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Pre-School Teacher Subject Matter Knowledge for Teaching Informal Geometry: A Pilot Study

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Abstract: ***Informal geometry which is part of the early childhood mathematics education course is relatively new in public universities in our country. Even in private universities where it has been taught for quiet sometime, varying attention has been paid to it from it not being offered to being offered as an elective to a full course. This article examined pre-school teacher subject matter knowledge for teaching informal geometry. The theory of pre-assessment was used. The philosophy used was realism. Mixed methods were used and the sampling criteria was explanatory-exploratory in nature. 111 participants were involved in this study. The findings were that the respondents under study had an average SMK. Although this varied depending on a question to question analysis and a focus group discussion where they could not separate SMK questions from PCK, classroom practice, challenges and opportunities that they spelled out during the focus group. Respondents were employing some play based teaching such as free, inquiry, collaborative, learning through games, stories, project, art, physical education. They also had personal, management and curriculum related challenges although local outdoor environments and conditions were favourable for informal geometry teaching. The study recommended pre-testing of students to tailor instruction on understanding, introduction of high level content in informal geometry to strengthen students informal geometry background, a guided student centred curriculum, further research in other content strands, further capacity building and technical services to teachers, teacher trainers/educators and managers in play based learning and informal geometry teaching during and post the Covid-19 era.***

Keywords: informal geometry, early childhood mathematics education, Mathematical knowledge for teaching, play based pedagogy, subject matter knowledge and mathematical quality of instruction

1. Introduction

Informal geometry which is part of the early childhood mathematics education (ECME) course is relatively new in public universities in Zambia (National Numeracy Framework -NNF, 2020; Chimfwembe- Gondwe, 2020 & 2021; Nagisa 2020). For curiosity and in a quest to gain knowledge and establishing the gap, this study searched university catalogs from two public universities and one private university. It was found that, this course has not been offered to pre-school teachers nor researched at any public university and yet the early childhood mathematics education teaching and research (Nagisa, 2017) has become a contentious issue in the international community (For example, Lin, 2013; Brandt, 2013; Clements & Sarama, 2009 quoted in Nagisa 2017). Zambian early mathematics instruction at all levels of education still lags. This is partially because Zambian Universities unlike The Texas University has paid differing attentions to this branch of Mathematics. (University of Zambia Catalogue- UNZA 2014-2016; Copperbelt University-CBU 2019/2020 courses); even in private universities where it has been taught for quiet sometime, varying attention has been paid to it (Zambia Open University-ZAOU 2021). From it not being offered to being taken as an elective (University of Texas Austin, n. D) to being part of the full course under the guise of pre-mathematics (ZAOU 2021 and ECME to numeracy development (Chalimbana University-CHAu 2013 & 2015; 2016 & 2018, 2020/2021) and a full course (Texas state university undergraduate Catalogue 2017-2018; 2020-2021 Catalogue) in a different school (UNZA Catalogue 2014-2016 & ZAOU 2021). It has not been offered has a full course (CHAU 2020 & 2021) but part of numeracy development (2018). The Zambian experience mirrors in many ways that of many countries (Morris, 1986). Historically, numeracy was a goal to be attained after the process of learning number and number and notation (Victorian state government education, n.D). Numeracy involves being flexible with number and number notation. While informal geometry involves exploring mathematical concepts such as size, shape, position and dimensions of objects. A large part of learning in informal geometry involves becoming familiar with and using numbers and words to describe objects and knowing the differences between objects while formal geometry involves verification of informal proofs in Euclidean geometry using a fragment of first-order logic called coherent logic and a corresponding proof representation (San Stojanovic-Durdevic, 2018).

According to Copley *(*2010*)* informal geometry at preschool level has the following content: shape, size, position, direction and movement and processes like problem solving, reasoning, connections, communication and representations and yet in Zambia only plane shapes are given more attention (MOE, 2014 & USAID, 2019). While according to Texas University Catalogue (TUC) 2020-2021 the content for university students has the following broad content areas: history of informal geometry (Mc Cartney,2012) an introduction to formal & informal geometry, mathematical reasoning, higher geometry, differential geometry, pre-Calculus & Calculus, college algebra, mathematics for business and economics, informal geometric analysis seminar (TUC 2020-2021). However, Morris (1986) warns that of all the decisions one must make in a curriculum development project with respect to choice of content, usually the most controversial and least defensible is the decision about geometry.it is a problem reconciling a logically developed course with the mental immaturity of young children and with the contemporary needs to understand geometric transformations, analytical methods and vector algebra.

