

A Review of Digital FIR Filter Design in Communication Systems

Kamal Hossain¹, Roni Ahmed², Md. Asadul Haque³, Muahmmad Towfiqur Rahman⁴

^{1,2,3,4}Department of Electrical and Electronics Engineering, Northern University Bangladesh

¹towfiqcse[at]gmail.com

Abstract: In signal processing, a finite impulse response (FIR) filter settles to zero in finite time. FIR filters can be discrete-time or continuous-time, and digital or analog. FIR filter is widely used in various signal processing and image processing applications because of less area, low cost, low power, and high operation speed. An FIR filter is usually implemented by using a series of delays, multipliers, and adders to create the filter's output. We concentrate on the following three categories: frequency sampling methods, windowing-based methods, and optimization-based methods. We also focus on the communications system, including transmissions equipment, relay stations, tributary stations, and other data terminal equipment. A communications system can even include other communications systems. A good example would be a regional emergency response communications system that connects several different cities and allows them to respond to a disaster by integrating systems they have installed for their police and firefighters. In the end, the performances of several FIR design methods are assessed. This article provides a comprehensive overview of the latest developments in finite impulse response (FIR) filter design methods in communication systems.

Keywords: FIR Filter; Communication system; frequency sampling methods; windowing based methods; optimization-based methods

1. Introduction

A finite impulse response (FIR) filter is a filter whose impulse response (or response to any finite length input) is limited because it settles to zero in finite time. This contrasts with infinite impulse response (IIR) filters, which may have internal feedback and may continue to respond indefinitely [1]. The communication system model describes a communication exchange between transmitter and receiver. Signals or information passes from source to destination through a channel [2]. Mainly FIR filters are using windowing based methods and optimization-based methods [3]. To design a Finite Impulse Response (FIR) filter with the desired frequency response, window functions to achieve a trade-off between ripples in the passband and the transition band's sharpness [4]. Digital filters form an important part of today's expanding field of Digital Signal Processing (DSP). Among them, the most used filter is Finite impulse response. FIR filters. FIR filters are used extensively to filter images, modulate frequency, precision arithmetic, and various other purposes. Thus, various optimization methods are employed for the designing of optimal digital FIR filters [5]. Various optimization techniques provide better results for different filter coefficients for control parameter, dependence, premature convergence, etc. They have many advantages, like simple design implementation, minimized error function, superior search capability, and fast convergence. This paper has reviewed the basics of the frequency sampling method for designing Finite Impulse Response (FIR) filters in a communication system. Moreover, FIR architecture, prototype for optimization techniques, hardware-based 2D FIR filter models are highlighted.

1.1 FIR Filter and Communication system

FIR filters are the most popular type of filters implemented in software. Filters are signal conditioners. Each function accepts an input signal, blocks pre-specified frequency components, and then passes the original signal minus these

components to the output. For example, a typical telephone line acts as a filter, limiting the frequency to a range much smaller than the frequency range that humans can hear. Therefore, listening to CD-quality music over the phone is not as pleasant as listening to music directly. [6]. Digital filter uses digital input, provides digital output, and is composed of digital components. A typical digital filtering application, software-based digital signal processor (DSP) Read input samples from the A/D converter, perform theoretically prescribed mathematical operations on the required filter type, and output the result through the D/A converter. In contrast, analog filters operate directly on the analog input and are composed entirely of analog components, such as resistors, capacitors, and inductors [7]. There are many filter types, but the most common are low pass, high pass, band pass, and bandstop filter. A low pass filter allows only low-frequency signals (below some specified cutoff) to its output, so it can be used to eliminate high frequencies. A low pass filter is convenient, in that regard, for limiting the uppermost range of frequencies in an audio signal; it's the type of filter that a phone line resembles. A high pass filter does just the opposite from lowpass by rejecting only frequency components below some threshold. An example of Qualcomm's application is to cut off the audible 60Hz AC power "hum", can be chosen up as noise accompanying almost any signal in the world [8]. The vendor of a cell phone or any other sort of wireless transmitter typically places an analog bandpass filter in its output RF stage to ensure that only output signals within its narrow, government-authorized range of the frequency spectrum are transmitted. Engineers can use the research band stop filters, which pass both low and high frequencies to block a predefined range of frequencies in the middle.

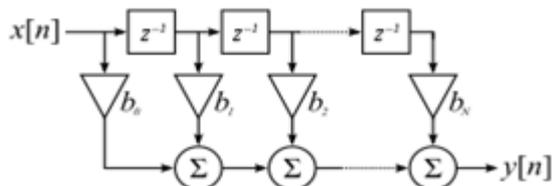


Figure 1: Finite impulse response (FIR) filter block diagram [8]

The Purpose of a communication system is to carry information from one point to another. A typical communication system consists of three main components as shown in fig. 2, that are source, channel, destination.



Figure 2: A basic communication system model.

Communication system is a system model that describes a communication exchange between two stations, transmitter, and receiver. Signals or information passes from source to destination through the communication channel, representing a way that signals carry information from the source toward the destination. Several stages must first process it to transmit signals in the communication system, beginning from signal representation to signal shaping until encoding and modulation. After preparing the transmitted signal, it passed to the channel's transmission line, and due to signal crossing this media, it faced many impairments such as noise, attenuation, and distortion.

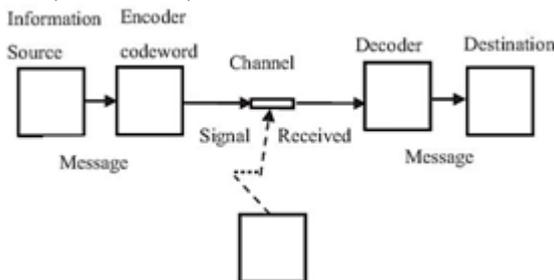
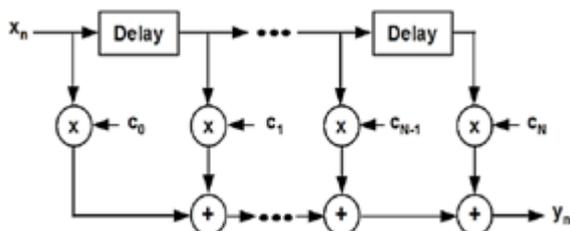


Figure 3: Typical communication system model [5]

1.2 FIR Filter architecture

FIR filter is called a non-recursive type filter because they require no feedback. Due to this no feedback, the impulse response of FIR filter remains finite.



The conventional design equation of a digital FIR filter can be written as:

$$y(n) = \sum_{k=0}^N b_k x(n - k) = b_0x(n) + b_1x(n - 1) + \dots + b_Nx(n - N + 1) \quad (1)$$

Where b_k represents the filter coefficients. The output $Y(m)$ can also be given in terms of the input signal $x(m)$ of order m whose transfer function is given as:

$$H(z) = \sum_{n=0}^N h(n)z^{-n} \quad n = 0, 1, \dots, N \quad (2)$$

Where, N is the order of filter and $h(n)$ is filter impulse response. The coefficients of $h(n)$ are symmetrical and thus, only $N/2+1$ number of $h(n)$ coefficients are needed to be optimized. The frequency response of the filter is given as:

$$H_d(e^{j\omega}) = \sum_{n=0}^N h(n)e^{-j\omega n} \quad (3)$$

Where, $h(n)$ represents the impulse response of the filter. For low pass filters, the frequency response is given as

$$H_i(e^{j\omega}) = 1 \text{ for } 0 \leq \omega \leq \omega_c \quad (4)$$

Where, $h(n)$ represents the impulse response of the filter. The frequency response is given for a low pass filter: Where ω_c is the cut-off frequency. The designing of an FIR filter is based on approximating the ideal filters according to the given designing conditions. For optimal design of digital FIR low pass filter, we use specific steps. These steps can be pictorially given as,

