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Number Theoretic Functions and Coordinate Geometry

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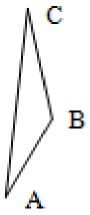
Abstract: ***In this paper, Author describes the Relation between Number Theoretic Functions and Coordinate geometry.***

1. Introduction

In these paper author describe how to find the number theoretic functions using area of triangle.

1)



(a):$q>p$,(b) z_1 , z_2 , z_3 ,z_4 greater than or equal to 0 , (c) a,b are the positive integers , (d) p^{a-1}, $p^{a+z}{}_{1}$, $p^{a+z}{}_{3}$ and q^{b-1},$q^{b+z}{}_{2}$,$q^{b+z}{}_{4}$ are in geometric progression, (e) a-1 ,$a+z_1$, $a+z_3$ and b-1,$b+z_2$,$b+z_4$ are in arithmetic progression having common difference d_1 and d_2 , $d_1= z_1+1$ and $d_2= z_2+1$ (f) d_1,d_2 are greater than or equal to 1 (g) $a+z_1$, $b+z_2$ are greater than or equal to 1. Vertices of triangle ABC are A(p^{a-1},q^{b-1}) , B($p^{a+z}{}_{1}$,$q^{b+z}{}_{2}$) and C ($p^{a+z}{}_{3}$,$q^{b+z}{}_{4}$)

Let consider triangle ABC is Eularian phi function triangle (because area of triangle ABC can be computed using eulars phi function)

Then area of triangle ABC =

$$\frac{\mathrm{Phi}(p^{a+z}{}_{1}.q^{b+z}{}_{2})\,.\,\mathrm{l}q^{d}{}_{2}\text{-}p^{d}{}_{1}\mathrm{l}\,.\,(p^{d}{}_{1}{}^{-1}+p^{d}{}_{1}{}^{-2}+\ldots..+1)\,.\,(q^{d}{}_{2}{}^{-1}+q^{d}{}_{2}{}^{-2}+\ldots..+1)}{2.p^{d}{}_{1}{}^{-1}.q^{d}{}_{2}{}^{-1}}$$

Relation between sigma function, eulars phi function and area of triangle or eularian phi function triangle.

Let area of eularian phi function triangle =A

Then, $\mathrm{phi}(p^{a+z}{}_{1}.q^{b+z}{}_{2})$ x $\mathrm{Sigma}(p^{z}{}_{1}.q^{z}{}_{2})$ = $(2.A.p^{z}{}_{1}.q^{z}{}_{2})$ / l $q^{z}{}_{2}{}^{+1}\text{-}p^{z}{}_{1}{}^{+1}$ l

If $\gcd(p^{a+z}{}_{1},q^{b+z}{}_{2})=1$, and $\gcd(p^{z}{}_{1},q^{z}{}_{2})=1$

Then, $\mathrm{phi}(p^{a+z}{}_{1})$. $\mathrm{phi}(q^{b+z}{}_{2})$. $\mathrm{Sigma}(p^{z}{}_{1})$. $\mathrm{Sigma}(q^{z}{}_{2})$ = $(2.A.p^{z}{}_{1}.q^{z}{}_{2})/\mathrm{l}q^{z}{}_{2}{}^{+1}\text{-}p^{z}{}_{1}{}^{+1}$ l { l l is mod }

Some inequality relations:

(a) $$A \geq \frac{[p^{z}{}_{1}{}^{-2}.q^{z}{}_{2}{}^{-2}\,.\,(p^{2}-1).(q^{2}-1).\,\mathrm{l}\,q^{z}{}_{2}{}^{+1}\text{-}p^{z}{}_{1}{}^{+1}\mathrm{l}\,.\,\mathrm{phi}\,(p^{a+z}{}_{1}.q^{b+z}{}_{2})]}{2.\,\mathrm{phi}(p^{z}{}_{1}.q^{z}{}_{2})}$$

(b) $$A \geq \frac{\mathrm{Sigma}(p^{z}{}_{1}.q^{z}{}_{2}).\,\mathrm{l}\,q^{z}{}_{2}{}^{+1}\text{-}p^{z}{}_{1}{}^{+1}\mathrm{l}\,.p^{a}.q^{b}}{2.\,\mathrm{Tau}(p^{a+z}{}_{1}.q^{b+z}{}_{2})}$$

According to number theoretic functions Tau(n) denote the number of positive divisors of n and Sigma(n) denotes the sum of these divisors .

