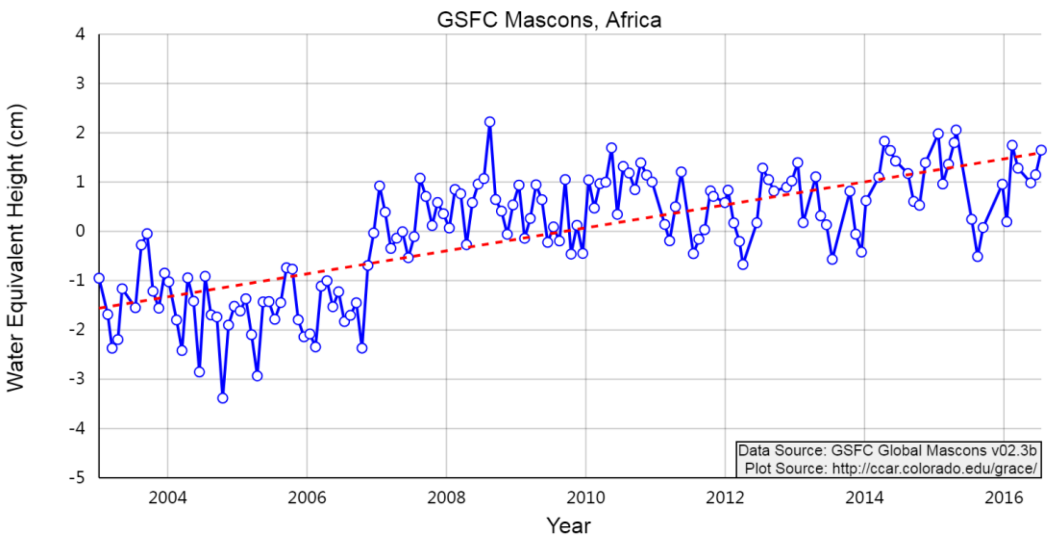
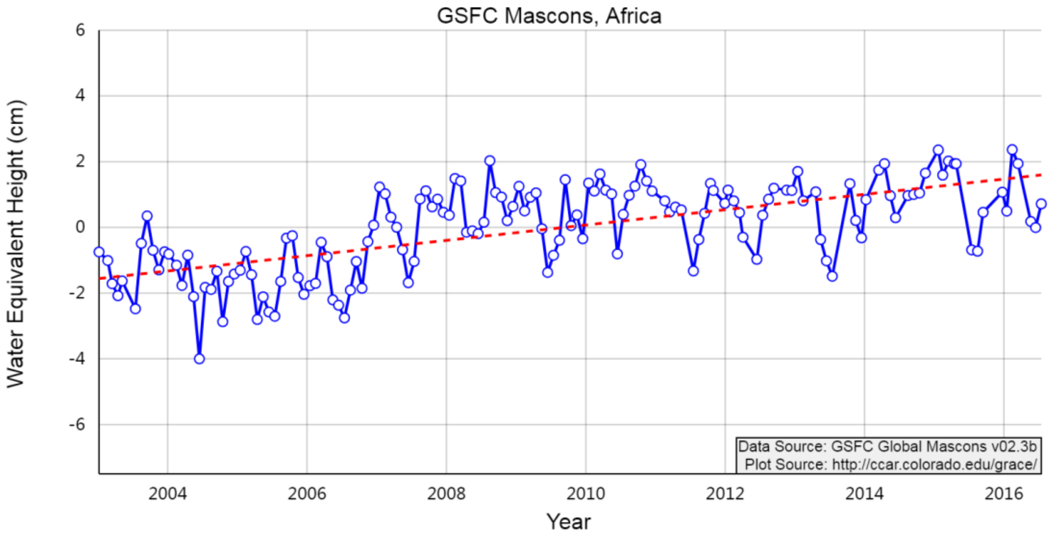
