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Chance Constraint Programming and PSO Techniques for a Multilevel Uncertain Transportation in Supply Chain Model

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Abstract: ***The traffic engineering management optimization problem has important component as supply chain optimization and transportation problem. In this investigation, we the shipping costs over a three tiered distribution system comprising of plants to the distribution corners to the customers, is optimized. Several plants produce multiple commodities that are shipped to distribution corners together with a fixed cost. From a distribution corner commodities are supplied to the customers. Evidently the variables such as supply capacities, demands in the model caries some uncertainty in the practical decision environment, this paper treated the supply capacities, demands. Then the uncertain transportation problem was reformulated as deterministic model by using different type of chance-constrained programming techniques. Soft computing technique like Particle swarm optimization (PSO) is used to solve numerically converting the model to deterministic model. To illustrate numerically the application and effectiveness of the proposed model.***

Keywords: Uncertain Random Programming; Chance constraint; Uncertain variables; Supply chain; transportation problem

1 Introduction

Along with transportation cost there are many additional cost. Also there are many additional cost called fixed charges associated with a transportation problem. Hirsh and Dantzig [10] introduced fixed change in a transportation problem by considering direct cost and fixed charge. While transporting in a particular place there might be toll charge, additional permit costs. The research area in TP is attracted by several researchers and developed to theory is different perspective such as Hitchock [9]. Chanas et al. [6] used fuzzy variables for demand and supply variables in a transportation problem and solved. Recently an optional solution is obtained by Fagad et al. [7] by taking interval and triangular fuzzy membership functions. Taking into account, the uncertainities in a variable we use fuzzy environment and it is clear that to determine the membership values is not easy task what is exact membership guide [5].
The transportation activities in a complex environment requires some significant variables those are taken as uncertain variable in real situation. As an example, suppose a transporter is planning for transporting in the next week, then to take a stock of source capacity, destination demands, product price, selling price, conveyance capacity. These stocks are to be judged by the transporter and a projection to be estimated with help of theory of probability and statistics based on prior information and/or statistical data. The uncertainity in the observations may be optimized may be optimized using the theory of uncertanity and methodologies. In this respect uncertain variables are introduced is standard transportation problems (STP) in the multiple supply chain model. To solve the multi objective optimization problem through optimization algorithm, there are many algorithms developed such as PSO, particles swarm optimization which is an optimization problem through optimization technique based on swarm intelligent developed as a course of bird blocking (cf. Kennedy and Eberhat [11]). PSO requires, like genetic algorithm (GA), initially swarms (a set of solutions) of the problem taken under consideration. The terms particles and food are used for individual solution in analogy to the optimal solution. Logically, the particle is moving in a multidimensional space and are adjusting every time according to the for survival. In continuous optimization field many studies are developing to improve PSO algorithm. (cf. Pedryez et al. [13], Sadeghi et al. [14], Koulinas et al. [12]). The possibility theory developed in fuzzy sets given an alternative idea to for addressing uncertainly in several modelling of real world problems involving with decision making. In recent years a new type of measure known as credibility measure, developed by Liu and Liu [4] that measures the possibility and necessity harmoniously.

In this model a transportation problem is considered with two fixed transportation stage in the supply chain network in an uncertain environment. The problem is modelled to maximize profit per units of transportation from plant to some distribution centre (DC) and from the distribution centre to business places to meet the demands from the retailer and then to the customer. The variable used in our model are uncertain variables. This variable used in our model are uncertain variables. This uncertain FCTPs are reduced to crisp FCTPs by two different techniques. The soft computing method PSO are used to solve the multi objective transporting problem. The results obtained from then two techniques are compared. Sensitivity analysis are performed at various optimistic situation of decision maker. Two motivations are explored in our problem with the uncertainity theory. First, it is more general to common sense to deal with some uncertain critical parameters. Secondly, an effective method is initiated to deal with uncertain parameters. An effective algorithm is designed to find optimal solution of the problem. This study develops two new defuzzification methods for uncertain variables. A numeral illustration is also given by three different soft computing technologies PSO and Lingo-140.0.