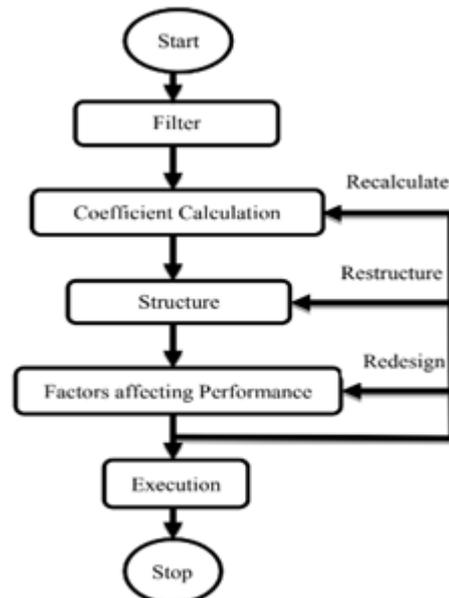


Figure 4: Design stages of digital FIR filters. [3]

- Filter Specification: This is the preliminary step used for deciding the type of filter, amplitude and phase requirements, sampling frequency, etc.

Table 1: Summary of frequency sampling methods

Methods	Comments	Pros and Cons
Bellanger's method	Design the filter parameter performance under the control stop band constraints and meet the Nudist standard.	Pros: Favorable stopband attenuation. Cons: Long filter length (long delay).
Viholainen's method	Optimize the filter parameters according to the following conditions. The evaluation criteria of different conditions are Nudist criteria.	Pros: Good stopband attenuation; flexible selection of appropriate filter parameters according to different evaluation standards. Cons: Need longer filter length.
Cruz-Roldán's method I	Frequency-based optimization scheme recommends sampling to obtain filter parameters.	Pros: low computational complexity; Design a filter of any length. Cons: It is difficult to select the sampling value of the transition zone to achieve fast convergence.
Cruz-Roldán's method II	It is recommended to improve the objective function based on the optimization algorithm to achieve multi-objective optimization.	Pros: The stop band energy and stop band attenuation are minimized at the same time. Cons: The initial parameters need to be appropriately selected to ensure performance.
Salcedo-Sanz's method	VLEP algorithm is based on classic and fast-evolving programming algorithm.	Pros: Can adapt to the initial conditions; the minimum value of the objective function can be obtained reliably. Cons: High computational complexity.

- **Coefficient Calculation:** The coefficients present in the transfer function, $H(z)$ are determined. They must follow the filter specifications.
- **Structure Realization:** The calculated coefficients result in a transfer function, which is then translated in a structure.
- **Factors affecting Performance:** Here, we study different factors that affect a filter's performance and increase the error percentage between the ideal and designed filter.
- **Execution:** This is the final step where the filter is tested and practically checked.

1.3 Reconfigurable digital FIR filter architectures

Digital FIR filters are used in mobile communication and multimedia applications, mostly for channel equalization, matched filtering, pulse shaping, videoconvolution functions, and signal preconditioning. The disadvantage of using digital FIR filters is that it involves many computations to process a signal. The proposed LUT based multiplier removes the need of using decoders in the FIR filter design. Also the complexity of the multiplier in a FIR filter is reduced using the memoryless computation. The multiplier block in memory based computation multiplier avoids the computation to obtain the possible product value [9]. Another new architecture has been demonstrated with a Virtex 2 xc2vp2-6fg256 FPGA with a precision of 8-bits, 12-bits, and 16-bits filter coefficients where a FIR filter architecture using dynamically reconfigurable multiplier block offers good area and speed improvement compared to existing reconfigurable FIR filter implementations [10]. M. Tirumala et al. showed a comparison between two different architectures, which are constant shift method (CSM) and vertical-horizontal binary common subexpression elimination (VHBCSE) for low complexity [11].

1.4 Memory Based FIR Filters

The memory-based filter approach uses physical memories such as RAMs and ROMs that store pre-calculated values for further operations. Memory-based FIR filters consequently are getting considerable acceptance in the DSP applications. These filters' results, performance throughput increases, and memory access latency are reduced comparing data computational time. They have much less

dynamic power consumption due to minimal switching activities associated with obtaining the output product/inner product values by memory read operations. There are two types of memory-based FIR filters. One technique is the direct memory-based implementation of FIR filters, while the other is based on distributed arithmetic (DA). In the direct-memory-based deployments [12], the multiplications of input values with the fixed coefficients can be replaced by a ROM or look-up-table (LUT), which contains the pre-computed product values for all possible values of input samples. Two new approaches are suggested for designing the LUT for LUT-multiplier-based implementation, where the memory size is reduced to nearly half of the conventional method [13].

On the other hand, DA is a bit-serial operation that implements a series of fixed-point MAC operations in a fixed number of steps, regardless of the number of terms to be calculated. DA is often preferred since it eliminates the need for hardware multipliers and can implement large filters with very high throughput. Croisier et al. had proposed the DA algorithm for digital filter implementations in 1973 [14]. The memory requirements ($2N$) of DA-based implementation for FIR filter increases exponentially with the filter order N . With the use of offset binary coding (OBC) the memory size can be reduced by half to $2N-1$ words [15].

2. FIR Prototype Filter Design

There are some design standards that can customize the performance of FIR, and there are some specific FIR design methods that correspond to these standards. Filter design methods are divided into four categories: frequency sampling methods, window-based methods, optimization-based methods and evolutionary optimization methods [16]. For each method, we will introduce their basic concepts and then discuss some typical design examples.

2.1 Frequency sampling methods

FIR filter design's frequency-sampling method is perhaps the simplest and most straightforward technique when a desired frequency response has been specified. It consists simply of uniformly sampling the desired frequency response, and

performing an inverse DFT to obtain the corresponding (finite) impulse response. However, the results may not be optimal because the response usually diverges from the samples and, it is essential to evaluate the final impulse response via a simulated DTFT (FFT with lots of zero padding) compared to the desired frequency response [16] initially.

The performances of different prototype filters using frequency sampling techniques measured by PSD and BER respectively. It was proved that the frequency sampling methods can achieve good PSD performance. The prototype filter designed by Bellinger's method shows the best performance compared to other frequency sampling techniques. Generally, a small number of parameters are simply designed in the frequency domain. Frequency sampling methods are inclined to make a narrowband filter design where a few non-zero values are needed. Furthermore, there are few side-lobes techniques that their resulting impulse response should be time aliased [17].

For time complexity, Bellanger's method has reached an order of magnitude $O(N \log N)$ as most of the other methods are its improved versions, and N denotes the Filter length. When the value of N increases, low sidelobes may appear in the frequency domain, and the interference of adjacent channels can be reduced. Therefore, while requiring low interference, frequency sampling methods can be considered. In addition, frequency sampling technology can meet NPR conditions. Research projects on 5G-related networks such as PHYDYAS and 5GNOW have considered frequency sampling technology modulated by FBMC. The frequency sampling method is also suitable for GFDM modulation. However, the frequency sampling method usually has a longer filter length, which leads to a longer waiting time, that is, a trade-off between high spectral efficiency and low waiting time. Moreover, the location of frequency control This point is limited by N sampling points in the frequency domain. This means that the sampling frequency can only be an integer multiple of $2\pi/N$, which makes it difficult to control the cutoff frequency of the filter. If you freely choose the cutoff frequency, you must increase the number of sampling points N , that is, increase the length of the filter, which is not conducive to short uplink burst communication in the 5G scenario.