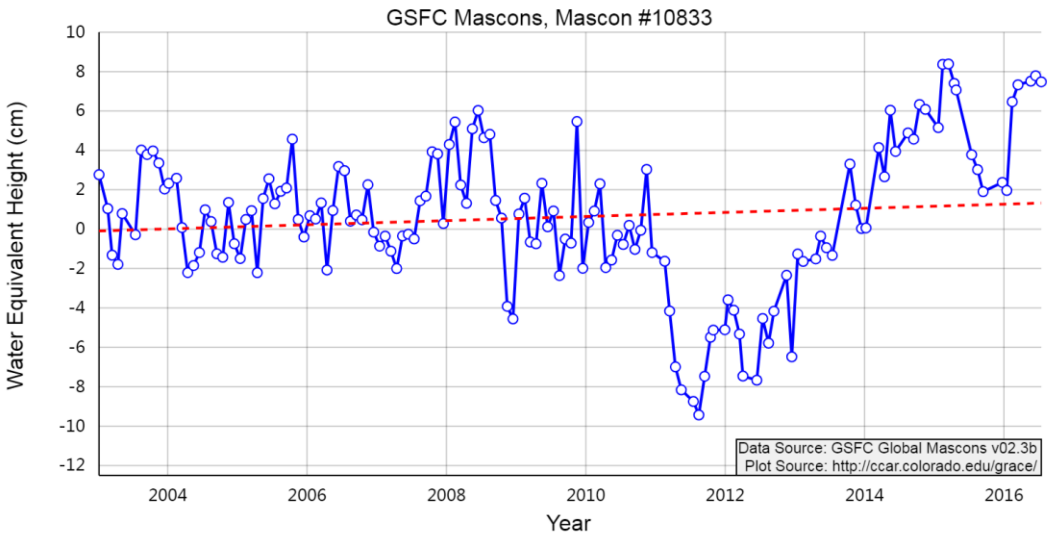
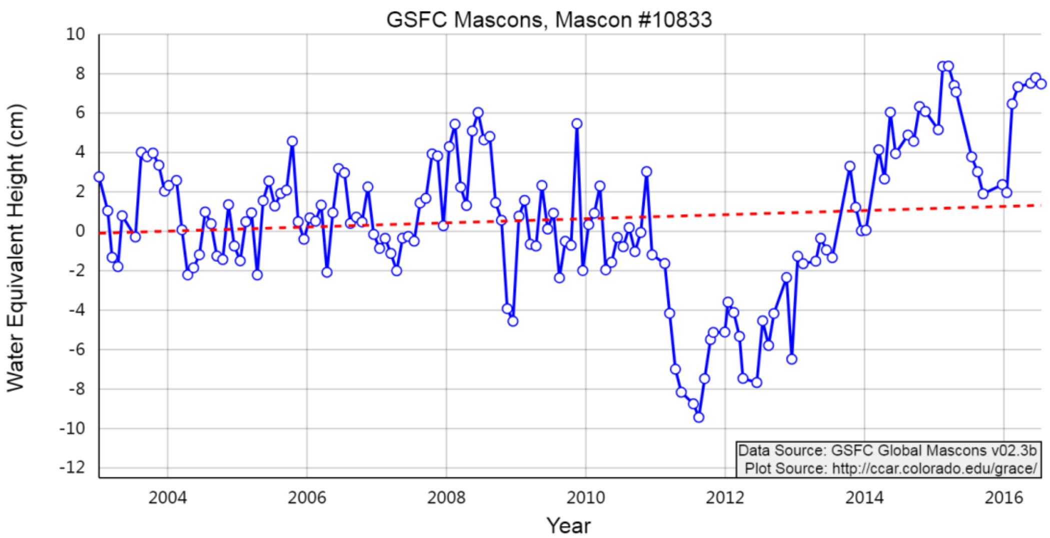