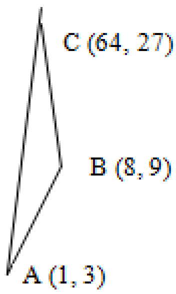
Statement: Area of eularian phi function triangle whose vertices are A (1,1), B (8, 27), C (64, 729) is Hardy Ramanujan number , that is 1729 .

Question: for what value of p, q, a, b, z_1 and z_2, Area of eularian phi function triangle is Hardy Ramanujan number .

Solution: If p=2, q=3, a=b=1 , and $z_1=z_2$ = 2 then, Area of eularian phi function triangle = Hardy Ramanujan number. {$z_3=z_4=5$}

Question: If a=1, b=2, p=2, q=3 z_1=2 and z_2= 0, find Sigma function for the above
Values using eulars phi function and area of eularian phi function triangle

Solution:



Area of eularian phi function triangle (ABC) = 105 Sq. unit.

$\mathrm{Sigma}(p^{z}{}_{1}.q^{z}{}_{2}) = [2.A.p^{z}{}_{1}.q^{z}{}_{2}]/[\ \mathrm{l}\ q^{z}{}_{2}{}^{+1}\text{-}p^{z}{}_{1}{}^{+1}\mathrm{l}\ .\mathrm{phi}(p^{a+z}{}_{1}.q^{b+z}{}_{2})\ ]$

$\mathrm{Sigma}(2^{2}.3^{0})$ = [2 x 105 x 2^2 x 3^0] / [l 3^1-2^3 l x $\mathrm{phi}(2^{3}.3^{2})$]
=[2 x 105 x 4] / [5 x 24] = 7 .

Hence, Sigma(4) = 7 , Sigma (4) , Sigma(4) = 1+2+4 .

Question: If the area of eularian phi function triangle is K, $\mathrm{phi}(p^{a+z}{}_{1}.q^{b+z}{}_{2})$=L , l $q^{d}{}_{2}$-$p^{d}{}_{1}$ l = M , $p^{d}{}_{1}{}^{-1}.q^{d}{}_{2}{}^{-1}$ = N , d_1 = 3 and d_2 = 5 . Find the product of two different polynomials and write the degree of both polynomials.

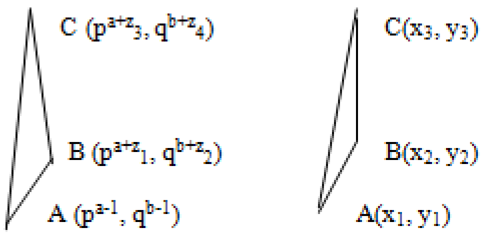
Solutions: According to (1) relation (2.k.N) / (L.M) = (P^2+P+1). $(q^4+q^3+q^2+q+1)$

Degree of first polynomial, that is p^2+p+1 is 2 (quadratic polynomial).

And degree of second polynomial, that is $q^4+q^3+q^2+q+1$ is 4 (Bi quadratic polynomial). Or degree of first polynomial = d_1-1 = 3-1 = 2, and degree of second polynomial =d_2-1 = 5-1 =4.

Statement : Let $A(p^{a-1},q^{b-1})$, $B(P^{a+z}{}_1,q^{b+z}{}_2)$,$C(p^{a+z}{}_3,q^{b+z}{}_4)$
A, B, C are never collinear, because $phi(p^{a+z}{}_1.q^{b+z}{}_2)$ never equal to zero .
Phi(n) never zero.
{phi(n) ≥ 1 } .
Hence A, B, C are never collinear.
A, B, C form a triangle

Statement:



If a=b and $z_1<x_1$, $z_1<y_1$
Then, $phi_{z1}(x_1.y_1) = phi_{2z1+1}(x_2.y_2) = phi_{3z1+2}(x_3.y_3)$
Note:
(a) $z1=z_1$
(a) If z_1=0 , then $phi_0(x_1.y_1) = x_1.y_1$
If z_1=1 , then $phi_1(x_1.y_1) = \left(\frac{phi(x_1.y_1)}{(p-1).(q-1)}\right)$
If z_1=2, then
$Phi_2(x_1.y_1)$ = phi[{phi($x_1.y_1$)}/{(p-1).(q-1)}] / {(p-1).(q-1)}

(b) $Phi_z(p^{a-1},q^{b-1})$, if a-1=b-1=z , then $phi_z(p^z.q^z)$ =1 .