2.2 Window-based method

PSD and BER's performances about different prototype filters using windowing-based techniques while the method in Reference performs relatively poorly. Compared with other methods. The window-based FIR filter is designed in the time domain. The basic idea is that the linear phase FIR filter is multiplied by a specific window function, so the time complexity is proportional to $O(N)$. Generally, window-based technology can be regarded as an improved version of the classic window function, but compared with the traditional window function, the performance improvement is not very obvious. Methods such as Hamming window, Henning window, from the perspective of actual use, window-based functions such as the Blackman window are easy to implement, and the computational complexity burden on the system is small. Therefore, they can be used

for prototype filter design in almost all multi-carrier modulation systems. However, it is difficult to accurately control the passband cutoff frequency of the filter [18]. The windowing process leads to a truncation effect and therefore a transition zone. Although increasing the length of the window can reduce the transition zone, the fluctuation range cannot be suppressed due to the Gibbs phenomenon. Changing the shape of the window can reduce the passband/stopband attenuation, but at the expense of increasing the width of the transition band.

2.3 Optimization-based methods

Using optimized technology, the performance of PSD and BER of different prototype filters. In recent years, evolutionary algorithms have been used in linear phase FIR filter design and can be considered in multi-carrier modulation applications. Five different evolutionary algorithms were used to simulate the FBMC-based system. By analyzing the filters' simulation with the same order, it is noted that different evolutionary algorithms achieve similar PSD and BER performances. Compared with frequency sampling techniques and windowing based techniques, the corresponding filter parameters of optimization-based methods, that are more concerned with the local optimization and algorithm convergence problem can be optimized depending on different evaluation criteria. The selected design parameters determine the effectiveness of the solution to the optimization problem.

The complexity of this type of design technique seems to be much higher than other two techniques. For instance, the PSO-based method's complexity is about the order of LPO (N^2), where L is the iteration size and P is the population size. This complexity is much less than QCQP based methods or non-convex optimization methods, of which the complexity is up to $O(N^3)$ or higher. Nevertheless, optimization-based methods are much more flexible as they focus on local optimization to support the constraints imposed by various scenarios such as cognitive radio, massive MIMO, etc. For example, when FBMC is applied for opportunistic spectrum sharing, to avoid interfering with other bands, well localized filters in time and frequency are prone to be designed to minimize Out of Band [19].

Similarly, there are some optimization-based methods for different waveform candidates, such as OFDM, FBMC, GFDM, and UFMC. Generally, the establishment and constraints on the objective function are usually highly nonlinear, which increases the computational complexity and sensitivity to selecting initial values. It is also possible to fall into the local optimal range and fail to identify optimal global solutions. This is one of the main problems to be solved based on the optimization design method. As mentioned earlier, because multi-carrier modulation can expand the system capacity and has relative immunity against multipath fading effects, it has become the mainstream of future communication technology. The key part of multi-carrier modulation is the FIR filter design. A good filter design can improve some important performance, including some key aspects such as PSD and BER, so as to improve the competitiveness of waveform candidates and improve the communication quality to adapt to various application scenarios of 5G or future networks [20]. FIR

filters have been widely used in many engineering applications involving Digital signal processing, such as communication signals, voice signals and medical signals. We can conclude without exaggeration that in the presence of FIR shadows, digital signal processing is required.

2.4 Evolutionary optimization methods

Recent times have seen a broad application of evolutionary optimization, based on frequency domain requirements, by researchers to design digital FIR filters. In the field of evolutionary optimisation-based FIR filter design, substantial growth has been recorded. To solve the filter design problem, optimization-based techniques are used by framing the design task as an error function, which is further solved to evaluate the filter coefficients that match the desired requirements. The nonlinear, non-differentiable, non-convex, multimodal nature of the associated optimization problem makes the task of design very difficult. [21].

2.5 Hardware efficient FIR filter

There is no doubt that in the past 30 years, the field of efficient FIR filter design has received many valuable contributions from researchers. It seems appropriate to summarize the concepts used over the years to solve this problem. Based on this goal, this work provides a detailed review of the evolution of the hardware efficient FIR design process. Section 5.1 describes the growth of mathematical programming in the design of linear phase powers-of-two FIR filters. The application of intelligent optimization techniques in multiplierless FIR filter design has been thoroughly investigated in Section 5.2. Section 5.3 accumulates all such approaches focusing on minimizing adder in hardware high-efficiency FIR filter design. Section 5.4 introduces the design strategy of two-dimensional hardware high-efficiency FIR filter. In Section 5.5, an extensive survey of the coefficient representation scheme was conducted.

2.6 Design of linear-phase powers-of-two FIR filter

The design of FIR filters in discrete power-of-two coefficient space has always been a part of active research. The first written article based on the field of signal processing was Lim et al., where the discrete coefficient was selected using integer programming. The frequency response $H(\omega)$ of any FIR filter of length N can always be expressed as a trigonometric function of the frequency variable ω .

$$|H(\omega)| = \begin{cases} h\left(\frac{N-1}{2}\right) + 2 \sum_{n=0}^{\frac{N-3}{2}} h(n) \cos\omega\left(\frac{N-1}{2} - n\right), & \text{for odd } N \\ 2 \sum_{n=0}^{\left(\frac{N}{2}\right)-1} h(n) \cos\omega\left(\frac{N-1}{2} - n\right), & \text{for even } N \end{cases} \quad (5)$$

Magnitude part of the response has been illustrated and the symmetricity of the impulse response $h(n)=h(N-1-n)$ is assumed in equation (5). The resulting phase response for both odd and even N may therefore have a form like:

$$\Theta(\omega) = \begin{cases} -\omega\left(\frac{N-1}{2}\right), & \text{if } H(\omega) < 0 \\ -\omega\left(\frac{N-1}{2}\right) + \pi, & \text{if } H(\omega) > 0 \end{cases} \quad (6)$$

The filter design problem is nothing but obtaining the set of coefficients $h(n)$ such that $H(\omega)$ is the best approximation to some desired function $D(\omega)$ concerning some optimality criterion. During their design using minimax strategy, the value of $H(\omega)$ is subject to the following constraint

$$D(\omega) - \delta k(\omega) \leq H(\omega) \leq D(\omega) + \delta k(\omega) \quad (7)$$

The $\delta k(\omega)$ in equation (7) represents the ripple to be minimized. One possible criterion for optimizing the filter coefficient $h(n)$ is to reduce the output error power. This can be expressed as

$$|E(\omega)|^2 = |V(\omega)|^2 |D(\omega) - H(\omega)|^2 \quad (8)$$

In the above formula, $E(\omega)$ and $V(\omega)$ represent the frequency spectrum of the error signal and the input signal, respectively. According to Parseval's theorem, the above equation can be regarded as the minimum value of variable J , as shown below:

$$J = \int_0^\pi |V(\omega)|^2 |D(\omega) - H(\omega)|^2 d\omega \quad (9)$$

The most convenient way to represent the coefficients of a hardware-friendly FIR filter is the power of two (SPT) diagram. For the fixed word length B of a DSP processor, the impulse response coefficient may have its general form like:

$$h(n) = \sum_{i=1}^B s_i 2^{-i} \text{ With } s_i \in \{-1, 0, 1\} \quad (10)$$

Some of these illustrations are provided in the literature, which usually focus on reducing hardware complexity. To this end, a mixed integer linear programming (MILP) technique was used wisely, which was developed to minimize the number of SPT terms for a given filter specification. The representation with the fewest SPT terms is the Canonical Symbolic Digital Code (CSDC) representation, where two SPT terms cannot be adjacent. The SPT representation in formula (6) can have the following alternative forms:

$$h(n) = \sum_{i=1}^B (S_{nj}^+ - S_{nj}^-) 2^{-i} \text{ with } S_{nj}^+, S_{nj}^- \in \{0, 1\} \quad (11)$$

The introduction of equation (11) has made the optimization, the objective function is linear, and it is possible that the system designer desperately tries to limit the number of SPT terms per coefficient.

