International Journal of Science and Research (IJSR)
ISSN: 2319-7064
SJIF (2019): 7.583

Neumann Boundary Condition on Taylor Series Method

Dr. Chitra Singh[1], Mukesh Yadav[2]

[1]Associate Professor Mathematics, Rabindranath Tagore University Bhopal, India
chitrasingh30[at]yahoo.in

[2]Research Scholar, Kendriya Vidyalay Sarani, India
mukeshy123[at]yahoo.com

Abstract: ***In this paper, a Neumann boundary condition for solving the Taylor's series method with constant coefficient and analytic initial condition in two & three independent variable is presented. The technique is based upon Taylor's expansion.The Taylor series may not converge if the solution is not analytic in the whole domain, however the present method can be applied on Neumann boundary condition for linear partial differential equation, when the solution is analytic in the interior of the domain and also a some open subsets for each distinct part of the boundary. The method is computationally attractive and application is demonstrated through illustrative examples.***

Keywords: Taylor's series, nthorder linear differential equation, Ordinary differential equation, Neumann boundary condition

Subject Classification Mathematics 2020: 30E05, 30E10

1. Introduction

Neumann boundary Condition on Taylor's series Method has been investigated by many authors .However there are few references on the solution of the Taylor's Series method of three variable and two variables on Neumann boundary Condition. One advantage of the method of using Taylor series is that a differentiable approximate solution is obtained. Whish can be replaced into the equation and the initial boundary value, Neumann boundary value conditions .In this manner the accuracy of the solution can be evaluated directly. In this way the problem is reduce to those of solving a system of algebraic equation .The modern theory of differential equation is based on the expansion of Taylor's series The striking analogy existing between the theory of algebraic equations and the theory of differential equations suggested the possibility of expressing the solutions of algebraic equations in series to be obtained by an application of Taylor's series.

1.1 Differential Equation

Differential equations first came into existence with the invention of calculus by Newton and Leibniz., Methodus fluxionum et SerierumInfinitarum, Isaac Newton listed three kinds of differential equations.

$$\frac{dy}{dx} = f(x)$$

$$\frac{dy}{dx} = f(x,y)$$

$$x_1\frac{\partial y}{\partial x_1} + x_2\frac{\partial y}{\partial x_2} = y$$

In all these cases, y is an unknown function of x (or of x_1 and x_2), and f is a given functions.

1.2 Taylor series

For two variables- $f(x_1,x_2) = \sum_{i_1,i_2=0}^{\infty} f(i_1,i_2)x_1^{i_1}x_2^{i_2}$.

For three variables- $f(x_1,x_2,x_3) = \sum_{i_1,i_2=0}^{\infty} f(i_1,i_2)x_1^{i_1}x_2^{i_2}$.

1.3 Neumann boundary condition

In mathematics, the Neumann (or second-type) boundary condition is a type of boundary condition, named after Carl Neumann. Carl Gottfried Neumann (also Karl; 7 May 1832 – 27 March 1925) was a German mathematician When imposed on an ordinary or a partial differential equation, the condition specifies the values in which the derivative of a solution is applied within the boundary of the domain.

A boundary condition which specifies the value of the normal derivative of the function is a Neumann boundary condition, or second-type boundary condition. For example, if there is a heater at one end of an iron rod, then energy would be added at a constant rate but the actual temperature would not be known.

Example

For an ordinary differential equation, for instance,

$$y'' + y = 0$$

The Neumann boundary conditions on the interval [a, b] take the form

$$y'(a) = \alpha, \quad y'(b) = \beta$$

Where α and β are given numbers.

2. Main result

2.1 Taylor's Series method of twovariable

If Ω is an open set of R^2, which contains the origin $0 = (0,0)$, we denote by $A(\Omega)$ the set of real analytic function in Ω. Then the neighbourhood of the origin , a function $f = f(x) \in A(\Omega)$, where $x = (x_1, x_2)$, can be expanded in power series as follows

$$f(x_1,x_2) = \sum_{i_1,i_2=0}^{\infty} f(i_1,i_2) x_1^{i_1} x_2^{i_2}. \quad (1)$$

The sum of right hand side of equation (1) by grouping the terms in a homogeneous part and then using a lexicographic order.

$$f(x_1,x_2) = f_{(0,0)} + f_{(1,0)}x_1 + f_{(0,1)}x_2 + f_{(2,0)}x_1^2 + f_{(1,1)}x_1x_2 + f_{(0,2)}x_2^2 \quad (2)$$