2. Purpose of the Study

The purpose of study was to assess teacher subject matter knowledge for teaching informal geometry

3. Statement of the Problem

Despite Government efforts to improve students' performance in mathematics, learner performance has continued to be below the desired level of 70% (NNF, 2020). A number of studies have been conducted to assess learner underperformance (Hambokoma,C; Nkhata B, Kostyuk,Vasly,2002-2016,Chimfwembe G,2011 and others). But, it seems no research has been conducted to investigate teacher subject matter knowledge for teaching of informal geometry at university T; so as to establish the challenges teachers face and use such challenges as opportunities to come up with further capacity building and technical support actions before they embark on university education. Learner underperformance has social, economic and political implications if it remains unchecked. The growth of Science, Technology, Engineering and Mathematics (STEM) also depends on a numerate society (Nkhata, etal, 2002, 2013 & 2019).

4. Theoretical Framework

Many theories exist to assess teacher understanding (Talsma, 1997). This study used the pre-assessment theory propounded by Terry Heick (2018). According to this theory, the teacher is expected to uncover what students know and how well they know it before a teacher could embark on teaching. Pre-assessments focus is any kind of evaluation, analysis or measurement of student understanding that occurs before the teaching/learning process begins. The purpose of pre-assessment varies-it can be to help plan lessons and activities, revise curriculum maps, create personalised learning pathways for individual students, and help inform grouping strategies, plan future assessments and many others.

5. Research Objectives

The research objective was to examine pre-school teacher subject matter knowledge for teaching informal geometry as they embarked on University education.

6. Research Questions

The research answered the following main question: Do pre-school teachers of this particular cohort have sufficient subject matter knowledge concerning informal geometry.

Significance of the Study

This study may bring out new ideas on how to improve the performance in mathematics in later grades that have been below the desired level of 70 %.(draft National Numeracy Framework (NNF), 2015) and Early Grade Mathematics Assessment report (2015). It may also improve the researchers knowledge on informal geometry and reduce on the questions own had concerning the gap in the literature concerning informal geometry. Although globally, there may be problems of reconciling a logically developed course with mental immaturity of the young children and with the contemporary needs to understand geometrical transformations, analytical methods and vector geometry (Morris, 1986) and many other types of geometries (Texas state university undergraduate Catalogue 2017-2018; 2020-2021 Catalogue, Sonnabend & Copley,2010)

7. Literature Review

7.1 Pre-school subject matter knowledge for teaching informal geometry

Hedwig Gasteiger, Julia Bruns, Christiane Benz, Esther Brunner & Priska Sprenger (2019) carried out a study on mathematical pedagogical content knowledge (MPCK) of early childhood teachers. This study highlighted what early childhood teaching is. Thus it includes some planned activities. It is also motivated and generated by situations that unfold spontaneously in play and everyday activities. Therefore, this field should be respected. Although respecting this field may fail to explain teacher's knowledge explicitly but indeed it may show implicit knowledge in their daily work. And that a measurement approach related to concrete situations may be suitable. Although qualitative MPCK interview instruments exist. But that there is lack of a standardized instrument suitable for large scale studies. And that developing a paper –pencil test with multiple-choice items measuring MPCK and respecting early childhood implicit knowledge could be an ideal way of dealing with this case. However, the current study focused on trying to develop the rightful content and instrument for early childhood mathematics teachers by examining teachers SMK (Chimfwembe, 2020) and mathematical reasoning (Chimfwembe, 2021) in lieu of interpreting the national policies on early childhood mathematics education (ZECF,2013 & ELDs ,2016). This policies are important because they provide guidelines on the way forward in early childhood mathematics education provision in Zambia. In most cases teachers are implementers and very important factors in children's learning, yet little is known about the specialized knowledge held by in-service teachers (Filgona Jacob, Sakiyo John and D.M Gwany, 2020). In any profession, there is a specialised professional knowledge that makes it unique and distinct with striking features entirely different from other professions. One of the characteristic of a good teacher is that they possess a substantial amount of subject matter knowledge (ibid). Subject matter knowledge which includes common content knowledge, specialised content knowledge and horizontal content knowledge (Ball etal, 2008 & 2011) has been the focus of research globally (Guoxing Yu,2007). Luneta (2014) also conducted a study in South Africa and found that, majority of student teachers had limited knowledge of basic geometry. Although her study was not on early child hood teachers but on student teachers starting tertiary education straight from high school. This study was on conceptual understanding of shapes such as polygons and polyhedrons and their properties. And Luneta's finding was that, the majority of student teachers have limited knowledge of basic geometry and require not remedial, but re-learning of basic concepts. In her study a cohort of 128 first year students registered for a foundation course which