JPL Mascons, Mascon #1984, land
Water Equivalent Height (cm)
30
20
10
0
-10
-20
-30
2004
2006
2008
2010
2012
2014
2016
Year
Data Source: JPL RL06M v01 Mascons
Plot Source: http://ccar.colorado.edu/grace/

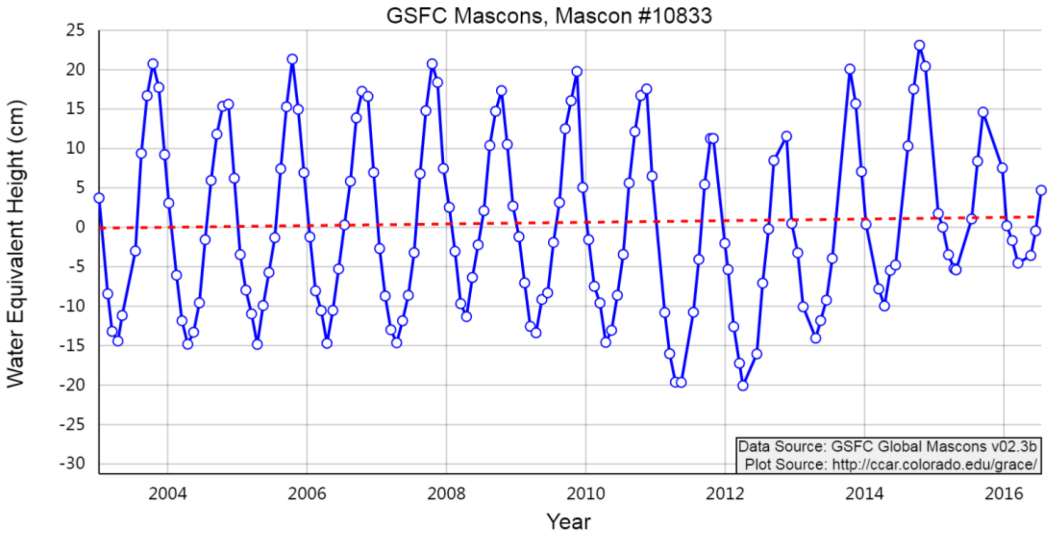

GSFC Mascons, Africa
Water Equivalent Height (cm)
4
3
2
1
0
-1
-2
-3
-4
-5
2004
2006
2008
2010
2012
2014
2016
Year
Data Source: GSFC Global Mascons v02.3b
Plot Source: http://ccar.colorado.edu/grace/


GSFC Mascons, Africa
Water Equivalent Height (cm)
4
3
2
1
0
-1
-2
-3
-4
-5
2004
2006
2008
2010
2012
2014
2016
Year
Data Source: GSFC Global Mascons v02.3b
Plot Source: http://ccar.colorado.edu/grace/


GSFC Mascons, Africa
Water Equivalent Height (cm)
Year
Data Source: GSFC Global Mascons v02.3b
Plot Source: http://ccar.colorado.edu/grace/


GSFC Mascons, Mascon #10833
Water Equivalent Height (cm)
Year
Data Source: GSFC Global Mascons v02.3b
Plot Source: http://ccar.colorado.edu/grace/


GSFC Mascons, Mascon #10833
Water Equivalent Height (cm)
Year
Data Source: GSFC Global Mascons v02.3b
Plot Source: http://ccar.colorado.edu/grace/



