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A Comparative Analysis for the Solution of Unbalanced Transportation Model by Various Methods

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Abstract: ***The transport problem has been studied in a number of fields as a key feature.As such in real life, it has been used in the computation of many problems.Therefore it was remarkably relevant for different disciplines to optimize the transportation problem of variables.An enhanced One Suffix Approach is used in this paper to solve the unbalanced transport problem and we compare these results with other various methods. Numerical examples are explained here to verify the viability of the proposed process. It's simple to understand, easy to implement.***

Keywords: Transportation Problem, Optimal Solution, Linear Programming Problem, Cost minimization, Enhanced One suffix method

1. Introduction

The transportation problem dates back to1941, when F.L.Hitchcock suggested that items be shipped from different outlets to various locations, and Koopmans and Dantzig were further developed (1949).The Simplex approach is not appropriate for transport problems because of its unique model structure, in particular for large-scale transport problems.In1954, for efficiency reasons, Charnes and Cooper invented the Stepping Stone process. Using the North-West Corner rule, Row minima, Column minima, Matrix minima, and Vogel's Approximation Method [Reinfeld and Vogel1958, Goyal's VAM edition], the Initial Basic Feasible Solution (IBFS) for transportation problem.To obtain a convenient, viable solution, a heuristic approach has been developed by Kirca and Stair. Sudhakar et al proposed a zero-suffice method in 2012 for specifically finding the optimal solution.A new, simple approach to solving the transportation problem is proposed in this paper. This presentation is considered to be the first innovative approach to the solution of unbalanced transportation problems.In order to find an optimal solution, a further iteration is also required in the above methods.The methodology algorithm is outlined in the paper and numerical examples are finally provided to demonstrate the approach and a minimized cost comparison table is provided. The paper is structured as follows: Mathematical representation and basic definitions in the first section, Different methods are summarized in the second section. Comparative analysis with numerical examples of the transport problem in the third section and the results are explained in the fourth section.Eventually, the best optimality is presented.The conclusion is being discussed.

1.1. Mathematical Formulation of a Transportation Problem:

In the transportation table O_1, O_2, ……, O_i, …, .O_m are sources from where goods are to be transported to destinations D_1, $D_{2, \ldots\ldots}D_j$, ….., D_n . Any of the sources can transport to any D_j for all i=1,2,..m and j=1,2,..n. x_{ij} is the quantity of products transported from ith source to jth destination.a_{ij} be the quantity of products available at the i^{th} source O_i and b_j, the demand at the destination Dj . The corresponding transportation problem is

$$\text{Min.z} = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}\, x_{ij}$$

$$\text{Subject to } \sum_{j=1}^{n} x_{ij} \le a_i \quad \text{for } i = 1,2,\ldots.,m$$

$$\sum_{i=11}^{m} x_{ij} \ge b_j \quad \text{for } j = 1,2,\ldots.,n$$

$$x_{ij} \ge 0 \ \text{ for } i = 1,2,\ldots.,m \text{ and for } j = 1,2,\ldots.,n.$$

This Linear Programming Problem is called Transportation Problem. It is said that the solution is feasible if$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$.

The transport problem can be expressed using a linear programming model and typically occurs in a transport table.The transport problem model can be represented in a simple tabular form with all related parameters.The transportation table (Typical TP is shown in the standard matrix form) where the availability of supplies (a_i) is shown in the far right column at each source and the destination specifications (b_j) are shown in the bottom row.One route represents each cell. The unit shipping cost (Cij) is shown in the upper right corner of the cell and the quantity of material shipped is shown in the center of the cell. Implicitly, the transportation table expresses the constraints of supply and demand and shipping.

1.2 Transportation Table

Origins (i)	Destination(j)				SUPPLY(ai)
	1	2	....	n	
1	X_{11} C11	X_{12} C12	....	C1n	a1
2	C21	C22	....	C2n	a2
3	C31	C32	....	C3n	a3
....	....	....	....	....	....
M	Cm1	Cm2	....	Cmn	am
Demand (bj)	b1	b2	....	b n	Σai = Σb j

1.3 Basic Definitions:

a) Feasible Solution(F.S)
A set of non-negative constraints $x_{ij} \geq 0$ that satisfies the constraint of rows and columns is referred to as the Feasible Solution.

b) Basic Feasible Solution(BFS)
A basic solution to problems with m-origin and n-destination is expected to be possible if the sum of the positive allocation is (m+n-1).

c) Degenerate Basic Feasible Solution
If less than (m+n-1) independently positive allocations is given in a feasible solution, then the solution is Degenerate Basic Feasible Solution.

d) Optimal Solution
A feasible solution (not necessarily basic) is said to be optimal if the total transportation cost is minimized.

e) Balanced and Unbalanced Transportation Problem
If the total supply from all sources is equal to the total demand at the destinations then it is balanced transportation problem. Otherwise it is unbalanced transportation problem.
Thus in the balanced transportation Problem $\sum a_i = \sum b_j$ and for unbalanced problem $\sum a_i \neq \sum b_j$

1.4 VAM (Vogel's Approximation Method):

The Vogel approximation method is an iterative method for computing an initial basic feasible solution to the transportation problem.