is similar to the number registered and participated in the current study. In her study it was found that while Grade 12 learners are expected to operate at levels 3 and4 on the van Hieles levels, the majority of the participants in the study were operating at level 1, the level of the leaners they will be teaching when they complete the course. Her study investigated the knowledge of geometry with which student teachers enter university which was the case with the current study. Luneta's study was predominantly qualitative which used interpretive descriptions to analyse data and Van Hiele's thought as a gauge while this study is mixed and used group work activities from Sonnabend (2010) and Copley (2010) course books as a gauge. This course books are recommended to be used especially if one is assessing the professional knowledge for teachers (NAEYC & NCTM, 2000) and they are in line with the current trends in ECME provision in the country (ZECF, 2013 & MOE, 2016). The two course books are full of playful activities such as mathematical investigations and problem solving.

8. Methodology

8.1 Study paradigm

This study used the philosophy of realism. The theory of reality and is best used in mixed method studies (Jannita, 2010). Teacher subject matter of informal geometry has an influence on learner performance. Although it is further argued that no strong correlation exists between teachers' knowledge of the subject matter and learner performance (NAP, 2009).

This study used mixed methods. Mixed methods research is a methodology for conducting research that involves collecting, analysing and integrating mixed (for example, experiments, surveys) and qualitative (for example, focus groups, interviews) research. And analysing data thematically and by use of minimal statistics (Cres well, 2003).

8.2 Study Design

The study was a pilot study (Hedwig Gasteiger et al, 2019). Because ECME had just started at the university (CHAu, 2013). The use of a pilot study provided information against which to monitor and assess an activities progress and effectiveness during implementation. A pilot study helps in testing the feasibility of the project proposal, recruitment of subjects, research tools and data analysis methods (Zaili Abu Hassan, Peter Scattner, and Danielle Mazza (2006).

8.3 Population and participant characteristics

The population comprised of Pre-school trained teachers with a minimum of a certificate in early childhood education (ECE) formally called pre-school. Most of them did their training from private institutions and others from two public institutions that were in existence before implementation of ECE in public colleges. The respondents in question had formal training based on the old early childhood curriculum which had less emphasis on (ELDs) and other mathematical content strands but basically on numeracy which was a foundation for learning informal geometry. In the case of pre-school teachers, the grades they got from colleges did not matter what was important was the professional certificate and a grade 12 certificate.

8.4 Sample and characteristics

The sample consisted of about 111 respondents. This number provided ample opportunity for the researcher to identify themes and pattern of cases that the researcher had in the literature review as well as conduct cross case theme analysis. Respondents had not received instruction in informal geometry at the University. But had basic ideas on plane shapes. Respondents chose the groups were they were comfortable.

8.5 Sampling method and Procedure

The study conveniently sampled the respondents because the study wanted to gather information to address critical issues that may arise from newly launched programs. It also wanted to collect feedback about the ECME program that had just been launched at a public institution.

8.6 Data collection instruments and procedure

The questions on the group work activity were open ended (Chimfwembe-Gondwe, 2020 & 2021). Because open ended questions makes interviewers to be free to express their views and to brainstorm while a survey ensures that data is collected from a large sample (Johan Malmqvist, Kristina Hellberg, Gunvie Mollas, Richard Rose and Michael Shevlin (2019). The focus group discussion guides on how to interrogate respondents deeply in order to get the required answers (Un Women, 2011). Data from the group work activity and focus group discussion was collected during the April-May 2018 residential.

8.7 Data Analysis Procedure

Data was collected and analysed firstly quantitatively then qualitatively Frequencies and percentages were used and latter content from the focus group discussion was mapped into themes, the challenges and opportunities were also mapped into themes. Commonalities and exceptions were also noted.

8.8 Reliability

Questions were selected from some methodological and content text books (Sonnabend, 2010 and Copley, 2010). Same findings emerged from the 2015/ 2016 cohort and the 2017/2018 cohort. In the cohort of 2015/2016 SPSS was used to analyse data and the result was the same as when results were analysed using excel in the current study of cohort 2017/2018. This was in line with the definition of the term reliability. Reliability is about consistency of a measure (Fiona Middleton, 2020)

8.9 Validity

This was considered in the earliest stages of this study through the way the researcher decided on data collection. Data was collected and analysed in a triangulated manner.