Design of FIR filters with the sum of SPT coefficients using the Lu proposed the semi-definite programming (SDP) problem that can be solved by polynomial time using an efficient SDP solver, the integer programming method has been relaxed for a long time. SDP is only a constrained optimization problem, in which the linear objective function is minimized under the constraints of a matrix that closely depends on the variable vector h . The typical form of the SDP problem is as follows: Cth, where

$$C(\omega) = [1, \cos\omega, \cos2\omega, \dots, \cos(N-1)\omega]^T \quad (12)$$

$$\text{Subject to: } F(h)E + \sum_{i=1}^r h_i F_i \succeq 0 \quad (13)$$

In the above formula, the matrix F_i with $0 \leq i \leq r$ is symmetric and represents a positive semidefinite. Recently, MILP simplified the discrete coefficient FIR filter, and then solved the problem through B&B technology. The filter design problem has been expressed as a minimization problem, such as:

$$\text{minimize } \gamma, \quad (14)$$

$$|H(\omega)| \leq \gamma \text{ for } \omega \in [\omega_s, \pi] \quad (15)$$

$$\text{Subject to: } |H(\omega) - 1| \leq \delta \text{ for } \omega \in [0, \omega_p] \quad (16)$$

γ represents the maximum allowable fluctuation of $H(\omega)$ in the passband and stopband regions of interest. The author pointed out the problem of minimizing the triangular semi-infinite constraint (TSIC). According to the Markov-Lukacs theorem, the linear TSIC in the variable $h \in \mathbb{R}^{N+1}$ can always be changed to a non-negative trigonometric polynomial. The filter design problem can therefore be expressed as:

$$A_i + h d_i \in C_i^* \quad (7), \quad i = 1, 2, 3, 4 \quad (17)$$

$$\text{With } A_1 = -A_2 = A_3 = -A_4 = I, d_1 = (\delta_p - 1, 0, \dots, 0)^T,$$

$$d_2 = (\delta_p + 1, 0, \dots, 0)^T, d_3 = (\delta_p, 0, \dots, 0)^T$$

Equation (17) introduces a new term C_i^* , derived from C_i , which is known to describe the triangle curve of TSIC and its polarity are opposite. Their mathematical illustrations are as follows:

$$C_{ab} = \{c(\omega): \cos \omega \in [\cos a, \cos b] \in \mathbb{R}^{N+1}\}$$

$$\text{Where } C_{N(\omega)} = (1, \cos \omega, \cos 2\omega, \dots, \dots, \dots, \cos N\omega)^T$$

$$C_{ab}^* = \{u: (u, v) \geq 0 \forall v \in C_{ab}\} \quad (18)$$

2.7 Optimization techniques of a multiplierless FIR filter

Several researchers as an optimization problem have discussed the successful design of the multiplier-less powers-of-two FIR filter. To solve the problem, some mathematical optimization algorithms such as MILP and SDP have been carefully employed. Due to its resourceful amalgamation with artificial intelligence, the last decade of the twentieth century is regarded as having a major effect on signal processing. One simulated annealing algorithm (SA) used to design linear phase quadratic numbers filters was proposed by Benvenuto and his co-researchers. New features concerning conventional SA algorithms have also been introduced to reduce computational complexity. The entropy-directed deterministic annealing (EDDA) optimization algorithm has been proposed to design digital filters with discrete coefficients [4] as an attempt to combat SA's large computation time. It uses conditional entropy estimates to prune the issue during optimization, thus reducing the computational time by 30 to 50 percent. The SA definition has recently been extended to maximizing the sum of powers-of-two by minimizing the total number of non-zero digits of the FIR coefficients. In addition to dealing with traditional filter requirements such as in-band ripple and stopband rejection, also in the transition band, additional uncommon shape constraints have also been considered. Besides, quite a few evolutionary and swarm optimization techniques have shown their competence in replacing many of the conventional optimization mechanisms that often fail

to work adequately in many engineering problems. Multiplierless FIR filter architecture has also been heavily affected by evolutionary optimization techniques; the most common genetic algorithm (GA) is among them. In connection with this, by using simple genetic operators such as replication, crossover and mutation to check for the discrete coefficient space of predefined power-of-two coefficients, Comes and Ait-Boudaoud initiated the GA-based power-of-two FIR filter design problem. Their strategy has outperformed conventional methods that limit their coefficients to single power-of-two terms. Two years later, by implementing a particular filter coefficient coding scheme, Gentile and his coworkers had thrown enough light on the same issue. The authors claim that their proposed method can achieve better or almost comparable results than other methods of interest (such as MILP, simulated annealing (SA), Parks McClellan method, proportional relationship retention (PRP) method). Due to its implicit parallel nature, GA-based methods can explore many possible solutions in each generation, and can be easily implemented in parallel machines. Design of a high-speed low-power FIR filter has also been facilitated by GA where the required goal has been achieved by factoring a long filter into several cascaded sub filters each with coefficient values constrained to the sum of SPT. With the help of GA, the filter can be implemented in a signed power-of-two under the condition that it is close to the global minimum and the hardware cost is low. Novel GA has recently been proposed for the design of multiplier-less linear phase FIR filters in both single-stage and cascade forms. Based on passband gains, the discrete search space is partitioned into smaller ones and the search efficiency has been enhanced by adaptively changing the crossover and mutation rate. The algorithm uses the filter's adder cost as the objective function, unlike the traditional GA, and penalties are applied when ripple criteria are not met. The idea proves to be selfish in terms of construction time and, in most situations, the hardware cost is saved. FIR filter optimization over the GA-based CSD coefficient space has been established. In combination with traditional crossover and mutation operators and a new local mutation operator, the proposed optimization technique exploits CSD numbers restoration. The GA application has been presented to optimize the filters created by the FRM technique. It has been shown that, as obtained from the linear optimization technique, GA is capable of producing better discrete coefficient solutions and is very similar to the continuous solution obtained from non-linear optimization techniques. Another new genetic algorithm for the design and discrete optimization over traditional CSD of FRM FIR optical filters and a new double base number system (DBNS) multiplier coefficient space is introduced. The proposed genetic algorithm is based on a pair of allowable CSD/DBNS number index lookup tables, whose indexes form a closed set under crossover and mutation operations. It will automatically lead to legal CSD/DBNS coefficients without any genetic repair during the optimization process. Finally, it has been successfully applied to design a pair of low-pass and band-pass FRM FIR digital filters. The author has proved that through appropriate design examples, in some cases, the performance of the final optimized CSD/DBNS filter is better than the corresponding infinite precision FRM FIR digital filter. Traditional GA (CGA) has proven to be a