For simplicity we adopt the following notations:

$$i = (i_1,i_2) \in N^2, x^i = x_1^{i_1}, x_2^{i_2}, f_i = f_{(i_1,i_2)}. \quad (3)$$

By using equation (3) , we can write equation (2) as

$$f(x) = \sum_{i\geq 0} f_i x^i, \quad (4)$$

Where

$$f_i = \frac{1}{i_1! i_2!} \frac{\partial^{i_1+i_2} f}{\partial x_1^{i_1}, \partial_{x_2}^{i_2}}(0)$$

For our subsequent development , we need to find the position of a term in equation (2) , we write $f(x_1,x_2)$ in equation (2) as

$$f(x_1,x_2) = a_1 + a_2x_1 + a_3x_2 + a_4x_1^2 + a_5x_1x_2 + a_6x_2^2 + a_7x_1^3 + \cdots \ldots \quad (5)$$

Example 2.2 : Find Taylor's expansion of the function $f(x_1,x_2) = x_1^2x_2 + 3x_2 - 2$ in powers of $(x_1 - 1)$ and $(x_2 + 2)$.

Solution: Taylor's theorem is used with $x_1 = 1, x_2 = -2$.

$f(x_1,x_2) = x_1^2x_2 + 3x_2 - 2$

$f(1,-2) = -10$

$f_{x_1}(x_1,x_2) = 2x_1x_2 f_{x_1}(1,-2) = -4$

$f_{x_2}(x_1,x_2) = {x_1}^2 + 3 f_{x_2}(1,-2) = 4$

$f_{x_1x_1}(x_1,x_2) = 2x_2 f_{x_1x_1}(1,-2) = -4$

$f_{x_1x_2}(x_1,x_2) = 2x_1 f_{x_1x_2}(1,-2) = 2$

$f_{x_2x_2}(x_1,x_2) = 0 f_{x_2x_2}(1,-2) = 0$

and all other higher order partial derivatives are zero.

$f(1,-2) = -10 - 4x_1 + 4x_2 - 4x_1^2 + 2x_1x_2 + \cdots \ldots$

2.3 Taylor's Series method of three variable

Similarly to the case of two variables, we assume Ω is an open set of R^3, which contains the origin $0 = (0,0,0)$, we denote by $A(\Omega)$ the set of real analytic function in Ω. Then the neighbourhood of the origin , a function $f = f(x) \in A(\Omega)$,where $x = (x_1,x_2,x_3)$ can be expanded in power series as follows

$$f(x_1,x_2,x_3) = \sum_{i_1,i_2=0}^{\infty} f(i_1,i_2) x_1^{i_1} x_2^{i_2}. \quad (1)$$

The sum of right hand side of equation (1) by grouping the terms in a homogeneous part and then using a lexicographic order.

By using a lexicographic order of Taylor series

$$f(x_1,x_2) = f_{(0,0)} + f_{(1,0)}x_1 + f_{(0,1)}x_2 + f_{(2,0)}x_1^2 + f_{(1,1)}x_1x_2 + f_{(0,2)}x_2^2 + f_{(3,0)}x_1^3 + f_{(2,1)}x_1x_2^2 + f_{(0,3)}x_2^3 + \cdots \ldots$$

we will get

$$f(x_1,x_2,x_3) = f_{(0,0,0)} + f_{(1,0,0)}x_1 + f_{(0,1,0)}x_2 + f(0,0,1)x_3 + f_{(2,0,0)}x_1^2 + f_{(1,1,0)}x_1x_2 + f_{(0,2,0)}x_2^2 + f_{(1,0,1)}x_1x_3 + f_{(0,1,1)}x_2x_3 + f_{(0,0,2)}x_3^2 + \cdots \quad (2)$$

For simplicity we adopt the following notations:

$$i = (i_1,i_2) \in N^2, x^i = x_1^{i_1}, x_2^{i_2}, f_i = f_{(i_1,i_2)}. \quad (3)$$

In order of the terms in equation. (2) implies an order on N^2 such that

$$i = (i_1,i_2) < i' = (i_1',i_2') \quad if\ either\ (i_1+i_2) < (i_1'+i_2')$$
$$(i_1+i_2) < (i_1'+i_2') \text{ and } i_2 < i_2'$$

By using equation (3), we can write equation (2) as

$$f(x) = \sum_{i\geq 0} f_i x^i, \quad (4)$$

Where

$$f_i = \frac{1}{i_1! i_2!} \frac{\partial^{i_1+i_2} f}{\partial x_1^{i_1}, \partial_{x_2}^{i_2}}(0)$$

2.4 Example

Find Taylor's expansion of the function $f(x,y,z) = e^{xyz}\ at\ (1,1,1)$third degree terms.