2 Uncertain multilevel supply chain model

In this model, we take three step distribution system comprising of manufacturing plants, distribution corners and customers. The objective is to maximize the total profit. The distribution corners incurs some fixed costs and customers are given choice from single distribution corners. The three tired transportation problem(TP) is formulated in uncertain environment as:

$$\max \widetilde{TP} = \Sigma_{i=1}^{P}\Sigma_{j=1}^{M}\Sigma_{k=1}^{N}\{(\tilde{s}_{ij} - \tilde{c}_{ijk})x_{ijk}\} - \Sigma_{i=1}^{P}\Sigma_{k=1}^{N}\Sigma_{l=1}^{R}\{\tilde{g}_{ijl} \times \tilde{d}_{il} \times Y_{kl}\} - \Sigma_{k=1}^{N}\{\tilde{f}_k \times Z_k\} \quad (1)$$

Subject to

$$\Sigma_{i=1}^{N}x_{ijk} \le \tilde{s}_{ij} \quad \forall i,j;$$
$$\Sigma_{j=1}^{N}x_{ijk} \le \tilde{d}_{il} \times Y_{kl} \quad \forall i,k,$$
$$\Sigma_{k=1}^{N}Y_{kl} = 1 \ \forall l \quad (2)$$

where a multi-product $P\ (i = 1,2,\cdots,P)$ is transported from M manufacturing plants $(j = 1,2,\cdots,M)$ to N distribution corners (DCs) $(k = 1,2,\cdots,N)$ for customers k $(l = 1,2,\cdots,R)$. Modes of transportation (conveyance) x_{ijk} decision variable denoting amount shipped, $\tilde{g}_{ijl}$ denotes cost or amount of item from DC to a customer and α_{ijk} is the percentage of damaged unit items due to delaying time or bad condition of roads or may be bad weather and others. For the objective function TC involving transportation cost like transportation cost per unit item from i-th origin to j-th destination for the k-th item $(\tilde{c}_{ijk})$, the associated fixed cost at each DC $(\tilde{f}_k)$, demand for a product by a customer $(\tilde{d}_{il})$, capacity for a product at a plant, which is an uncertain variable $(\tilde{s}_{ij})$, representing zigzag uncertain numbers. Y_{kl} denotes each customer is served by on DC, indicated by Y. Z_k is an indicator can take 0 or 1 by the decision maker.

3 Crisp equivalences for multivariate supply chain model

Here, we have used two different techniques to determine the deterministic models from uncertain models.

3.1 An uncertain CPP model 1 (UCCP-1)

Let us consider that the zigzag variables $\tilde{c}_{ijk}$, $\tilde{s}_{ij}$, $\tilde{f}_k$, $\tilde{d}_{il}$ are all mutually independent. Apply possibility measure for chance constraint (Liu [3], Dubois et al. [1]) the uncertain transportation problem given in Eq.(1) subject to the constraints given in Eq. (2) we obtain the equivalent crisp problem as:

$$\max f$$

subject to

$$M\{[\Sigma_{i=1}^{P}\Sigma_{j=1}^{M}\Sigma_{k=1}^{N}\{(\tilde{s}_{ij} - \tilde{c}_{ijk})x_{ijk}\} - \Sigma i=1P\Sigma k=1N\Sigma l=1Rgijl\times dil\times Ykl - \Sigma k=1Nfk\times Zk \ge f \ge \alpha (2)$$

$$M\{\Sigma_{i=1}^{N}x_{ijk} \le \tilde{s}_{ij}\} \ge \beta_{ij} \quad \forall\, i,j$$
$$M\{\Sigma_{j=1}^{N}x_{ijk} \le \tilde{d}_{il} \times Y_{kl}\} \ge \gamma_{il} \quad \forall\ i,l$$
$$\Sigma_{k=1}^{N}Y_{kl} = 1 \ \forall l \quad (3)$$

where α, β_i, γ_j, η_k constants signify the different optimistic levels chosen by decision makers.