(c) $Phi_z(p^{a-1}.q^{b-1})$, if a-1 = b-1 is not equal to z , then $phi_z(p^{a-1}.q^{b-1}) = P^{a-1-z}.q^{b-1-z}$

Example : $A(2^1,3^1)$, $B(2^3,3^3)$, $C(2^5,3^5)$
$A (p^{a-1},q^{b-1})$, $B(p^{a+z}{}_1,q^{b+z}{}_2)$, $C(p^{a+z}{}_3,q^{b+z}{}_4)$
$A (2^{2-1}, 3^{2-1})$, $B(2^{2+1},3^{2+1})$, $C(2^{2+3},3^{2+3})$
a=b=2, and z_1=1.
$Phi_{z1}(x_1.y_1)=phi_1(x_1.y_1)$= [phi($x_1.y_1$)] / [(p-1).(q-1)]
=[phi(2x3)] / [(2-1). (3-1)]=1
$Phi_{2z1+1}(x_2.y_2)= phi_3(x_2.y_2)$= phi[phi[Phi($2^3.3^3$)/2]/2]/2 = 1
Similarly, $phi_{3z1+2}(x_3.y_3)= phi_5(x_3.y_3)$ = 1.{note: z1 = z_1}
Hence, $phi_1(x_1.y_1) = phi_3(x_2.y_2) = phi_5(x_3.y_3)$.

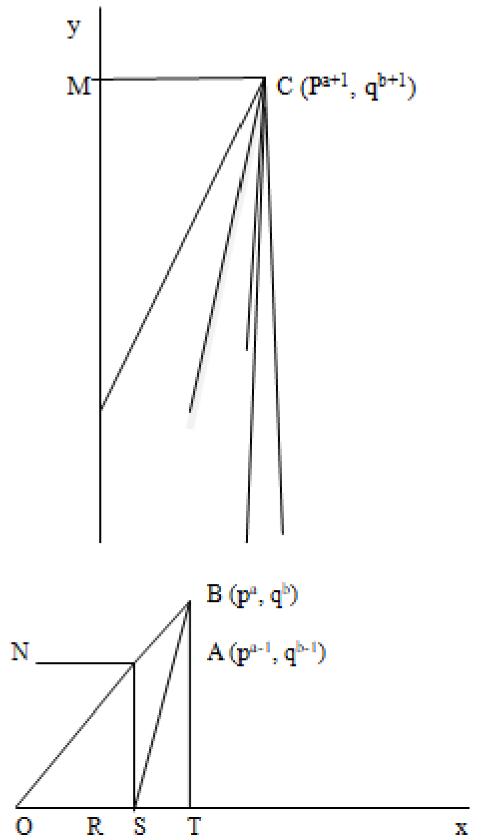
Statement: [(t_n+t_{n+1}) / 2] = phi(t_{n+2}) .
For above statement only [ ] is the Greatest Integer Function.
T_n is a triangular number , n = 1,2,3,4 .

Statement: Let T_1,T_2,T_3T_4,…. Are the tringular numbers and P_1,P_2,P_3,P_4… Are the pentagonal numbers then,
$(T_K+T_{K+1}) -(P_m+P_{m+1})$ =[{ LCM(T_K,T_{K+1})}/ {HCF(T_K,T_{K+1})}] – [{LCM(P_m,P_{m+1})} / {phi(m) x HCF(P_m,P_{m+1})}] .
Where K is odd ≥ 1 and m is odd <5 .

Statement: $phi_{n-2}(P) = [phi_{n-2}(T)]^2 = 4$, where P is pentagonal number and T is tringular number . 5<P<51 and 3<T<21 .
$Phi_{n-2}(P)$ means applying Eulars phi function $(n-2)^{th}$ times for P .
n is the number of terms (In series).

Example: Let P = 35 and T = 15
5<35<51 and 3<15<21
35 = 1+4+7+10+13, n =5
n-2 = 3. $Phi_3(35)$ = phi(phi(phi(35))) = 4
15 = 1+2+3+4+5, n=5, n-2 =3.
Phi_3 (15) = phi(phi(phi(15))) = 2 .
$[Phi_3 (15)]^2$ = 4
Hence, $phi_3 (35) = [phi_3(15)]^2$ = 4 .