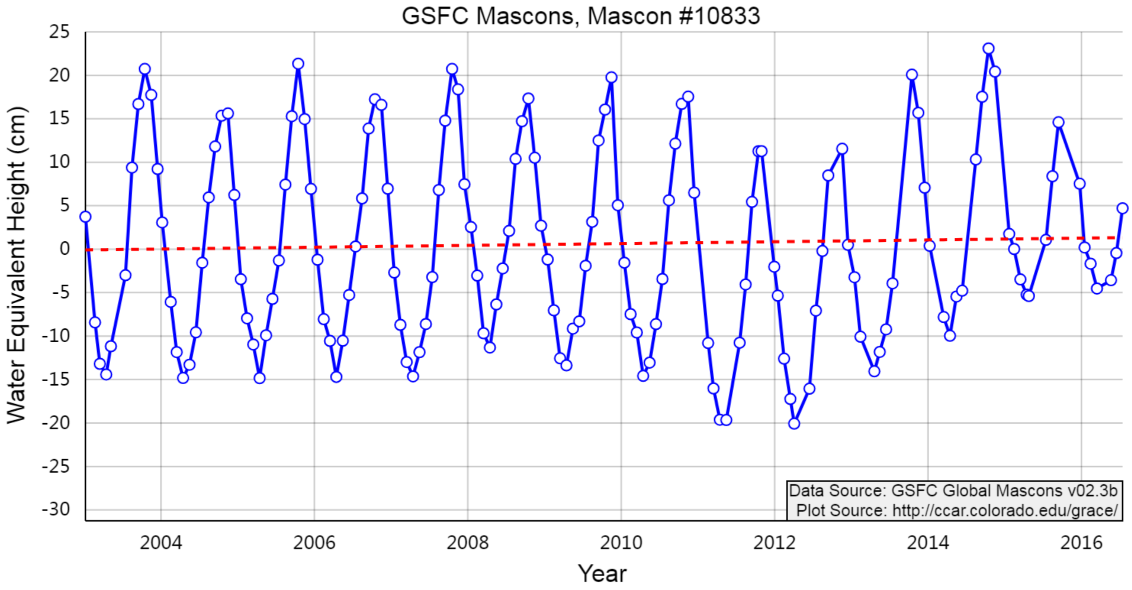


Figure 4.3: b Differents models variatins of water storage over the years.

4.4 DDK filtering

DDK filtering is a decorrelated smoothing filter, which is similar to fan-shaped filtering. It is also a non-isotropic filtering function. The smoothing function of non-isotropic filtering is related to the degree of spherical coefficients and the order of spherical coefficients. Related. The smooth kernel function is:

$$W_a = (E^{-1} + aS^{-1})^{-1} E^{-1}$$

where E is the error covariance matrix and S is the signal covariance matrix.

The spherical coefficient after DDK filtering can be expressed as:

$$\begin{cases} C_{nm}^{filt} = \sum_{n'=n_{\min}, n' \in parity(n)}^{n_{\max}} w(n,n',m,a) C_{n'm} \\ S_{nm}^{filt} = \sum_{n'=n_{\min}, n' \in parity(n)}^{n_{\max}} w(n,n',m,a) S_{n'm} \end{cases}$$

Where $n' \in parity(n)$ means that n' and n have the same parity, and the left side of the equation is the ball harmonic coefficient after ddk filtering.

The existing ddk filters mainly include ddkl, ddk2, ddk3, ddk4, and ddk5. The latter two have been used in the new version of the RL05 data. The agencies have released the coefficients after DDK filtering, which can be downloaded from the relevant website ICGEM.

4.5 Conclusion

The formal equivalence of Kalman filtering and smoothing techniques to generalized least squares. Smoothing and filtering equations are presented for the case where some of the parameters are constant. And shows that generalized least squares will produce consistent estimates of those parameters that are not time varying. Starting from the principle of Grace gravity satellite measuring the change of the earth's mass, this chapter focuses on discussing several filtering methods to remove strip errors, including Gauss filtering, sector filtering, Non filtering, decorrelation filtering and DDK filtering. The calculation formulas of each filtering method are deduced, and their respective characteristics are analyzed and compared.

Author Statement

Josette-Gila ABA-BIKOUMOSSALI-MBESSO: Conceptualization, Methodology, Software, Writing-Original Draft, Visualization, Resources.
YueDongjie: Supervision, Writing- Reviewing and Editing,.
Data availability: All data included in this study are available upon reasonable request from the corresponding author.