The problem (2) under the constraints (3) can be solved by two different soft computing technique PSO, where the confidence level α is also safety margin predetermined by the decision-makers. Liu [3] used expectation of objective function with n independent uncertain variables with uncertainty distributions $\Phi_1^{-1}, \Phi_2^{-1}, \cdots, \Phi_n^{-1}$.

$$\max f$$

Subject to

$$\Sigma_{i=1}^{P}\Sigma_{j=1}^{M}\Sigma_{k=1}^{N}\left\{\Phi_{s_j}^{-1}(\alpha) - \Phi_{c_{ijk}}^{-1}(1-\alpha)x_{ijk}\right\} - \Sigma_{i=1}^{P}\Sigma_{k=1}^{N}\Sigma_{l=1}^{R}\{\Phi_{g_{ikl}}^{-1}(1-\alpha) \times \Phi_{d_{il}}^{-1}(1-\alpha) \times Y_{kl}\} - \Sigma_{k=1}^{N}\{\Phi_{f_k}^{-1}(1-\alpha) \times Z_k\} \ge f$$
$$\Sigma_{k=1}^{N}x_{ijk} \le \Phi_{s_{ij}}^{-1}(\beta_{ij}), \forall i,j$$
$$\Sigma_{k=1}^{M}x_{ijk} \ge \Phi_{d_{ij}}^{-1}(1-\gamma_{il}) \times Y_{kl}, \forall\, i,l$$
$$\Sigma_{k=1}^{N}Y_{kl} = 1, \forall\, l$$

in which $\Phi_{s_j}^{-1}(\alpha)$ is the inverse of uncertainty distribution of s_j Then $\Phi_{s_j}^{-1}(\alpha)$ can be further calculated by $\Phi_{s_j}^{-1}(\alpha) = (1-\alpha)a_j + \alpha b_j$ considering linear uncertain variable $\mathcal{L}(a_i, b_i)$.

3.2 An uncertain CCP model 2 (UCCP-2)

Let us consider that the zigzag variables $\tilde{c}_{ijk}$, $\tilde{s}_{ij}$, $\tilde{f}_k$, $\tilde{d}_{il}$ are all mutually independent. Apply expected value method for chance constraint (Liu [3]) the uncertain transportation problem given in Eq.(1) subject to the constraints given in Eq. (2) we obtain the equivalent crisp problem as:

$$\max \Sigma_{i=1}^{P}\Sigma_{j=1}^{M}\Sigma_{k=1}^{N}\left\{\left(E(\tilde{s}_{ij}) - E(\tilde{c}_{ijk})\right)x_{ijk}\right\} - \Sigma i=1P\Sigma k=1N\Sigma l=1RE(gijl)\times Edil\times Ykl - \Sigma_{k=1}^{N}\{E(\tilde{f}_k) \times Z_k\} \quad (4)$$

$$M\{\Sigma_{i=1}^{N}x_{ijk} \le \tilde{s}_{ij}\} \ge \beta_{ij} \quad \forall\, i,j$$
$$M\{\Sigma_{j=1}^{N}x_{ijk} \le \tilde{d}_{il} \times Y_{kl}\} \ge \gamma_{il} \quad \forall\ i,l$$
$$\Sigma_{k=1}^{N}Y_{kl} = 1 \ \forall l \quad (5)$$

The problem (4) under the constraints (5) can be solved by two different soft computing techniques PSO, where the confidence level α is also safety margin predetermined by the decision-makers. Liu [3] used expectation of objective function with n independent uncertain variables with uncertainty distributions $\Phi_1^{-1}, \Phi_2^{-1}, \cdots, \Phi_n^{-1}$.