The methods and measurement technique targeted to measure mathematics (informal geometry) which has been the researchers teaching subject. The measurement technique was verified by the supervisor of the research and two other students.

8.10 Assumptions of the Study

The researcher assumed that the respondents in the study had enough subject matter knowledge in informal geometry to embark on university education.

8.9 Ethical dilemmas and considerations

Ethically the study was permitted by MOE and the university under study. Confidentiality and anonymity were ensured by using pseudonyms of personal names and locations, and omitting certain details in the descriptions (Bryman, 2008). In this study in cases where information concerned well-being, information was divulged to relevant authorities but with pseudonyms used.

8.10 Trust worthiness of the study

The researcher took a sequential explanatory-exploratory direction and cared about hypothesis testing and providing information because ECE is relatively new in government schools including university T. And little is known about it. Furthermore, trust worthiness depended on the degree to which the theoretical, methodological choices and process on analysis and data collection were made visible.

This study used thick description to show that the research study's findings are applicable to other contexts, circumstances, and situations. And documented all the activities -provided an audit trail-that highlighted every step of data analysis that was made in order to provide a rationale for the decisions made. This helped the researcher to establish that the research study's findings accurately portray participants' responses. The study used an inquiry audit in order to establish dependability. Two outside persons were used to review and examine the research process and the data analysis in order to ensure that the findings are consistent and could be repeated.

9. Findings and Discussions of the Study

9.1 Teacher subject matter knowledge for teaching informal geometry

Figure 1 below gives a summary of teacher subject matter knowledge on informal geometry before they embarked on university education.

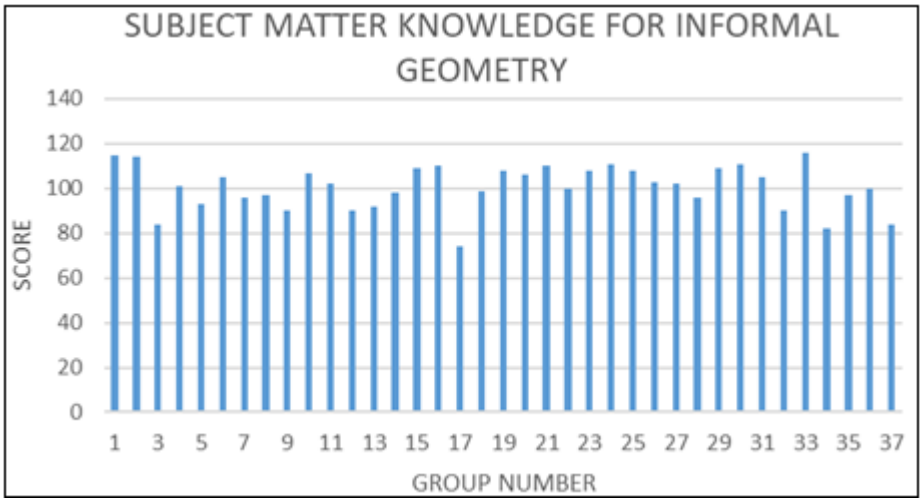

Figure 1

The cut-off point was 79 scores out of 158 total marks. According to the information on the graph most of the respondents scored above the cut-off point and below an ideal score of 118. This meant that the respondents of the cohort in question had generally an average SMK. This was sufficient SMK where the instructor could initiate instruction from as they needed retraining not remedial work (Luneta, 2014; Nagisa 2019 & Chimfwembe, 2020). Group 03, did not respond to questions 35, 37, 38 and 43. These questions were on completing a table on the number of triangles formed and the sum of interior angles using the principle of inductive reasoning; explaining some geometrical software that a teacher can use to teach geometry; showing that the equilateral triangle tessellate and when a line and a plane are perpendicular. While group 06 did not respond to questions 19, 32, 33, 34, and 38 and was the weakest group. This questions were on defining and giving examples and non-examples of a polygon, geo-board and diagonal; naming a time of the day when the minute and hour hands of a clock form a 75^{o} angle and devising a plan and solving the problem; verifying the principle that, the sum in degrees of angle measures of any triangle is always the same and what the sum of three angle measures appear to be and later finding the angle measures of a convex quadrilateral; Also checking for misconceptions and errors in a learner who says that the sum of angle measures of a quadrilateral is 720^{0} and not 360^{o} and draws a kite with two diagonals then makes the assumption that the interior angle measure of the kite add up to 4x180,and goes on computing the sum of interior angle measures of the quadrilateral using four triangles. Further, group 06 could not show by explaining that, the equilateral triangle tessellates. It seemed

