International Journal of Science and Research (IJSR)
ISSN: 2319-7064
ResearchGate Impact Factor (2018): 0.28 | SJIF (2019): 7.583

Forecasting Students Performance: A Case Study at Tabuk University

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Abstract: ***The focus of this article is to use the absorbing Markov Chain model as a forecasting tool to investigate the student's progression at the Mathematics Program, Tabuk University.In this article we consider the case of students populations that are structured only by their academic stages. Our sample consist of the archived academic records of all students enrolled at the Mathematics Program during the academic year 2016-2020, we then followed students progression across the different academic stages over the course of four years.***

Keywords: Markov Chains, student's progression, transition matrix

1. Introduction

Student's progress through educational systems is stochastic process, and therefore Absorbing Markov Chains can be used to model the probabilities of student's progression and thus predicting the likelihood of future performances [1]-[5]. The early detection of student's performance deficiencies is quite important in preventing any undesired future consequences that might affect their migration through the system and also feeds directly in the strategic planning of the institution.

Recently in Saudi Arabia student's performance in higher education has been of growing concern. Therefore the Higher Council of Education has established a system for quality assurance and accreditation, with the objective to support continual quality improvement at all institutions in post-secondary education. As a result the institutions has taken the responsibility to keep a high quality standards through enhancing good practices, performance mentoring, reporting and making action plans for continual improvements. One of the important good practices for quality assurance is reviewing student's performance quantitatively as well as qualitatively.

2. Review

A stochastic process is the counterpart to a deterministic process, given the initial stage of a stochastic system its future evolution is indeterministic, it can only be described by probability distributions. Markov process is a stochastic process with a property called Markov property. Given a probability space (Ω, F, P), where Ω is the stage space (in this article we willassume that Ω finite or countable set of stages, F is the σ-algebra and P is a probability measure. A stochastic or random process defined on (Ω, F, P) is a collection of random variables indexed by a set of numbers, usually viewed as points in time, giving the interpretation of a stochastic process representing an observed quantity that evolves randomly in time.
Mathematically stochastic process is defined as,

$$X = \{X_n : n \epsilon \mathrm{T}\} \qquad (1)$$

WhereX_n is a random variable representing a value observed at time n. Usually X_n is called the stage of the system at time n. The index set T is the set of all times we wish to define for the process. When T is discrete we obtain a discrete-time stochastic process and when T is continuous we get a continuous-time stochastic process. In this article we will focus on the class of discrete-time stochastic processes. An important questions about stochastic processes is how to find the new probability distribution for X_n in each stage at a time n if we are given the distribution for X_0 at the initial time 0. Another important question is about the asymptotic behavior of the stochastic process, that is what happen to the distribution of X_n as $n \to \infty$. The most simple stochastic model which deals with these kind of questions is the Markov chain.

Markov chains are used to study stochastic processes by viewing events occurring as stages transitioning into other stages, or transitioning into the same stage as before.

Markov chain is defined as follows:

Stochastic processes $\{X_0, X_1, \ldots., X_n\}$ with finite stage space $\Omega = (s_0, s_1, \ldots, s_n)$, such that the conditional distribution of each variable given the past only depends upon the value of the immediately preceding variable is called a Markov chain or Markov process, that is,

$$P(X_n = s_n | X_0 = s_0, X_1 = s_1, \ldots \ldots, X_{n-1} = s_{n-1}) = P(X_n = s_n | X_{n-1} = s_{n-1}) \qquad (2)$$

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Equation 2 says that the distribution of any X_n, $n > 0$, depends only on X_{n-1} and is conditionally independent of the past of the system. In other words given the present stage, the past does not provide any extra information on the future evolution of the system.