References

[1] A. Tarantola Inverse Problem Theory Elsevier, Amsterdam (1987) 613 pp.
[2] B.D. Tapley, S. Bettadpur, M. Watkins, C. Reigber The gravity recovery and climate experiment: mission overview and early results Geophys. Res. Lett., 31 (2004), p. L09607 http://dx.doi.org/10.1029/2004GL019920View Record in Scopus, Citing articles (942)
[3] B.D. Tapley, S. Bettadpur, J.C. Ries, P.F. Thompson, M. Watkins GRACE measurements of mass variability in the Earth system Science, 305 (2004), pp. 503–505 CrossRef | View Record in Scopus| Citing articles (829)
[4] B. Rudolf, H. Hauschild, W. Rueth, U. Schneider Terrestrial precipitation analysis: operational method and required density of point measurements , in: M. Desbois, F. Desalmond (Eds.), Global Precipitations and Climate Change, NATO ASI Series I, vol. 26, , Springer-Verlag (1994), pp. 173–186 CrossRef| View Record in Scopus
[5] B. Rudolf, T. Fuchs, U. Schneider, A. Meyer-Christoffer Introduction of the Global Precipitation Climatology Centre (GPCC)Deutscher Wetterdienst, Offenbach a.m. (2003), p. 16 View Record in Scopus
[6] C. Jekeli, Alternative Methods to Smooth the Earth's Gravity Field, Rep. 327. Dep. of Geod. Sci. and Surv, Ohio State Univ., Columbus, 1981. Citing articles (1)
[7] D. Werth, R. Avissar The regional evapotranspiration of the Amazon J. Hydrometeorol., 5 (2004), pp. 100–109 Cross Ref| View Record in Scopus | Citing articles (53)
[8] D. Stammer, R. Tokmakian, A. Semtner, C. WunschHow well does a 1/4° global circulation model simulate large-scale oceanic observations?J. Geophys. Res., 101 (1996), pp. 25799–25811
[9] D. Stammer, C. Wunsch, R. Giering, C. Eckert, P. Heimbach, J. Marotzke, A. Adcroft, C.N. Hill, J. Marshall The global ocean circulation during 1992–1997, estimated from ocean observations and a general circulation model J. Geophys. Res., 107 (C9) (2002), p. 3118 http://dx.doi.org/10.1029/2001JC000888CrossRef | View Record in Scopus | Citing articles (1)
[10] G. Ramillien, A. Cazenave, O. Brunau Global time-variations of hydrological signals from GRACE satellite gravimetry Geophys. J. Int., 158 (2004), pp. 813–826 CrossRef | View Record in Scopus| Citing articles (66)
[11] G. Ramillien Gravity/magnetic potential of uneven shell topography J. Geod., 76 (2002), pp. 139–149 http://dx.doi.org/10.1007/s00190-002-0193-5 CrossRef| View Record in Scopus| Citing articles (18)
[12] H. Douville, E. Bazile, P. Caille, D. Giard, J. Noilhan, L. Peirone, F. Taillefer Global Soil Wetness Project: forecast and assimilation experiments performed at Meteo-France J. Meteorol. Soc. Jpn, 77 (1999), pp. 305–316View Record in Scopus| Citing articles (22)
[13] H. Douville, F. ChauvinRelevance of soil moisture for seasonal climate predictions; a preliminary study Clim. Dyn. (2000), pp. 16719–16736
[14] J. Wahr, M. Molenaar Time variability of the Earth's gravity field: hydrological and oceanic effects and their possible detection using GRACEJ. Geophys. Res., 103 (B12) (1998), pp. 30205–30229CrossRef| View Record in Scopus | Citing articles (779)
[15] J. Wahr, S. Swenson, V. Zlotnicki, I. Velicogna Time-variable gravity from GRACE: first results Geophys. Res. Lett., 31 (2004), p. L11501 http://dx.doi.org/10.1029/2004GL019779 View Record in Scopus, Citing articles (367)