potential search tool, multiplierless FIR filter design, since the repetitive evaluations of a large population of candidate solutions are relatively low, it needs comparatively huge computational time. Cen and Lian, who had introduced a new variant of GA, known as micro-GA (μ GA), into the same design problem, later addressed this issue. For its implementation, the μ GA-based algorithm needs small population sizes, making the convergence speed of μ GA is faster than that of CGA. However, due to the small population, μ GA may fall into a local optimum. This problem has been solved by appropriately modifying μ GA by changing the probability of crossover and mutation during evolution, so it is called modified μ GA. The author claims that compared with CGA, the modified μ GA can significantly speed up the optimization process. Experimental research has substantiated this because updated μ GA is around seven times faster than CGA and provides a better solution than the design based on MILP. In cascade type multiplierless FIR filters, a new version of GA, called an orthogonal genetic algorithm (OGA)[22], was implemented in cascade form multiplierless FIR filters, which explored two objective functions based on a single and multiple criterion for amplitude response. The OGA approach leads to an improved amplitude response compared to an analogous direct-form cascade filter obtained using the Remez exchange algorithm, the authors have argued. Experimental experiments have substantiated this because the revised μ GA is about seven times faster than CGA and offers a better solution than the MILP-based design. A new version of GA, called an orthogonal genetic algorithm (OGA)[22], was implemented in cascade form multiplier-less FIR filters, which explored two objective functions based on a single and multiple amplitude response criterion. Compared to an analogous direct-form cascade filter obtained using the Remez exchange algorithm, the OGA approach leads to an improved amplitude response, the authors have argued. [23]. AGA serves as the basis of the hybrid algorithm with varying population sizes and various probabilities of genetic operations. The usage of SA is to help AGA escape and avoid premature convergence from the local optima. By reducing search space according to FIR filter coefficients' properties, the definition of tabu has been implemented to speed up convergence. Unlike the other GA, the GST approach increases the efficiency and decreases the machine effort. With some advanced computational algorithms that have outperformed GA and its various variants in several benchmark issues, the FIR filter's power-of-two architecture has recently been achieved. Some differential evolution (DE) algorithm was used to design a multiplier-less, the [24]. Power-of-two coefficient FIR filter. The influence of different DE mutation strategies was subsequently studied in the design process, and a new self-adaptive DE algorithm was also proposed for the purpose of the design. The same problem was later solved through the genetic algorithm of self-organizing random immigrants (SORIGA), and its superiority over previous design strategies was developed. [25]. With some sophisticated computational algorithms that have outperformed GA and its numerous variants in several benchmark issues, the FIR filter's power-of-two architecture has recently been achieved. To design a multiplier-less [26], the differential evolution (DE) algorithm was used. Power-of-two coefficient FIR filter. The effect of various DE mutation

techniques was subsequently analyzed in the design process, and a new self-adaptive DE algorithm was also proposed for the benefit of the design. The same issue was later solved through the genetic algorithm of self-organizing random immigrants (SORIGA), and its superiority go beyond previous design strategies was developed [27]. The design of the non-uniform filter bank multiplexer has been realized, in which the filter coefficients are synthesized in CSD format, and the ABC algorithm has been used for optimization. The simulation results show that the performance of the algorithm is better than that obtained by the rounding algorithm. filter's continuous coefficients to the nearest CSD number.

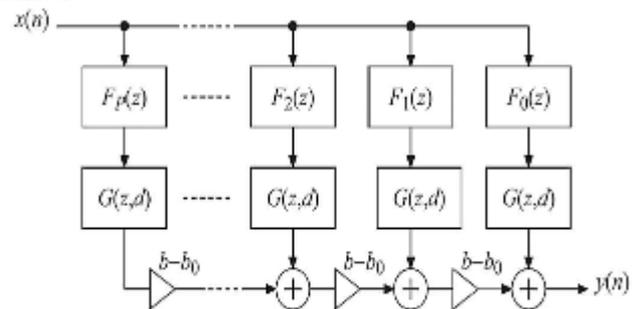


Figure 4: Block diagram of multiplierless FIR filter with simultaneously variable bandwidth [21]

2.8 Hardware proficient FIR filter design

The complexity of the design due to the implementation of a non-recursive digital filter without any built-in multiplier in embedded custom or semi-custom circuits is often measured in terms of the number of additional operations used to perform the multiplication process. To mitigate this complexity, circuit designers have used CSD representation for a long time for this purpose. 1995 marked a major advancement in circuits and systems, and many researchers developed their innovative ideas in constructing filters to minimize adder costs. Some of them have also demonstrated that multiplier blocks to leverage coefficient redundancy result in a large decrease in the sophistication of CSD representation, which is less complicated than regular binary representation. Three such modern algorithms were then proposed, consisting of a good modification of an existing algorithm, a new improved results algorithm, and a hybridization of both of these algorithms that exchange performance against computation time. The authors explored the shortcomings of the popular BH algorithm by Bull and Horrock, which used multiplier blocks to minimize the expense of implementing FIR filters [28]. Compared with the original design, the performance of Bull and Horrocks produced the same results. The original design used several single-coefficient multipliers with fewer adders and subtractors, so that all products of input samples can be generated simultaneously. The shortcomings and the related solutions that have greatly increased the findings obtained are readily accessible. Dempster and Macleod also implemented n-dimensional simplified adder graph (RAG-n) algorithm as part of their main contribution to reducing adder costs, which is split into two parts [29]. The first section is optimal because it guarantees minimal adder costs given the set of coefficients has been thoroughly synthesized by this part of the algorithm. The second part heuristic algorithm uses two MAG algorithm-generated look-up

tables, covering a range from 1 to 4096. The cost lookup table contains the optimal single-coefficient multiplication cost for each coefficient value, specified range, and the simple look-up-table provides various sets of fundamentals that can be used at optimal cost to enforce multiplication. It has been well known that the algorithms BH [30] and modified BH (BHM) are compared with the RAG-n algorithm, the speed of small-size sets is much faster. It has been proved that compared with the well-known BH algorithm, the heuristic RAG-n multi-coefficient cost multiplier block design algorithm increases the length by about 20% on average in the contribution of five coefficients in the 12-bit word-related field. The BHM algorithm [31] is considered to be less efficient than RAG-n because of its higher cost. Compared with the 12-bit word length BH algorithm and hybrid algorithm, its efficiency is even improved by 10%. However, for small coefficient sets, RAG-n algorithm is slower than BHM is slower than BHM, but for large sets it is faster, in which case the computing time for BH and BHM has a growth rate of square law with a set size relative to linear growth for RAG-n. In the same year, one approach was proposed by Li in order to reduce the complicity of fixed-point multipliers with fixed or programmable multiplicands, which received considerable interest in the related sector. Their solution is to figure out the minimum number of adders for a multiplier of a given multiplicand to be applied. CSD words were typically used for multiplicands that had been particularly difficult prior to this paper's proposal by the proposed minimum number of shift-add operations (MNSAO) as far as the number of adders in the structure is concerned [32]. The MNSAO expression greatly increases the largest representable contiguous range and the number of representable integers in a given range, thus reducing the mean approximation error relative to CSD expressions under no more than the same number of shift-add operations (SAOs). Therefore, the concept of multiplier-less digital filters was subsequently implemented, subject to certain pre-specified implementation costs calculated. The total number of adders in the entire filter. The results show that the filter designed in Reference 70 is significantly better than the filter designed through MILP programming and simulated annealing (SA), which stipulates that the number of SPT terms for each coefficient does not exceed two. Li et al. The research in [33] shows that the designed filter can reach 4.2 dB Smaller normalized peak ripple (NPR). Use optimized conversion to the total number of adders in the entire filter. The results show that the filter designed in Reference 70 is significantly better than the filter designed through MILP programming and simulated annealing (SA), which stipulates that the number of SPT terms for each coefficient does not exceed to Li et al. The research in [33] shows that the designed filter can reach 4.2 dB Smaller normalized peak ripple (NPR). Use optimized conversion to decrease the number of additions and subtractions for a given range of filter coefficient values and coefficient representation schemes is seen in another promising paper. The number of additions has been reduced by removing the typical sub-expressions in the coefficients' binary representation for a direct form FIR filter structure. In the transposed form, the reduction of adders. The authors have also taken care of the MCM-based FIR filter by updating. They have already proposed the algorithm. It has been clearly shown that by