$$\max \Sigma_{i=1}^{P}\Sigma_{j=1}^{M}\Sigma_{k=1}^{N}\left\{\left(E(\tilde{s}_{ij}) - E(\tilde{c}_{ijk})\right)x_{ijk}\right\} - \Sigma i=1P\Sigma k=1N\Sigma l=1RE(gijl)\times Edil\times Ykl - \Sigma_{k=1}^{N}\{E(\tilde{f}_k) \times Z_k\} \quad (6)$$

$$\Sigma_{i=1}^{N}x_{ijk} \le \Phi_{s_{ij}}^{-1}(\beta_{ij}) \ \forall\, i,j$$
$$\Sigma_{j=1}^{M}x_{ijk} \ge \Phi_{d_{il}}^{-1}(1-\gamma_{il}) \times Y_{kl} \quad \forall\ i,l$$
$$\Sigma_{k=1}^{N}Y_{kl} = 1 \ \forall l \quad (7)$$

3.3 An uncertain CPP model 3 (UCCP-3)

Let us consider that the zigzag variables $\tilde{c}_{ijk}$, $\tilde{s}_{ij}$, $\tilde{f}_k$, $\tilde{d}_{il}$ are all mutually independent. Apply possibility measure for chance constraint (Liu [3]) the uncertain transportation problem given in Eq.(1) subject to the constraints given in Eq. (2) we obtain the equivalent crisp problem as:

$$\max \Sigma_{i=1}^{P}\Sigma_{j=1}^{M}\Sigma_{k=1}^{N}\left\{\left(E\left(\tilde{s}_{ij}\right)-E\left(\tilde{c}_{ijk}\right)\right)x_{ijk}\right\}-$$
$$\Sigma i=1P\Sigma k=1N\Sigma l=1RE(gijl)\times Edil\times Ykl \qquad (6)$$
$$-\Sigma_{k=1}^{N}\{E(\tilde{f}_k)\times Z_k\}$$
$$\Sigma_{i=1}^{N}x_{ijk} \le E(\tilde{s}_{ij})\ \forall\ i,j$$
$$\Sigma_{j=1}^{M}x_{ijk} \ge E(\tilde{d}_{il})\times Y_{kl} \quad \forall\ i,l$$
$$\Sigma_{k=1}^{N}Y_{kl} = 1\ \forall l$$

where α, β_i, γ_j, η_k constants signify the different optimistic levels chosen by decision makers.

4 Numerical experiment

Soft computing methodology PSO (Particle Swarm Optimization [2]) is used to solve our model.

4.1 Input data

Let us assume a system with $P=2$, number of products, $M=3$, number of plants, $N=4$, number of distribution corners, $R=5$, number of customers. Let us consider the unit transportation cost, fixed charge cost, demand and supply are linear uncertain in nature and are given in Table 1 to Table 5.

Table-1: Capacities of Plant $\tilde{s}_{ij}$ (in ton)

$\mathcal{L}(60,100)$	$\mathcal{L}(30,50)$	$\mathcal{L}(60,90)$
$\mathcal{L}(15,25)$	$\mathcal{L}(30,90)$	$\mathcal{L}(70,80)$

Table-2: Unit transportation costs $\tilde{c}_{ijk}$ (in $)

