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Adaptation of the Short Time Fourier Transform for Analysis of Nonlinear Time-Varying Continuous Systems

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Abstract: ***The most commonly used tool for system analysis is the Fourier Transform,whichcan be applied to linear continuous systems. If a linear system is time-varying, the Short-Time Fourier Transform can be used. In another hand, the Volterra transform can consider an extension of the Fourier transform that is applied basically to continuous nonlinear systems. By the use of the Associated Linear Equations that are parametric models of Volterra operators, the present workobtains the Short-Time Fourier Transform for continuous nonlinear systems. The Short-TimeHigher-Order Frequency Response Function is also defined***

Keywords: Nonlinear systems, time-invariant systems, Frequency Response Function, Associated Linear Equations, Short-Time Fourier Transform, Volterra series

1. Introduction

One of the most widely usedmethods for signal analysis is the Fourier Transform (FT). It has been used for both linear and nonlinear systems analysis, e.g., in agriculture [1], material structure detection [2], and [3] vision analysis [4]. The theory for nonlinear applications has been widely studied[5]. The mainlimitation oftheFT is that it cannot detect changes in time. For linear systems, this deficiency has been overcome by using the Short-Time Fourier Transform (STFT),of which many examples can be found in the literature, e.g.,[7] and [8].

The FTfornonlinear systems wasdeveloped based on the Volterra series [6]; the close relationshipbetween the FT and Volterra series has been madeevident in such works as [9], [10], and [11].

The appropriate version of the FT fortime-varying systems is the STFT,which introduces a wavelet(window) in the FT integral [12]. There are a variety of wavelets, from the simpleHaar[13] andDaubechies [14],to the flexible Mexican hat (the Ricker wavelet) [15],as well as many more, each with its strengths and limitations. The main criterion for selectinga wavelet isbased on the characteristics of the system response. To present general results, we will use a Mexican hat in this work, asit is easy to implement and is widely used.

The use of the STFT is restricted to linear systems due to the complexity of the nonlineargeneration of harmonics, even in the simple case of Volterra systems. The identification and analysis of nonlinearsystems is simplyimpossible,as there is no clear relationship between the different harmonic components. The appropriate tool for the analysis of nonlinear systems is Associated Linear Equations (ALEs) [13]. Since ALEs are linear models, each one produces a particular order of Volterra operators. This kind of orthogonalization allows us to analyze in detail the generation of harmonics because of the effect of the window functions onnonlinear continuous systems.

The objective of this work is to find an expression that allows us toanalyzethe frequency domain of a nonlinear system that is also time-varying. This expression is obtained by the unit impulse response of each ALE.The effect of the windowfunction on the output can then be easily visualized, and it is possible to findanexpression for a Short-Time Higher Order Fourier Transform (STHFT) and a Short-Time Higher-Order Frequency Response Function (STHFRF).

The work is structured as follows. Section 2 states the basicdefinitions; section 3 analyses how to incorporate thewindow functions in the higher-order kernels; section 4 develops the appropriate expression for the STHFRF and the STHFT; section 5 presents a simulated second-orderDuffing oscillator from which the STHFRF and the STHFTareobtained. The last section presents the conclusions.

2. Background

This section presents the basicdefinitions required for subsequent development of the expressions for the STHFRF and the STHFT. The output of a continuous nonlinear system may be obtained usingthe Volterra series, as follows,

$$y(t) = \sum_{0}^{\infty} y_n \tag{1}$$

Each term of the Volterra series is the response of the$n$-th order operator.

The Associated Linear Equations (ALEs) [14] are a linear model of each operator. As a linear system, the output of any operator can be obtained from theunit impulse response function$h_{1(n)}(\tau)$ through the following convolution [15],

$$y_n(t) = \int_{-\infty}^{\infty} h_{1(n)}(\tau) x_n(t-\tau)\, d\tau \quad (2)$$

The subscript 1 means that $h_{1(n)}(t)$ is a linear operator; $x_n(t)$ is the corresponding n-th order input signal vector. It is also possible to obtain $y_n(t)$ as,

$$y_n(\tau_1, \tau_2, \ldots \tau_n) = \int_{-\infty}^{\infty} h_n(\tau_1, \tau_2, \ldots \tau_n) x(t_1 - \tau_1) x(t_2 - \tau 2 \ldots xtn - \tau n\, d\tau 1 d\tau 2 \ldots d\tau n \quad (3)$$

The variable $h_n(\tau_1, \tau_2, \ldots \tau_n)$ is known as the n-th order kernel; it is a multilinear impulse response.
The Frequency Response Function of each ALE can be obtained from the following equation,

$$H_{1(n)}(\Sigma^n \omega) = \frac{1}{\pi} \int_{-\infty}^{\infty} h_n(\tau) e^{-i(\Sigma^n \omega)\tau}\, d\tau \quad (4)$$

where the summation representsthe sum of n-th input frequenciesinthe system.