(2): Eulars phi function phi(n) , n= $p^a.q^b$ where p,q are the primes (q>p) and a,b are the positive integers using Area of 8 triangles formed under MCTO (excludind Area of triangle ABC) are :



Vertices of M are $(0,q^{b+1})$, vertices of N are $(0,q^{b-1})$, vertices of o are (0,0), Vertices of R are $(p^{a-1},0)$, $S(p^a,0)$ and T are $(p^{a+1},0)$
n = $p^a.q^b$

2. Constructions

a) Construct the straight lines from the vertices A and C (of triangle ABC) to X and Y axis
b) Construct a straight line from the vertice B to X axis, not on Y axis because it cover some portion of triangle ABC.

Let the Area of 8 triangles are Ar.= Area and Tr.= Triangle .
Ar.(Tr.MNC) = T_1, Ar.(Tr.NAC) = T_2 , Ar.(Tr.ONA) = T_3 , Ar.(Tr.ORA) = T_4 ,
Ar.(Tr.RAB) = T_5 , Ar.(Tr.RSB) = T_6 , Ar.(Tr.SBC) = T_7 and Ar.(Tr.STC) = T_8 .

Note: $T_3=T_4$
If a, b≥ 2 , then , phi(R_1) / phi(R_2) = R_1 / R_2 .
R_1 = Area of Rectangle MOTC, and R_2 = Area of Rectangle NORA .

Note:
(a): Eularian phi function triangle is a scalen triangle .
(b): Eularian phi function triangle is an obtuse triangle .

Some Results:
a) $\text{phi}(n) = [2.T_2.T_5.T_7] / [T_3(T_7+T_8)]$.
b) $\text{phi}(n) = [2.T_2^{3/2}.T_7^{3}] / [T_1^{1/2}.T_3.T_8.(T_7+T_8)]$.
c) In terms of all 8 triangles.
d) $\text{Phi}(n) = [2/(T_7+T_8)]\text{x} [\{(T_4.T_5.T_6.T_2^{11}.T_7^{21})\}/\{(T_1^{3}.T_3^{9}.T_8^{7})\}]^{1/8}$.

e) Inequality Relations :
$\text{Phi}(n) \leq \{2 .T_1.T_3.T_8^{2}\} / \{T_2^{1/2}.T_7.(T_7+T_8).(T_1^{1/2}+T_2^{1/2})\}$

f) In terms of all 8 triangles:

$$\text{Phi}(n) \leq \frac{[2.T_1.T_2.T_4.(T_5+T_6)]}{[(T_2-T_1).(T_7^2-T_8^2)]} \text{ X } \frac{(T_7.T_8)^2}{T_3^2.(T_7+T_8)^2}$$

(g): $q-p \geq \dfrac{(T_1-T_2).(T_7^2-T_8^2).(T_1^{1/2}.T_7-T_2^{1/2}.T_8)}{T_1.T_2^{1/2}.T_7.T_8^2}$

.

h) phi(n) in terms of triangles and summation of 8 triangles (K) { excluding area of triangle ABC }
Phi(n) =

$$\frac{2.T_2^{1/2}.T_7.[\text{phi}^{-1}\{\text{phi}(2.T_3).(T_1/T_2).((T_8/T_7)^2)\}-k]}{T_8.T_2^{1/2}-T_7.T_1^{1/2}}$$

For, a,b≥2 and $T_1+T_2+\ldots\ldots+T_8 = k$, $(n=p^a.q^b)$

i) phi(R1) / phi(R2) = R1 / R2 =

$$\frac{2.K+\text{phi}(n).[T_8.T_2^{1/2}-T_7.T_1^{1/2}]}{T_2^{1/2}.T_7}$$

$2.\text{phi}^{-1}[(T_2/T_1).((T_7/T_8)^2).\text{phi}\{(2.T_1.T_3.T_8^2)/(T_2.T_7^2)\}]$
a, b≥2

j) phi(R_1)/R_1 = phi(R_2)/R_2
If gcd(2,T_3) =1 , then phi(R_1)/R_1 = [phi(T_3)] / [2.T_3] , a,b ≥2 .