By defining the one step transition probabilities from stage s_i to stage s_j as,

$$p_{ij}(1) = P\left(X_1 = s_j \middle| X_0 = s_i\right) \quad (3)$$

From this one step transition probabilities p_{ij} we can define the one step transition matrix $P(1) = [p_{ij}(1)]$.

$$P(1) = [p_{ij}(1)] = \begin{bmatrix} p_{11} & \cdots & p_{1n} \\ \vdots & \ddots & \vdots \\ p_{n1} & \cdots & p_{nn} \end{bmatrix} \quad (4)$$

The i^{th} row of $P(1)$ is the conditional distribution of X_n given that $X_{n-1} = s_i$, therefore each row of $P(1)$ sums up to 1 that is,

$$\sum_{j \in \Omega} p_{ij} = \sum_{j \in \Omega} P\left(X_n = s_j \middle| X_{n-1} = s_i\right) = 1 \quad (5)$$

In general the probabilities p_{ij} are not just functions of the stages s_i and s_j they could be functions of time n as well. When p_{ij} are independent of time, then the Markov chain is called time homogeneous. All Markov Chains considered in this article will be time homogeneous.

The probability of the Markov chain traversing the path $(s_0, s_1, \dots\dots, s_n)$ starting from the stage s_0 is given by,

$$P(X_0 = s_0, X_1 = s_1, \dots\dots, X_n = s_n) = \pi^{(0)} P(X_1 = s_1 | X_0 = s_0) P(X_2 = s_2 | X_1 = s_1) \dots . P(X_n = s_n | X_{n-1} = s_{n-1}) = \pi^{(0)} p_{01} p_{12} \dots \dots p_{n-1\,n} \quad (6)$$

Where the row vector $\pi^{(0)} = (P(X_0 = s_0, P(X_0 = s1, \dots\dots, P(X0 = sn))$ represent the initial distribution, which tells us how the Markov chain starts. Since $\pi^{(0)}$ represents a probability distribution, we have,

$$\sum_{j \in \Omega} \pi_i^{(0)} = 1 \quad (7)$$

Using Chapman-Kolmogorov relation,

$$P(m+n) = P(m)P(n) \quad (8)$$

the one-step transition probabilities can be generalized to n-step transition probabilities,

$$P(n) = P(1)^n \quad (9)$$

The distribution of Markov chain at any time n is given by,

$$\pi^{(n)} = \pi^{(0)} P(1)^n \quad (10)$$

It is clear that the matrix $P(1)$ provides a complete characterization of Markov chain. Once we know the initial stage $\pi^{(0)}$ and the transition probability $P(1)$ we can determine the new stage of where the system will be after any n time units. As n becomes large enough the distributions stabilizes. Markov chain is widely used in studying various real-world phenomenon. In the application of Markov Chains to student's performance, the likelihood of the future progression of the students is predicted using Markov transition matrix. The main step in applying Markov chain analysis is determining the transition matrix between the different academic stages. In many applications the transition matrix is estimated by making use of the past student's academic records.

3. The model

At the undergraduate mathematics program at Tabuk University, students must complete 120 credit hours to be eligible for graduation and earn a bachelor's degree. The program runs throughout the whole year and is divided into three semesters. Key events such as graduation only depend on the completion of the required number of credit hours, where students starts to graduate from the third level onwards. Differences in timing of graduation of students in the cohort are due to the different choices students make on the number of credit hours they take each semester. Students are also allowed to withdraw and reenroll again at any time in the future.

At any point in time cohort population comprised of students of different academic stages, such as withdrawal, dismissal, graduate, enrolled. In this article we are interested in studying the underlying cohort population dynamics which govern the change in size and the transitions of students between these stages overtime.

Estimation of the cohort population dynamics is grounded on the transition probabilities between the different academic stages over a time step. In this model we assume that the transition probability matrices are stationary. Reshuffling of students among the stages is governed by the matrix $P(1)$, where $P(1)$projects the initial vector $\pi^{(0)}$ of cohort population of students forward through time.Given the current stage of student enrolled in the program the transition matrix govern the future stage at any point in time. The time evolution of the student's stage starting from some initial stage defines a trajectory in the stage space of the model. Each student trajectory we obtain in the stage space is a possible realization of the Markovian process that governs the overall student's progression in the program.