The n-dimensional Fourier Transform is defined by [16],

$$\mathcal{F}\, f(t_1, t_2, \ldots t_n) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(t_1, t_2, \ldots t_n)\, e^{-i\omega_1 t_1} e^{-i\omega_2 t_2} \ldots e^{-i\omega_n t_n} dt_1\ dt_2 \ldots\ dt_n \quad (5)$$

This equation allows one to obtain the Higher Frequency Response Function (HFRF) of order n as a function of$h_n(\tau_1, \tau_2, \ldots \tau_n)$,

$$H_n(\omega_1, \omega_2, \ldots \omega_n) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h_n(\tau_1, \tau_2, \ldots \tau_n)\, e^{-i\omega_1 t_1} e^{-i\omega_2 t_2} \ldots e^{-i\omega_n t_n} dt_1\ dt_2 \ldots\ dt_n \quad (5)$$

The relationship between $h_{1(n)}(\tau)$ and $h_n(\tau_1, \tau_2, \ldots \tau_n)$is obtained from[15] (for the Duffing Oscillator),

$$h_n(\tau_1, \tau_2, \tau_3, \ldots, \tau_n) = \frac{n!}{r_1! r_2! \ldots r_n!} \int_{-\infty}^{\infty} h_{1(n)}(\tau) \sum_{p=1}^{p=n} \sum_{i_1=1}^{F(\frac{n}{p})} \sum_{i_2=1}^{F(\frac{n-i}{p-1})} \sum_{i_3=1}^{F(\frac{n-i-j}{p-2})} \ldots \sum_{i_s=i_{s-1}}^{F(\frac{n-i-j-\ldots}{2})} h_{i_1}(\tau_1 - \tau,, \tau_2 - \tau, \ldots \tau_{i_1} - \tau hi2\tau i1+1-\tau,, \tau i1+2-\tau, \ldots \tau i2+i1-\tau hi3\tau i2+i1+1-\tau,, \tau i2+i1+2-\tau, \ldots \tau i1+i2+i3-\tau \ldots hn-k=1p-1ik(\ldots, \tau n-\tau) d\tau \quad (6)$$

This is a simple convolution.

The Short-time Fourier Transform (STFT) is obtained from,

$$W_\Psi\left(f(t)\right) = \int_{-\infty}^{\infty} f(t)\, \Psi_{a.b}\, t\, dt \quad (7)$$

where $\Psi_{a.b}$ is a function of two parameters; a and ba wavelet,

$$\Psi_{a.b} = \emptyset(a, t-b) e^{-i\omega t} \quad (8)$$

The wavelet used in this work is known as the Mexican hat (Ricker wavelet),

$$\emptyset(a,b) = \frac{2}{\sqrt{3b}\pi^{\frac{1}{4}}} \left(1 - \frac{(x-a)^2}{b^2}\right) e^{-\frac{(x-a^2}{2b^2}} \quad (9)$$

3. The Effect of Temporal Windows on Volterra Kernels

Our objective is to study the effect of unit impulse responses ofALEs on the Volterra kernels when windows are introduced. Since the kernel itself is expected to vary in time, it is necessary to know how to detect the changes and how past states affect the present response of the system.

To analyze the response of the second-order (harmonic) Volterra operatorof a second-order second Duffing Oscillator, we consideredthe following model,

$$y\ddot{(t)} + A_1 y\dot{(t)} + A_2 y(t) + A_3 y^2(t) = Bx(t) \quad (10)$$

The second order Associated Linear Equation (ALE) is (see [14]),

$$\ddot{y}_2(t) + A_1 y_2\dot{(t)} + A_2 y_2(t) = -A_3 y_1^2(t) \quad (11)$$

Still from [14], the second-order kernel is obtained fromthe following unit impulse response function:

$$h_2(\tau_1, \tau_2) = \int_{-\infty}^{\infty} h_{1(2)}(\tau) h_1(\tau_1 - t) h_1(\tau_2 - \tau) d\tau \quad (12)$$

If one assumes that the Duffing oscillator varies with time, equation (10) may be expressed as

$$y\ddot{(t)} + A_1(t)\dot{y}(t) + A_2(t) y(t) + A_3(t) y^2(t) = B(t) x(t) \quad (13)$$