merging the CSE algorithm, the total number of addition and subtraction operations for direct structures is reduced by 35%, and the total number of subtraction operations for transposed structures is reduced by 38%. In fact, the total number of additions and subtraction operations has been reduced by an average factor of 2.2 compared to 1.43, as achieved in Reference. Pearson and Parhi proposed a novel approach to the design of low power FIR filters by parallel or block processing with hardware replication. At the expense of doubling the number of adder elements, they have accomplished a significant decrease in multiplier elements. However, because the area required to incorporate a multiplier element is considerably greater Compared with adder components, reduced multiplier implementations can reduce hardware costs and reduce power consumption usage. Following this, an adjacent coefficient sharing based substructure sharing approach was subsequently used to reduce the hardware expense of parallel FIR filters along with the maximal absolute difference quantization technique. Based on the examples presented, the authors have demonstrated that their solution results in a 45 percent reduction in hardware costs relative to conventional parallel filtering methods. The system algorithm proposed by Kaakinen and Sara maki found a suitable place in the literature[34]. In the optimization process, a linear programming algorithm is initially used to evaluate the parameter space of infinite precision coefficients and the feasible space of the filter to meet the specified amplitude requirements. The second step is to locate the filter parameters in this space in this way, so that the simplest coefficient representation type is used, and as a result the filter satisfies the condition. Compared to other current approaches, the key benefit associated with the method is that it seeks all the solutions to reach the criteria of magnitude. Since multiplier blocks' complexity was substantially reduced by introducing techniques such as decomposing multiplication into basic shifts and addition operations and exchanging common sub-expressions, the multiplier latency was reduced. Under the specified delay constraints, Kang and Park have proposed new algorithms to minimize multiplier blocks' complexity. To incorporate filters that can fulfill the number of adder-step criteria, authors have merged three suggested BHM, and RAG-n algorithms approaches. By altering the delay constraints, a trade-off between delay and hardware complexity is allowed. Experimental studies have shown that the algorithm can minimize the multiplier block delay at the expense of a slight increase in complexity. It took several years for researchers to decrease the word length of the coefficient actively and the amount of non-zero bits in the filter coefficients to reduce the adder step. The authors have changed the filter coefficient representation so that the number of full-adders arising from the application of the hardware is equal to just the signal word length product and the number of adders. The use of this new algorithm results in positive results for filters of up to 500 taps. Authors also showed the superiority of their suggested strategy over certain current ones in terms of multiplier block adders and multiplier block total adders. More specifically, although the proposed algorithm reduces the number of MB adders by 25% to 44%, the achieved result is as high as 67%. However, the new algorithm called HCUB has been developed to boost outcomes over RAG-n[35]. Both are

algorithms of the adder graph, separated into two phases, an optimal part and a heuristic part. It is possible to consider the heuristic part as adding additional coefficients to be realized so that the simple operation can proceed in the optimal part. It is specifically claimed that the HCUB algorithm seeks solutions that need up to 20% fewer additions and subtractions than the solutions defined by the best algorithm previously known, such as RAG-n and BHM. With a novel heuristic inspired by various algorithm groups, an adder graph style algorithm has been implemented to solve the MCM problem. It does not depend on look-up tables for its execution, unlike the previous algorithms. The suggested heuristic has been shown to have better or equivalent outcomes than RAG-n. The algorithm is marginally higher on average for most of the conditions compared to HCUB. Popular subexpression sharing is quite fruitful during the optimization of the multiplier-less filter coefficients. With mutual shifter adders, the coefficient multipliers are represented as a multiplier block (MB). As long as MBs' power consumption is concerned, not only the overall number of adders is concerned, but even the adder depth of each coefficient requires a major contribution. To maximize the filter coefficients subject to the depreciation of ripples in the filter frequency response and a restriction on the total number of adders and the permitted maximum adder depth, a MILP based technique was used a few years ago. Via a design illustration that shows that the proposed algorithm produces filters using less adders with minimal adder depth than the solution, authors have identified their proposed algorithm's superiority.

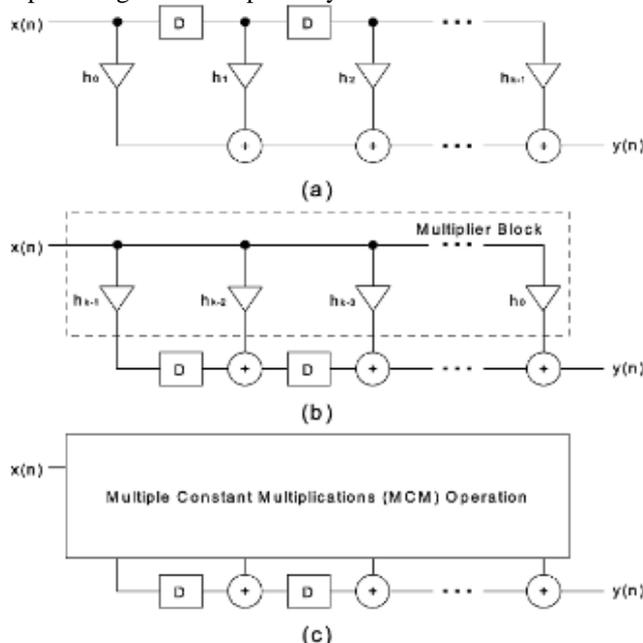


Figure 6: FIR filter implementations. (a) Direct Form (b) Transposed Form (c) Transposed Form with MCM Block

For FIR filter implementation, a truncated MCM using pattern modification technology (PMT) has recently been developed [36]. This algorithm will truncate any node adders generated by various MCM algorithms in DAG, and has a universal concept of ensuring the same weight for every two inputs to the same node. PMT supremacy has been developed because it reduces the area cost by 35 percent relative to non-truncated MCM algorithms without raising the quantization error [6].

2.9 Two-dimensional hardware efficient FIR filter

Over the last few decades, researchers have also paid serious attention to the nature of two-dimensional multiplier-less filters. Since 1987, Pei and Jaw have taken a pioneering move, that is, when a special type of coefficientless one-dimensional filter is used as a coefficient to represent a 2D coefficientless optical FIR filter, the coefficient is the number of 2 or the difference of the power, Great changes have taken place in this regard. area. The author has integrated McClellan transform, which can map a one-dimensional filter to a two-dimensional filter. In terms of the hardware implementation of these filters, they are very attractive for high-speed operation, efficient and reliable computing. The framework proposed by Pei, however, is true only for the original McClellan first-order transformation. In conjunction with this, Kwan and Chan have proposed a new theoretical approach to evaluate the first-order McClellan transformation [37]. The author also defined the change from the theoretical method to the original method by comparing the findings with those of the original first-order McClellan transformation. For the construction of 2D linear phase FIR optical filters, the use of a generalized McClellan transition of more than one order has been seen. To optimize the transformation width of 1D FIR filters according to the 2D frequency domain's inequality constraints, the architectural problem is formulated as a linear programming (LP) optimization problem. Finally, local search methods have been implemented to effectively locate the required square coefficients [38]. This optimization algorithm avoids the pitfalls of heavy computational cost and huge memory storage caused by traditional LP-based algorithms. Aiming at the properties of circular symmetrical broadband and multiple 2D FIR filters, three simple and effective conversion methods are proposed [39]. The first transformation was assumed to be the k th order variant of the original McClellan transformation, and two other transformations were generated on the basis of the McClellan transformation of the k th order. The diagrams presented have thoroughly highlighted the feasibility and versatility of the transformations proposed. The author also concludes that this strategy has resulted in substantial savings in the number of multiplications compared to other conversions, but at the expense of a slightly larger adder and delay. For 2D FIR digital filters with limited precision coefficients and linear processes, an optimized minimax architecture is developed. This algorithm combines linear programming and branch and bound methods, and consists of depth-first search and breadth-first search. The two methods are compared, namely depth-first search and hybrid strategy. In order to illustrate the usefulness of the system for designing 2D filters with various parameters and sizes, several design examples are given. For the minimal and maximal design of a two-dimensional multiplier-free FIR filter whose coefficients have been written as the sum or difference of two binomial powers, a design technique based on simulated annealing (SA) is proposed. The algorithm turned out to be fundamentally very versatile. In the form of video filters, a variety of filter architecture samples have shown the technique's usefulness. The Minimax architecture problem with continuous and discrete coefficients of two-dimensional linear phase FIR filters was later defined. The

writer initially regarded the problem of the maximum-minimum continuous coefficient structure as an LP subject with inequality constraints. An effective method for constructing 2D filters with power-of-two coefficients in the spatial domain was proposed based on the obtained continuous coefficients. In the 2D multiplier-less filter

design process, several artificially intelligent optimization strategies have found their appropriate use. Related to this, the first paper was published in 1995, when Sri Ranganathan and his colleagues designed two FIR filters with circular symmetry and diamond-shaped low-pass linear phase quadratic FIR filters. GA.