Solution: Here$f(x,y) = e^{xyz}$

$f(1,1,1)$ =e

$\frac{\partial f}{\partial x} = yze^{xyz}, \frac{\partial f}{\partial x} = xze^{xyz}, \quad \frac{\partial f}{\partial z} = xye^{xyz}$

$\frac{\partial^2 f}{\partial x^2} = y^2z^2e^{xyz}, \quad \frac{\partial^2 f}{\partial x \partial y} = (1 + xyz)ze^{xyz}, \frac{\partial^2 f}{\partial y^2} = x^2z^2e^{xyz}$

$\frac{\partial^3 f}{\partial z \partial x \partial y} = e^{xyz}(1 + 3xyz + x^2y^2z^2), \frac{\partial^2 f}{\partial z^2} = x^2y^2e^{xyz}.$

$at \ (1,1)$.

Paper ID: SR21220113016 DOI: 10.21275/SR21220113016

$\frac{\partial f}{\partial x}=\frac{\partial f}{\partial y}=\frac{\partial f}{\partial z}=e,$
$\frac{\partial^2 f}{\partial x^2}=\frac{\partial^2 f}{\partial y^2}=\frac{\partial^2 f}{\partial z^2}=e,\frac{\partial^2 f}{\partial x\partial y}=2e,\frac{\partial^3 f}{\partial z\partial x\partial y}=5,$

Hence the required Taylor expansion about the point (1,1,1) by using a lexicographic order as

$$f(x_1,x_2,x_3)=f_{(0,0,0)}+f_{(1,0,0)}x_1+f_{(0,1,0)}x_2+f(0,0,1)x_{3+}f_{(2,0,0)}x_1^2+f_{(1,1,0)}x_1x_2+f_{(0,2,0)}x_2^2+f_{(1,0,1)}x_1x_3+f_{(0,1,1)}x_2x_3+f_{(0,0,2)}x_3^2+$$

……….

$$f(1,1,1)+e+ex+ey+ez+ex_1^2+2ex_1x_2+ex_2^2+\cdots..$$

This is required Taylor's expansion.

3. Result

In this paper we introduce the basic properties of Taylor series in two and three variables. By using a Neumann Boundary conditions we can easy to solve Taylor series expansion of two and three variable method.

References

[1] R. Barrio, Sensitivity analysis of ODEs/DAEs using the Taylor series method, SIAM J. Sci. Comput. 27 (6) (2006) 1929–1947.
[2] R. Barrio, M. Rodriguez, A. Abad, F. Blesa, Breaking the limits: the Taylor series method, Appl. Math. Comput. 217 (2011) 7940–7954.
[3] H.R. Marzban, M. Razzaghi, Solution of multi-delay systems using hybrid of block-pulse functions and Taylor series, J. Sound Vib. 292 (2006) 954–963.
[4] R.D. Neidinger, Directions for computing truncated multivariate Taylor series, Math. Comput. 74 (249) (2004) 321–340.
[5] R.D. Neidinger, An efficient method for the numerical evaluation of partial derivatives of arbitrary order, ACM Trans. Math. Software 18 (2) (1992) 159–173.
[6] M. Razzaghi, M. Razzaghi, Taylor series analysis of time-varying multi-delay systems, Internat. J. Control 50 (1) (1988) 183–192.
[7] M. Razzaghi, M. Razzaghi, Solution of linear two-point boundary value problems with time-varying coefficients via Taylor series, Int. J. Syst. Sci. 20 (11) (1989) 2075–2084.
[8] G. Groza, N. Pop, Approximate solution of multipoint boundary value problems for linear differential equations by polynomial functions, J. Difference Equ. Appl. 14 (12) (2008) 1289–1309.
[9] B. Bülbül, M. Sezer, Taylor polynomial solution of hyperbolic type partial differential equations with constant coefficients, Int. J. Comput. Math. 88 (3) (2011) 533–544.
[10] M. Dehghan, On the solution of an initial–boundary value problem that combines Neumann and integral condition for the wave equation, Numer. Methods Partial Differential Equations 21 (1) (2005) 24–40.
[11] N. Liu, E.B. Lin, Legendre wavelet method for numerical solutions of partial differential equations, Numer. Methods Partial Differential Equations 26 (1) (2010) 81–94.