$\mathcal{L}(0.5,1.5)$	$\mathcal{L}(2,4)$	$\mathcal{L}(2.5,3.5)$	$\mathcal{L}(4.5,5.5)$	$\mathcal{L}(2,6)$
$\mathcal{L}(3.5,5.5)$	$\mathcal{L}(0.5,2.5)$	$\mathcal{L}(1.5,6.1)$	$\mathcal{L}(1.5,2.5)$	$\mathcal{L}(4.3,2.3)$
$\mathcal{L}(1.3,3.1)$	$\mathcal{L}(2.5,3.9)$	$\mathcal{L}(0.5,1.5)$	$\mathcal{L}(1.5,2.5)$	$\mathcal{L}(1.8,2.2)$
$\mathcal{L}(3.5,6.5)$	$\mathcal{L}(1.5,6.5)$	$\mathcal{L}(1.4,7.8)$	$\mathcal{L}(0.5,2.1)$	$\mathcal{L}(1.9,5.1)$
$\mathcal{L}(0.7,2.9)$	$\mathcal{L}(1.9,4.1)$	$\mathcal{L}(1.8,4.2)$	$\mathcal{L}(1.7,2.3)$	$\mathcal{L}(2,5)$

Table-3: Fixed charge costs $\tilde{f}_{ijk}$ (in $)

$\mathcal{L}(70,130)$	$\mathcal{L}(100,200)$	$\mathcal{L}(60,260)$	$\mathcal{L}(132,146)$

Table-4: Demand for a product $\tilde{d}_{il}$ (in $)

$\mathcal{L}(23,27)$	$\mathcal{L}(19,41)$	$\mathcal{L}(43,57)$	$\mathcal{L}(11,19)$	$\mathcal{L}(15,35)$
$\mathcal{L}(23,27)$	$\mathcal{L}(2.9,13.1)$	$\mathcal{L}(2.5,3.5)$	$\mathcal{L}(21,39)$	$\mathcal{L}(20,40)$

4.2 Optimum results

Table-5: Optimistic results via PSO for UCCP-1

Optimistic labels			Received amount	Total Profit
α	β_{ij}	γ_{ij}		
0.2	0.2	0.2	244	1287
	0.15	0.15	242	1287
	0.10	0.10	241	1285
	0.05	0.05	239	1282
0.15	0.2	0.2	238	1238
0.1			238	1236
0.5			238	1231

Table-6: Optimistic results via PSO for UCCP-2

Optimistic labels			Received amount	Total Profit
α	β_{ij}	γ_{ij}		
0.95	0.95	0.95	260	2156
	0.90	0.90	259	2155
	0.85	0.85	258	2152
	0.80	0.80	254	2067
0.90	0.95	0.95	255	2065
0.85			255	2065
0.80			256	2527

The optimistic value of UCCP-2 has been given in Table-7 for different optimistic labels.

Table-6: Optimistic results via PSO for UCCP-2

Optimistic labels		**Received amount**	**Total Profit**
β_{ij}	γ_{ij}		
0.95	0.95	260	2156
0.90	0.90	259	2155
0.85	0.85	258	2152
0.80	0.80	254	2152
0.95	0.95	255	2067
	0.90	256	2033
	0.80	256	2517

The optimized results has been ordained by two different soft computing techniques GRG and PSO. From this Table 7, it is observed that PSO results is better than the GRG results.

Table-7: Optimistic results via PSO for UCCP-3

PSO	
Amount	**Total Cost**
244.72	**2126.53**

Table-7 contains the best solution (maximumprofit), standard deviation for 20 times running of the PSO, lower bound, percentage gapbetween the best solution and the lower bound, upper bound and percentage gap between the bestsolution and the upper bound for each instance.

5 Conclusions

This paper focuses on generating the optimal solutions of the multilevel distribution in a supplychain transportation problem under uncertain environment, in which the supply capacities, demandsand transportation capacities, unit transportation cost and fixed charge cost are supposedto be zigzag uncertain variables due to the instinctive imprecision. The total profit of the uncertain transportation problem is reformulated as the chance-constrained programming model with theleast expected transportation cost. Numerical experiments are implemented to illustrate the applicationand effectiveness of the proposed approaches. To give a modelling framework for optimizationproblems with multi-fold uncertainty, different reduction methods were proposed to transform auncertain variable into deterministic variable. This paper attempted to propose some methodsof reduction for uncertain variables, and applied the methods to the transportation problem with uncertain inputs.

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