Table 2: Comparison and analysis of frequency response of hardware high-efficiency FIR filters

Method	Length of the filter	Transition-band attenuation (dB) at different frequency points (rad/pi)			Stop-band attenuation (dB) at different frequency points (rad/pi)			Minimum stop-band attenuation (dB)
		0.35	0.4	0.45	0.65	0.75	0.85	
Samueli	25	4.279	15.03	34.02	98.19	87.92	112.2	84.66
Chen and Willson	28	2.958	13.48	39.58	135.2	118.1	124.6	115.8
Kaakinen and Saramaki	29	1.002	4.239	15.76	45.8	50.33	53.6	30.28
Jheng, Jou and Wu	30	2.945	13.77	40.6	162.1	121	168.1	117.9
Xu, Chang and Jong	28	3.732	13.8	38.3	150.2	116.7	89.06	80.25
Feng and Teo	34	2.41	13.71	44.2	154.4	147	143	130.6
Chandra and Chattopadhyay	29	22.28	40.93	70.05	120.5	125.2	120.5	110.7

The author uses the minimax error criterion to minimize the weighted ripple in the passband and stopband. Related to this, the first paper was published in 1995, when Sri Ranganathan and his colleagues designed two FIR filters with circular symmetry and diamond-shaped low-pass linear phase quadratic FIR filters. GA. The author uses the minimax error criterion to minimize the weighted ripple in the passband and stopband. It has been found that the designed filter has better performance or comparable performance than the filter designed with the aid of LP and SA. Another powerful GA-based multiplier-free 2D state-space digital filter (SSDF) architecture method has been proposed, which is attractive for high-speed operation and fast implementation. The architectural problem is defined and formulated by Roesser's local state space model and depends on the stability of the final filter. In order to find the periodic shift variable (PSV) coefficients of the 2D filter, Thamvichai et al. have integrated two independent GA forms, namely binary GA and integer GA [40]. The specification involves finding the impulse response of a 2D PSV filter in closed form, and then using GA to find the coefficients of the filter. Another powerful GA-based multiplier-less 2D state-space digital filter (SSDF) architecture approach has been suggested, which is found to be attractive for high-speed operation and fast implementation. The architecture issue is defined and formulated by the local state-space model of Roesser, subject to the stability of the resulting filter. In order to find the periodic displacement variable (PSV) coefficient of the 2D filter, Thamvichai et al. have integrated two independent GA forms, namely binary GA and integer GA [41]. The specification involves finding the impulse response of a 2D PSV filter in a closed form, and then using GA to find the coefficients of the filter. Filters with good response characteristics have been developed by Adaptive GA while reducing the error requirements and Processor time substantially. In accordance with Singular Value Decomposition (SVD), GA is used to construct 2D FIR filters, and the function of GA is to optimize the design of one-dimensional filters. SVD can be enhanced by changing the order of the 1D filters in each branch according to its specific value. By reducing the number of coefficients by 20% and passing appropriate passband and stopband errors, this development has led to more efficient architectures. Recently, by designing a multiplier-free one-dimensional

linear phase FRM FIR filter and then performing a multiplier-free transformation, the design of a two-dimensional multiplier-free linear phase FIR filter has been completed. Convert the generated 1D filter into CSD space using a new discrete optimization based on the improved gravity search algorithm (GSA) and improved harmony search algorithm (HSA) [42]. GSA and HSA have been adapted so that candidate solutions turn out to be integrated throughout the optimization and effective discovery and utilization of the search field. A method with characteristics of lower computational complexity and time is given. With the help of DE algorithm, a new multiplier-free image filter architecture technology is introduced [43]. Therefore, use the built-in filter to minimize the influence of Gaussian noise from the standard test image, and analyze the relevant parameters to obtain the result. By comparing certain standards with other design methods, the authors assert that their design is supreme. A comparative analysis of evolutionary algorithms for 2D FIR filter design is established. It also explored several random methods that can handle large spaces. Finally, someone proposed a new genetic algorithm, in which certain ideas were adopted to maximize the trade-off between the genetic pool of diversity and elites.

This section aims to shed sufficient light from multiple viewpoints on the power-of-two FIR filter architecture's success and effect. The design process aims to achieve the optimal filter specification with the lowest possible hardware cost. Few output parameters such as passband ripple, transition and stopband attenuation, transition-band width, etc. have governed the engineered filter's frequency characteristics. The DEMLFIR filter's dominance can easily be defined as it yields a higher attenuation value in the frequency response transition band. On the other hand, in terms of the frequency functions' stopband behavior, it outperforms the other architecture algorithms by a wide margin. It greatly defeats multiple programming algorithms. However, in addition to this architecture, the remaining two second-order FIR filters create proper stopband behavior, that is, the minimum stopband attenuation value is always greater than 80 dB. In order to compare the hardware complexity of FIR filters without multipliers, similar indicators are measured as different orders per unit length of the filters.