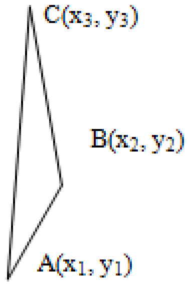
$[\text{Phi}\{(2.T_1.T_8^{2}.T_3) / (T_2.T_7^{2})\}] \text{ x } (T_2/T_1) = \text{phi}(2.T_3) \text{ x } (T_8/T_7)^2$

k) If vertices of eularian phi function triangle are A(2^{m-1},1) , B(2^m, 3), C(2^{m+1}, 9)
Where m is a positive integer, then
Phi(phi(T_1)) – phi(T_2) = phi[phi(T_1) – phi(T_2)]

l) phi(n) , n= $p^a.q^b$ can also be written as
$\text{Phi}(n) = [2(R_1-K).T_2^{1/2}.T_7] / [T_8.T_2^{1/2}-T_7.T_1^{1/2}]$

Where, $k=T_1+T_2+\ldots..+T_8$, and R_1 = Area of Rectangle MOTC

m) For eularian phi function triangle



$\text{Phi}(x_1.y_1) = \text{phi}[\{\text{phi}(x_2.y_2)\} / \{(p-1).(q-1)\}] = \text{phi}[\text{Phi}[\{\text{Phi}(x_3.y_3)\} / \{(p-1).(q-1)\}] /\{ (p-1).(q-1)\}]$.
a,b ≥2 .

In terms of triangles:
$(X_1 .y_1).(T_1^{1/2}-T_2^{1/2}).(T_8-T_7) = \text{phi}(x_2.y_2).T_2^{1/2}.T_7 = \text{PHI}[\{\text{Phi}(x_3.y_3).T_2^{1/2}.T_7\}/\{(T_1^{1/2}-T_2^{1/2}).(T_8-T_7)\}] \text{ x } (T_2^{1/2}.T_7)$
(m) : If the vertices of triangle ABC are A(1,1) , B(p,5), C(p^2, 25) , where p is a prime, p<5.

Then, phi $[\{\text{phi}(T_1+T_2).\text{phi}(T_5+T_6).\text{phi}(\text{Area of triangle ABC})\}^{1/2}] = \text{phi}(5.p)$

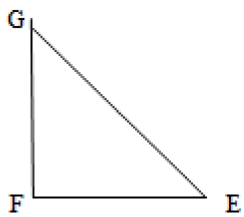
Question: If $T_1+T_2+\ldots\ldots+T_8$ = K = 752 and T_8 = 200, Find phi(n), a and n . {n is of the form $2^a.5$ } .

Solution: phi(n) = phi[K+4.T_8] / 24 = phi[752+4x200] / 24

phi(n) = 32 . n is of the form $2^a.5$, therefore a=4 and n = 80 .

Statement: If the vertices of scalen triangle ABC are A(p^{a-1},q^{b-1}) , B(p^a,q^b), C(p^{a+1},q^{b-1}) ,where p, q are the primes (q>p) and if phi($p^a.q^b$), (q-p) are the base and height of a right angle triangle EFG then ,

Area of eularian phi function triangle ABC = Area of triangle EFG.



If EF = phi($p^a.q^b$) and FG = q-p

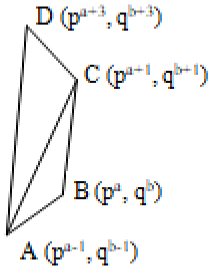
Then, Angle E =

$$\tan^{-1}\left[\frac{2 \times \text{Area of triangle ABC}}{\{phi(p^a.q^b)\}^2}\right]$$

and Angle G =

$$\cot^{-1}\left[\frac{2 \times \text{Area of triangle EFG}}{\{phi(p^a.q^b)\}^2}\right]$$

Statement: If ABCD form a quadrilateral whose vertices are



Then, Area of above quadrilateral ABCD (In terms of eulars phi functions) is given below:
[(q-p) x {p.q.phi($p^a.q^b$) + phi($p^{a+1}.q^{b+1}$).(p+1).(q+1).(p+q)}] / [2.p.q].

Benefits: It should encourage students to think more about this topic.

References

[1] Elementary number theory by David. M.Burton.
[2] First and second year BSC mathematics textbook.

Author Profile

Chirag Gupta is studying in S.Y.BSC SICES Degree College of Arts, Science and Commerce (ambernath). His mathematical works can be seen at you tube channel “Chirag Gupta”. Name of the topics are special form of phi function of area of Rectangle and some results. GIF, Leonard phi function, tringular numbers, Right angle triangle and etc. He has published two papers on eulars phi function in IJSR, in month of August and October .