In this process, in each column, we write down the differences between least cost and next least costs, below the corresponding column, and write down the similar differences between each row. Instead of the lowest cost in each row or column, these individual differences can be viewed as a penalty for allocating the second-lowest-cost cell. We now pick the row/column for which the penalty is the highest and assign the maximum amount possible to the lowest cost cell in that cell. If there is more than one huge penalty row or column, then in the lowest cost cell, pick the row /column where we can assign more numbers.Then we cross the row or column in which the requirement (or demand) has been met and construct the reduced matrix. We continue this process on the reduced matrix, until all allocations have been made.

1.5 Improved Zero Suffix Method

The steps involved of solving a transportation problem to find the optimal solution is as follows:
Step 1: Construction of a transportation problem.
Step2: Subtract the minimum element of every row in the matrix from every element in the corresponding row.
Step3: Also, deduct from each matrix element of the corresponding column the minimum element of each column.
Step 4: Find the suffix values of all the 0"s in the reduced cost matrix by the following formula

$$S = \frac{\text{Sum of non}-\text{zero costs in the ith row and jth column}}{\text{No. of zeroz in the ith row and jth column}}$$

Step 5: Check for the highest suffix value of 0"sIf it is unique, it is reuqired to allocate the minimum demand and supply to the corresponding cell, and if it has more than one maximum value, it is possible to allocate the minimum cost to the cell. Now, delete row/column with supply/demand is exhausted and the reduced table can be found. Then go to Step2.

Step 6: Repeat step 2 to step 3 until all the demand and supply are exhausted.

1.6 Improved One's Suffix Method

Step1: Divide each row with the minimum cost and divide each column with the minimum cost so that each row and column has at least one 1 "s.

Step 2: Find the suffix values of all the 1"s in the reduced cost matrix by the following formula

$$S = \frac{\text{Sum of non}-\text{one costs in the ith row and jth column}}{\text{No. of ones in the ith row and jth column}}$$

Step3: Check for the highest suffix value of 1"s. If it is unique, the minimum demand and supply must be allocated to the corresponding cell, and if it has more than one maximum value then allocation can be done to the cell whose cost is minimum. Now, delete row/column with supply/demand is exhausted and the reduced table can be found. Then go to Step2.

Step 4: Repeat step 2 to step 3 until all the tasks have not been assigned to the persons.

Note: In order to solve an unbalanced transport problem, we first turn it into a balanced transport problem by adding a fictitious source or destination that will provide surplus supply or demand.It is assumed that the cost of shipping a commodity from a fictional source is zero.After converting the unbalanced transport problem into a balanced transport problem, by the inclusion of a fictitious source or destination, it is resolved using earlier methods.

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2. Numerical Examples

Example 1:

Consider the following Transportation Problem

Warehouse ↓	Customer 1	2	3	4	Supply
1	3	6	8	5	20
2	6	1	2	5	28
3	7	8	3	9	17
Demand	15	19	13	18	

Solution: In the given problem, $\sum a_i = \sum b_j$. Hence it is balanced transportation problem.

On applying row reduction & column reduction we get the following table.

Ware House	Customer 1	2	3	4	Supply
1	3	6	8	5	20
2	6	1	2	5	28
3	7	8	3	9	17
Demand	15	19	13	18	65

By VAM the final allocation is given in the following table:

3(2)	6	8	5 (18)
6 (9)	1(19)	2	5
7(4)	8	3(13)	9

The net cost of transportation = Rs. (2x3+18x5+9x6+19x1+4x7+13x3) = Rs. 236

By Modified Zero Suffix method the final allocation is given in the following table:

3(15)	6	8	5 (5)
6	1(19)	2	5(9)
7	8	3(13)	9(4)

The net cost of transportation = Rs. (3x15+5x5+19x1+9x5+13x3+4x9) = Rs. 209

By Enhanced One Suffix method the final allocation is given in the following table:

3(15)	6	8	5 (5)
6	1(19)	2	5(9)
7	8	3(13)	9(4)

The net cost of transportation = Rs. (3x15+5x5+19x1+9x5+13x3+4x9) = Rs. 209

Optimality Test

Using Initial Simple feasible solution obtained from VAM, find the set of $u_i(i=1,2,3)$, $v_j(j=1,2,3,4)$ such that $c_{ij} = u_i + v_j$ for occupied cells and entering u_i+v_j and d_{ij} in the unoccupied cells, the table giving the optimal solution is as follows:

3(11)	6	8	5 (9)
6	1(19)	2	5(9)
7(4)	8	3(13)	9

The net cost of transportation = Rs. (3x11+5x9+1x19+5x9+7x4+3x13) = Rs. 209

Example 2:

There are three open hearth furnaces and five steel rolling mills.The cost of transport (in rupees are shown in the following table for shipping steel from furnaces to rolling mills). We need to find an optimum shipping schedule.

Furnaces ↓	M_1	M_2	M_3	M_4	M_5	Capacities (in quintal)↓
F_1	4	2	3	2	6	8
F_2	5	4	5	2	1	12
F_3	6	5	4	7	3	14
Requirements→	4	4	6	8	8	

Solution: Here total requirement of mills = 30 quintals
And total capacity of all furnaces= 34 quintals

Since total capacity is 4 quintals more than the total requirement and hence it is an unbalanced transportation problem. Since the total capacity is 4 quintals more than the total requirement, so we convert this problem to a balanced one by introducing a fictitious mill M_6 of requirement 4 quintals with all transportation costs to this mill as zero.

The balanced transportation problem is given by the following table:

Furnaces ↓	M_1	M_2	M_3	M_4	M_5	M_6	Capacities (in quintal)↓
F_1	4	2	3	2	6	0	8
F_2	5	4	5	2	1	0	12
F_3	6	5	4	7	3	0	14
Requirements→	4	4	6	8	8	4	

By VAM, we get the following initial basic feasible solution of the problem:

Furnaces ↓	M_1	M_2	M_3	M_4	M_5	M_6	Capacities (in quintal)↓
F_1	4	2(4)	3	2(4)	6	0	8
F_2	5	4	5	2(4)	1(8)	0	12
F_3	6(4)	5	4(6)	7	3	0(4)	14
Requirements→	4	4	6	8	8	4	

The net cost of transportation = Rs.(2x4+2x4+2x4+1x8+6x4+4x6+0x4) = Rs. 80

By Modified Zero Suffix method the final allocation is given in the following table:

Method 1:

In this method first, we apply the One Suffix method by ignoring the dummy column, i.e. M6, and finally we allocate the remaining requirement/capacity to the respective cell of the dummy column.

Method 2:

In this method first, we calculate zero suffixes to all zeros of the dummy column and then minimum demand/supply must be allocated to the cell whose suffix value is maximum in dummy column. Now, we delete the dummy column and then apply one suffix method for the reduced matrix.

The final allocation table is:

Furnaces ↓	M_1	M_2	M_3	M_4	M_5	M_6	Capacities (in quintal) ↓
F_1	4	2(4)	3	2(4)	6	0	8
F_2	5	4	5	2(4)	1(8)	0	12
F_3	6(4)	5	4(6)	7	3	0(4)	14
Requirements→	4	4	6	8	8	4	

The total transportation cost =
Rs.(2x4+2x4+2x4+1x8+6x4+4x6+0x4) = Rs. 80

When we apply optimality test, we get the same solution for this example.

Comparative Study

Here, we compare the proposed method with the VAM method and the Zero Suffix method.The table below summarizes our study in this regard.

Table: A Comparative study of different methods

Examples	Optimum Value	Number of iterations required using					
		Vogel's Approximation Method		Zero Suffix method		Enhanced One's Suffix method	
		Value obtained	No. of Iterations	Value obtained	No. of Iterations	Value obtained	No. of Iterations
1	209	236	6	209	4	209	4
1.	80	80	6	80	7	80	6

This table clearly shows that the proposed method requires less number of iterations, than the other three, to get the optimalsolution.

3. Conclusion

In this paper, we proposed a different approach to finding an optimal solution to an unbalanced transport problem.With the help of numerical examples, the solution of this method is explained and tested with an optimality test. The technique suggested here is very basic, easy to understand, and easy to apply. Thus the proposed approach will help decision-makers deal with transportation problems of an unbalanced type arising in real-life situations and have an optimal solution in a simple manner.

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