the question on the equilateral triangle tessellating was difficulty to most of the groups. Furthermore, groups 03,13,17,18, 28 and 36 got the maximum number of zeroes on certain questions. This finding is consistent with what Luneta (2014) found Luneta's study found that, majority of student teachers had limited knowledge of basic geometry. Her methodology was mainly qualitative with a lot of interpretations while this study is mainly mixed with minimal statistics such as frequencies and percentages. The current study differs from Luneta"s study in the sense that, the current study did not find out the level at which the students were operating at as put forth by Van Hiele (1957) because the focus was to find out the general information in-service teachers had concerning informal geometry and not pre-service, how they were interpreting children's misconceptions of geometric concepts, how they understood children's thinking, how they taught this children and how they understood the curriculum and their understanding of geometry based on pedagogy and content they exhibited as they answered questions from the group activity questionnaire and during focus group discussions. In the current study there was no treatment group as was the case with Fatma & Thomasenia (2015). And no repeated measure ANOVA results showed a significant change in the treatment group participant's geometry content knowledge but only a bar chart and tables were used with minimal frequencies and percentages because this will be handled in the next article as this article is just part of a long journal on early childhood mathematics education in Zambia. However the performance in the current study varied in the sense that those who failed certain questions did well in other questions (Olivia N Saracho & Bernard Spodek, 2007)

Table 3 below shows the questions per group that three groups got wrong. While table 4 shows the questions that they got correct. Each group had the same questions except that what is reflected in the tables 3 and 4 are the questions each of these groups got wrong and correct respectively.

Table 3: Questions per group where they got a Zero mark

Group	Question
3	• In which year did the Greeks develop an important new approach to geometry and what was this approach? • Name a time of the day when the minute and hour hands of a clock form 75^o. Devise a plan and solve the plan.* • Verify the following principle" The sum in degrees of angle measure of any triangle is always the same." What does the sum of three angle measure appear to be? Find the angle measures of a convex quadrilateral. • Give examples in everyday life of something that approximates a line that is perpendicular to a plane. • What are the possible relationships between two planes in three dimensions • You asked a second –grade class how many edges a cube has. A learner responds," Each face has 4 edges. There are 6 faces. The total would be 4+4+4+4+4+4=24." How would you explain the correct solution to the learner? • Describe properties that prisms have that pyramids, spheres and cones do not have?
17	• Name the oldest recorded examples of geometry • In which year did the Greeks develop an important new approach to geometry and what was this approach? • Where do geometric ideas such as, a line come from? • Complete the following statements o Two intersecting lines have exactly------- point in common o Two lines are parallel if and only if they lie in the same plane and they do not-------- • Complete a table and find the number of diagonals for a polygon with N-sides • Name the time of day when the minute and hour hands of a clock form a 75^o. Devise a plan and solve the problem.* • Verify the following principle" The sum in degrees of angle measure of any triangle is always the same." What does the sum of three angle measure appear to be? Find the angle measures of a convex quadrilateral. • A learner says that the sum of angle measure of a quadrilateral is $720^o$and not 360^o and draws a kite with two diagonals then makes the following assumptions that the interior angle measure of the kite add up to 4x 180^o=720^o • What is wrong with the learners reasoning • How could you compute the sum of interior angle measures of the quadrilateral using the four triangles
	• With examples, explain some geometrical software that a teacher can use to teach geometry. • What are the possible relationships between two planes in three dimensions? • You asked a second –grade class how many edges a cube has. A learner responds," Each face has 4 edges. There are 6 faces. The total would be 4+4+4+4+4+4=24." How would you explain the correct solution to the learner?
28	• Name the oldest recorded examples of geometry**.** • What do we need to consider to figure out which regular polygons tessellate the plane? • Name the time of day when the minute and hour hands of a clock form a 75^o. Devise a plan and solve the problem.* • Verify the following principle" The sum in degrees of angle measure of any triangle is always the same." What does the sum of three angle measure appear to be? Find the angle measures of a convex quadrilateral • •A learner says that the sum of angle measure of a quadrilateral is 720^o and not 360^o and draws a kite with two diagonals then makes the following assumptions that the interior angle measure of the kite add up to 4x 180^o=720^o • a).What is wrong with the learners reasoning • b).How could you compute the sum of interior angle measures of the quadrilateral using the four triangles • When is a line and a plane perpendicular? • You asked a second –grade class how many edges a cube has. A learner responds," Each face has 4 edges. There are 6 faces. The total would be 4+4+4+4+4+4=24." How would you explain the correct solution to the learner? • Describe properties that prisms have that pyramids, spheres and cones do not have?