The general standard form of the probability transition matrix of an absorbing Markov chain model with r absorbing and t transient stages is,

$$T = \begin{bmatrix} I & 0 \\ R & Q \end{bmatrix} \quad (11)$$

Where Q is a $t \times t$ matrix of transition between the transient stages, R is a $t \times r$ matrix representing the transition from the transient to the absorbing stages and the $r \times r$ identity matrix I reflect the fact that student who enters any one of the absorbing stages then the probability of staying at that stage is 1, while and the $r \times t$ zero matrix reflects the fact that once student is at an absorbing stage then he is not allowed to join the program again.

In the long run the transition matrix T stabilizes and reach an equilibrium stage given by,

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$$\bar{T} = \lim_{n\to\infty} T^n = \begin{bmatrix} I & 0 \\ F\,R & 0 \end{bmatrix} \quad (12)$$

Where F is called the fundamental matrix and is given by,

$$F = (1 - Q)^{-1} \quad (13)$$

Classification of Stages: The proposed Markov system has five stages, three transient and two absorbing stages.

The transient stages are:
1) The student is currently enrolled in the program.
2) The student is inactive.
3) The student has withdrawn from the program.

The absorbing stages are:
G: The student has graduated.
W: The student is dismissed.

To drive the transition matrix between the different academic stages we have collected data from 97 student's archived records. The transition matrix is given by

$$T = \begin{array}{c} \\ G \\ S \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccccc} G & S & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0.081 & 0.049 & 0.758 & 0.053 & 0.060 \\ 0.010 & 0.054 & 0.030 & 0.879 & 0.026 \\ 0.042 & 0.048 & 0.173 & 0.053 & 0.685 \end{bmatrix} \end{array} \quad (14)$$

The transition diagram of the model is shown in figure 1.

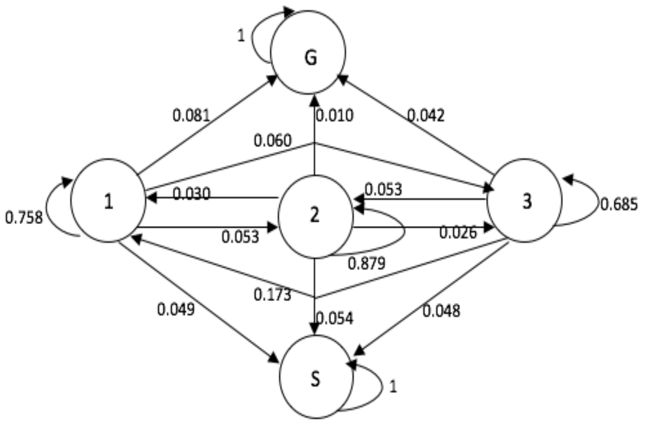

Figure 1: The transition diagram.

Given the initial stage s_0 of student's enrollment in the first year the matrix T transform s_0 to the final stage after completing the four years program. The Q and R matrices are given by,

$$Q = \begin{bmatrix} 0.758 & 0.053 & 0.060 \\ 0.030 & 0.879 & 0.026 \\ 0.173 & 0.053 & 0.685 \end{bmatrix} \quad (15)$$

$$R = \begin{bmatrix} 0.081 & 0.049 \\ 0.010 & 0.054 \\ 0.042 & 0.048 \end{bmatrix} \quad (16)$$

The fundamental matrix is therefore,

$$F = (I - Q)^{-1} = \begin{bmatrix} 5.38 & 2.89 & 1.25 \\ 2.05 & 9.70 & 1.81 \\ 2.29 & 3.20 & 4.06 \end{bmatrix} \quad (17)$$

4. Analysis

After long run the transition matrix T stabilizes and reach an equilibrium stage, which is given by,

$$\bar{T} = \lim_{n\to\infty} T^n = \begin{array}{c} \\ G \\ S \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccccc} G & S & 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0.519 & 0.481 & 0 & 0 & 0 \\ 0.316 & 0.684 & 0 & 0 & 0 \\ 0.469 & 0.531 & 0 & 0 & 0 \end{bmatrix} \end{array} \quad (18)$$