References

- [1] Sharma, I., B. Kuldeep, A. Kumar, and V. K. Singh. "Performance of swarm based optimization techniques for designing digital FIR filter: A comparative study," *Engineering science and technology, an international journal* 19, no. 3 (2016): 1564-1572.
- [2] Dwivedi, Atul Kumar, Subhojit Ghosh, and Narendra D. Londhe. "Review and analysis of evolutionary optimization-based techniques for FIR filter design." *Circuits, Systems, and Signal Processing* 37, no. 10 (2018): 4409-4430.
- [3] Singh, Neelam, and Anjali Potnis. "A review of different optimization algorithms for a linear phase FIR filter." In *2017 International Conference on Recent Innovations in Signal processing and Embedded Systems (RISE)*, pp. 44-48. IEEE, 2017.
- [4] [4] Chandra, Abhijit, and Sudipta Chattopadhyay. "Design of hardware efficient FIR filter: a review of the state-of-the-art approaches." *Engineering Science and Technology, an International Journal* 19, no. 1 (2016): 212-226.
- [5] [5] Jiang, Lei, Haijian Zhang, Shuai Cheng, HengweiLv, and Pandong Li. "An Overview of FIR Filter Design in Future Multicarrier Communication Systems." *Electronics* 9, no. 4 (2020): 599.
- [6] [6] Jiang, Lei, Haijian Zhang, Shuai Cheng, HengweiLv, and Pandong Li. "An Overview of FIR Filter Design in Future Multicarrier Communication Systems." *Electronics* 9, no. 4 (2020): 599.
- [7] [7] S. Fu Hsiao and J. H. Zhang Jian, "Low Cost FIR Filter Designs based on faithfully Rounded Truncated Multiple Constant Multiplications," *IEEE Trans. on Circuits and Systems-II: Expression Briefs*, vol. 60, no. 5, pp.287-291, May, 2013
- [8] [8] D. L. Maskell, "Design of Efficient Multiplierless FIR filters," *IET Circuits Device System*, vol.1, no. 2, pp. 175-180, May 2007.
- [9] [9] L.MAZHER IQBAL et.al "New Approach to Memory Less Design and Look-Up-Table Realization for Low-Complexity Reconfigurable Digital FIR Filter Architectures", *WSEAS TRANSACTIONS on systems* Feb 13 2013.
- [10] [10] Iqbal, Mazher&Sourirajan, Varadarajan. (2013). High Performance Reconfigurable FIR Filter Architecture Using Optimized Multiplier. *Circuits, Systems, and Signal Processing*. 32. 10.1007/s00034-012-9473-3.
- [11] [11] Tirumala, Mohan. (2017). Vertical-Horizontal Binary Common Sub-Expression Elimination for Reconfigurable Transposed Form FIR Filter. *International Journal for Research in Applied Science and Engineering Technology*. V. 1276-1280. 10.22214/ijraset.2017.8181.
- [12] [12] H.-R. Lee, C.-W. Jen, and C.-M. Liu, "On the design automation of the memory-based VLSI architectures for FIR filters," *IEEE Trans. Consumer Electronics*, vol.39, no.3, pp. 619-629, Aug. 1993.
- [13] [13] P. K. Meher, "New approach to lookup table design and memory based realization of FIR digital filter", *IEEE Transactions on circuit and systems-I*, Vol.57, NO.3, March 2010.
- [14] [14] A. Croisier, D. J. Esteban, M. E. Levilion, and V. Rizo, "Digital filter for PCM encoded signals," U.S. Patent 3 777 130, Dec. 4, 1973.
- [15] [15] K. K. Parhi, *VLSI Digital Signal Processing Systems: Design and Implementation*. New York: Wiley, 1999.
- [16] [16] Luo, Jiufei, ZhijiangXie, and Ming Xie. "Interpolated DFT algorithms with zero padding for classic windows." *Mechanical Systems and Signal Processing* 70 (2016): 1011-1025.
- [17] [17] Bellanger, M.G. Specification and design of a prototype filter for filter bank based multicarrier transmission. In *Proceedings of the IEEE International Conference Acoustics, Speech, and Signal Processing*, Salt Lake City, UT, USA, 7-11 May 2001; Volume 4, pp. 2417-2420.
- [18] Salcedo-Sanz, S.; Cruz-Roldan, F.; Heneghan, C.; Yao, X. Evolutionary Design of Digital Filters With Application to Subband Coding and Data Transmission. *IEEE Trans. Signal Process.* **2007**, *55*, 1193-1203
- [19] Smith, Mark JT. "A novel FIR filter design method based on windowing." *IEEE International Symposium on Circuits and Systems*, IEEE, 1989.
- [20] Konar, Aritra, and Nicholas D. Sidiropoulos. "Fast approximation algorithms for a class of non-convex QCQP problems using first-order methods." *IEEE Transactions on Signal Processing* 65.13 (2017): 3494-3509.
- [21] Jiang, Lei, et al. "An Overview of FIR Filter Design in Future Multicarrier Communication Systems." *Electronics* 9.4 (2020): 599.
- [22] Simon, Dan. *Evolutionary optimization algorithms*. John Wiley & Sons, 2013.
- [23] Ahmad, Sabbir U., and Andreas Antoniou. "Cascade-form multiplierless FIR filter design using orthogonal genetic algorithm." *2006 IEEE International Symposium on Signal Processing and Information Technology*. IEEE, 2006.
- [24] Cen, Ling. "A hybrid genetic algorithm for the design of FIR filters with SPoT coefficients." *Signal Processing* 87.3 (2007): 528-540.
- [25] Youssef, Habib, Sadiq M. Sait, and Hakim Adiche. "Evolutionary algorithms, simulated annealing and tabu search: a comparative study." *Engineering Applications of Artificial Intelligence* 14.2 (2001): 167-181.
- [26] Karaboga, Nurhan, and Bahadir Cetinkaya. "Design of digital FIR filters using differential evolution algorithm." *Circuits, Systems and Signal Processing* 25.5 (2006): 649-660.
- [27] Tinós, Renato, and Shengxiang Yang. "A self-organizing random immigrants genetic algorithm for dynamic optimization problems." *Genetic Programming and Evolvable Machines* 8.3 (2007): 255-286.
- [28] Karaboga, Dervis, and BahriyeAkay. "A modified artificial bee colony (ABC) algorithm for constrained optimization problems." *Applied soft computing* 11.3 (2011): 3021-3031[55] Yli-Kaainen, Juha, and TapioSaramaki. "Multiplication-free polynomial-based FIR filters with an adjustable fractional delay."

- Circuits, Systems and Signal Processing 25.2 (2006): 265-294.
- [29] Dempster, Andrew G., and Malcolm D. Macleod. "Use of minimum-adder multiplier blocks in FIR digital filters." *IEEE Transactions on Circuits and Systems II: Analog and Digital Signal Processing* 42.9 (1995): 569-577.
- [30] Meyer-Baese, Uwe, et al. "A comparison of pipelined RAG-n and DA FPGA-based multiplierless filters." *APCCAS 2006-2006 IEEE Asia Pacific Conference on Circuits and Systems*. IEEE, 2006.
- [31] Dempster, Andrew G., and Malcolm D. Macleod. "Use of minimum-adder multiplier blocks in FIR digital filters." *IEEE Transactions on Circuits and Systems II: Analog and Digital Signal Processing* 42.9 (1995): 569-577.
- [32] Kang, Hyeong-Ju, and In-Cheol Park. "FIR filter synthesis algorithms for minimizing the delay and the number of adders." *IEEE Transactions on Circuits and Systems II: Analog and Digital Signal Processing* 48.8 (2001): 770-777.
- [33] Li, Dongning. "Minimum number of adders for implementing a multiplier and its application to the design of multiplierless digital filters." *IEEE Transactions on Circuits and Systems II: Analog and Digital Signal Processing* 42.7 (1995): 453-460.
- [34] Li, Dongning, Jianjian Song, and Yong Ching Lim. "A polynomial-time algorithm for designing digital filters with power-of-two coefficients." *1993 IEEE International Symposium on Circuits and Systems*. IEEE, 1993.
- [35] Yli-Kaakinen, Juha, and Tapio Saramaki. "A systematic algorithm for the design of lattice wave digital filters with short-coefficient wordlength." *IEEE Transactions on Circuits and Systems I: Regular Papers* 54.8 (2007): 1838-1851.
- [36] Negi, Vivek. *Analysis and Design of Power Efficient Multiplierless Digital FIR Filter using Radix-2r Algorithm*. Diss. 2018.
- [37] Guo, Rui, Linda S. DeBrunner, and Kenny Johansson. "Truncated MCM using pattern modification for FIR filter implementation." *Proceedings of 2010 IEEE International Symposium on Circuits and Systems*. IEEE, 2010.
- [38] Kwan, H. K., and C. L. Chan. "Circularly symmetric two-dimensional multiplierless FIR digital filter design using an enhanced McClellan transformation." *IEE Proceedings G (Circuits, Devices and Systems)* 136.3 (1989): 129-134.
- [39] Jamshed, Shamoan. *Using HPC for Computational Fluid Dynamics: A guide to high performance computing for CFD engineers*. Academic Press, 2015.
- [40] Siohan, Pierre, and Acyl Benslimane. "Finite precision design of optimal linear phase 2-D FIR digital filters." *IEEE transactions on circuits and systems* 36.1 (1989): 11-22.
- [41] Chandra, Abhijit. "Evolutionary Algorithms for the Efficient Design of Multiplier-Less Image Filter." *Bio-Inspired Computing for Image and Video Processing* (2018): 267.
- [42] Khazali, A. H., and M. Kalantar. "Optimal reactive power dispatch based on harmony search algorithm." *International Journal of Electrical Power & Energy Systems* 33.3 (2011): 684-692.
- [43] Chandra, Abhijit. "Evolutionary Algorithms for the Efficient Design of Multiplier-Less Image Filter." *Bio-Inspired Computing for Image and Video Processing* (2018): 267.