Content for which questions were formulated adopted from Thomas Sonnabend (2010)

From the table above all the three groups failed the question on the time of the day when the minute and the hour hands of a clock form a 75^{o} and devising a plan to solve the problem. This could be seen by the * that is reflected in all the three groups.

Table 4: below shows the questions per group that six groups got right.

Group(S)	Question
13 &17	What do we need to consider to figure out which regular polygons tessellate the plane?
03,13,17,28 & 36	With examples define a chord, radius, diameter and central angle.
03,13,18 &36	List the properties of a rectangle, square and a rhombus
13,17,18,28 & 36	How many faces does a cube have? How many edges does a cube have? How many vertices does a cube have?

Content for which questions were formulated adopted from Thomas Sonnabend (2010)

From the table above although these groups got the maximum number of zeroes as deduced from table 3, they also answered certain questions well as seen in table 4 above. This performance reflects a situation where it is not always that when a person fails one question then it means that, that person cannot pass the other questions. This finding is consistent with (Olivia N Saracho & Bernard Spodek, 2007). These researchers carried out a study where they critically analysed 40 studies on early preparation of early childhood education teachers and quality of their educational programs that were published within a 15 year period (1989-2004). Using a literal, allegorical and interpretive critical analysis, they explored the importance of high level of education for early childhood education teachers and recommended professional development, importance of a bachelor's degree and educational standards for early childhood teachers

10. Conclusions and Recommendations

The researcher's conclusion from the current study was that, respondents of this particular cohort had an average subject matter knowledge although this varied from one group to the other. And that regarding subject matter knowledge, all the decisions one must make in a curriculum development project and high level program for early childhood education teachers with respect to choice of content, usually the most controversial and least defensible is the decision about geometry. It is a problem reconciling a logically developed course bearing in mind that the teachers in question will be teaching young children who may be the mentally immature and with the contemporary needs to understand geometric transformations, analytical methods and vector algebra among the so many geometries (TUC, 2020-2021 catalog, Sonnabend, 2010 & Copley, 2010, Moris, 1986). Nevertheless, the issue of a quality informal geometry program is critical for teachers of early learners (Olivia N Saracho & Bernard Spodek, 2007).

Therefore, this study recommends that;

- The university should be pretesting students before lecturers embark on teaching. This will enable them to tailor content towards teaching for understanding (Talsma, 1997). And ensure high quality informal geometry is offered to teachers of early learners (Olivia et al, 2007)
- Besides strengthening students informal geometry background, courses in mathematics should develop skills that are essential for teachers but not generally taught in other informal geometry course such as the ability to give clear explanations, evaluate children's mathematical work, solve problems with differential methods, select appropriate models to develop concepts, and sequence a series of problems or topics (Sonnabend,2010 & Matthew Maning, Susan Garvis, Christopher Flemmings, Gabriel T.W (2017)).
- Content under informal geometry should include history, polygons, triangles, quadrilaterals circles and angle measure of polygons, three dimensional geometry, viewing and drawing solid figures, geometry in grade one to eight, transformation and congruence, constructions, symmetry, similarity and dilation (TUC, 2020-2021 Catalog), history of mathematics (Mc Cartney, 2012) an introduction to formal & informal geometry, mathematical reasoning, higher geometry, differential geometry, pre-Calculus & Calculus, college algebra, mathematics for business and economics, informal geometric analysis seminar (2020-2021 Catalogs. Mathematics (MATH) TEXAS STATE) and Course Catalog 2014-2016-UNZA .This content links well with our pre-school syllabus content (ZECF,2013,School syllabus,2013 & ELDs,2016) and has a lot of geometry in line with mathematical investigations, problem solving and other mathematical processes and its content trajectory is very clear and is of high quality (Olivia & Spodek,2007).
- Managers and other teachers need orientation on play – based inquiry, free play, collaborative play, playful learning and learning through other contexts as most of them are just interested in seeing learning goals and success criterion on the wall.
- The early childhood curriculum at university T should be based on guided student centred-ness and play.
- Since the current study only focused on SMK of informal geometry, further research is needed to find out where mathematics is in play in other content areas such as number, measurement, patterns, algebra, functions, data and probability(ibid). Also, there should be a study to find the level at which respondents were operating as put forth by Van Hiele.

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