This limiting matrix $\bar{T}$ clearly shows how the population of the students in the stages 1, 2 and 3 get divided between the two absorbing stages G and S. That is in the long run 52% of the enrolled students graduates, and 48% get dismissed. 32% of students at stage 2 will graduate and 68% will be dismissed. And for students at stage 3, 47% will graduate and 53% will be dismissed. The expected time before student in one of the transient stages fall in one of the absorbing stages is given by summing the rows of the matrix N as follows:

$$N.1 = \begin{bmatrix} 5.38 & 2.89 & 1.25 \\ 2.05 & 9.70 & 1.81 \\ 2.29 & 3.20 & 4.06 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 9.53 \\ 12.93 \\ 10.54 \end{bmatrix} \quad (19)$$

- For students in stage 1 It takes an average of 9.53 years before they either graduate or get dismissed
- For students in stage 2 It takes an average of 12.93 years before they either graduate or get dismissed
- For students at 3 It takes an average of 10.54 years before they either graduate or get dismissed

Figures 2 shows the time evolution of the probability of occupying the different academic stages, It is clear that the absorbed stages G and S reach an equilibrium stage in the long run.

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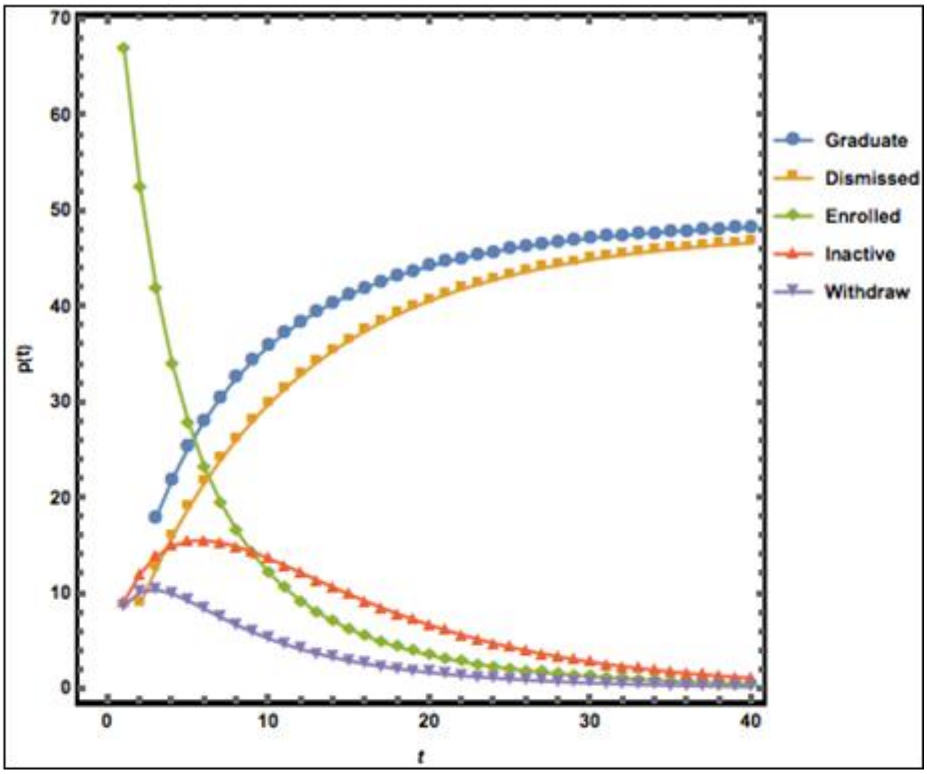

Figure 2: Time evolution of students stages over forty years period

5. Conclusion

Although this approach might lead to unreliable results, but as long as the same influences that caused past students progression continue to be present, future progression will probably be similarly influenced. Therefore by studying the past students progression patterns we will be able to some level of confidence forecast their future progression using Markov Chain.

The archived records data of student's progression at the Mathematics Program show an alarming level of dismissal of students, high withdrawal rate at the third year and high inactive rate at the sixth year from the beginning of the cohort enrollment. To understand the underlying factors leading to the observed student's performance deficiencies more quantitative and qualitative analysis needs to be developed.

In the future work, we plan to use factor analysis model to get more insights about the underlying factors affecting students progression.

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