Let's assume that there is a small $\Delta t$for which the coefficients of the systems show negligible variation. Then there is a window function thin enough to capturethe response with negligible distortion. Aunit impulse response function$n_1$can be defined as

$$\eta_1(a, b, t-\tau) = h_1(t-\tau) \emptyset(a, t-b) \quad (14)$$

This unit impulse response function can be handled as if it weretime-invariant. Referring to equation (12), the argument of the second-order kernel are two different unit impulse response functions, and the integration is over the entire time domain. Sinceunit impulse responseschange with time, it is necessary to know to which state of the impulse response is the second-order kernel responding. As the integration is over the entire time domain, the twoimpulses may indicate two different states. This problem is solved by using a window for each impulse response,whichcan be equal or different to each other. The convolution is now

$$\int_{\infty}^{\infty} h_{1(2)}(\tau) h_1(\tau_1 - t) \emptyset(a_1, t - b_1) h_1(\tau_2 - \tau) \emptyset(a_2, t - b_2) d\tau$$

As the system is time-variant, the second order impulse response also changes;therefore, equation (12) has to include an additional window,

$$\eta_2(a_1, a_2, a_3, b_1, b_2, b_3, \tau_1, \tau_2,) = \int_{\infty}^{\infty} h_{1(2)}(\tau) \emptyset(a_3, t - b3h1\tau 1 - t\emptyset a1, t - b1h1\tau 2 - \tau\, \emptyset a2, t - b2 d\tau \quad (15)$$

Equation (3) reproduces an output signal that is not only a function of two times, but also of three different windows,

$$\eta_2(a_1,a_2,a_3,b_1,b_2,b_3,\tau_1,\tau_2)=\int_{-\infty}^{\infty}\eta_{1(2)}(a_3,b_3,\tau)\eta_1(a_1,b_1,\tau-\tau_1)\eta_1(a_2,b_2,\tau-\tau_2)d\tau \qquad (16)$$

Forthe nth order response, the number of windows increases according to the number of components of the input signal. For the general second-order response,one has,

$$y_2(t_1,t_2,a_1,a_2,a_3,b_1,b_2,b_3)=\int_{-\infty}^{\infty}\eta_2(a_1,a_2,a_3,b_1,b_2,b_3,\tau_1,\tau_2)x(t_1-\tau_1)x(t_2-\tau 2\, d\tau 1 d\tau 2 \qquad (17)$$

The output of the second-order operator, whent_1=t_2,must be

$$y_2(t)=\sum_j^n\sum_k^n\sum_i^n\ y_2\left(a_j,a_k,a_i,b_j,b_k,b_i,t,t\right) \quad (17)$$

Analogous to equation (6), the *n*th-order Volterra kernel fortime-variant nonlinear systemsis

$$\eta_n(a_1,a_2,\ldots\ldots,a_{n+1},b_1,b_2,\ldots\ldots b_{n+1},\tau_1,\tau_2,\ldots\ldots\tau_n)=\frac{n!}{r_1!\ r_2!\ldots r_n!}\int_{-\infty}^{\infty}\eta_{1(n)}(a_{n+1},b_{n+1},\tau)\sum_{p=1}^{p=n}\ \sum_{i_1=1}^{F\left(\frac{n}{p}\right)}\sum_{i_2=1}^{F\left(\frac{n-i}{p-1}\right)}\ \sum_{i_3=1}^{F\left(\frac{n-i-j}{p-2}\right)}\ldots$$

$$\sum_{i_s=i_{s-1}}^{F\left(\frac{n-i-j-..}{2}\right)}\eta_{i_1}\left(a_1,a_2,\ldots\ldots,a_{i_1+1},b_1,b_2,\ldots\ldots b_{i_1+1},\tau-\tau_1,\tau-\tau_2,\ldots\ldots\tau_{i_1+1}\right)$$

$$\eta_{i_2}\left(a_1,a_2,\ldots\ldots,a_{i_2+1},b_1,b_2,\ldots\ldots b_{i_2+1},\tau_1-,\ldots\ldots\tau_{i_2+i_1}-\tau-\tau_{i_1}\right)$$

$$h_{i_3}\left(\tau-\tau_{i_2+i_{1+1}},,\tau-\tau_{i_2+i_1+2},\ldots\tau-\tau_{i_1+i_2+i_3}\right)\ldots$$

$$\eta_{n-\sum_{k=1}^{p-1}i_k}(a_1,a_2,\ldots\ldots,a_{n+1},b_1,b_2,\ldots\ldots b_{n+1},\tau-\ \tau_1,-\tau,\ldots\ldots\tau-\tau_n)d\tau(18)$$

And the output of the *n*-th order operator is then

$$y_n(t)=\underbrace{\sum_{j=1}^n\sum_{k=1}^n\sum_i^n\ldots\sum_{m=1}^n\ y_n\left(a_j,a_k,a_i,\ldots\ldots.a_m.b_j,b_k,b_i,\ldots\ldots b_m,t,t,\ldots t\right)}_{n+1\text{ summations}} \qquad (17)$$

4. The short time higher order frequency response function and the short time higher order fourier transform

After the previous section, we have now all thatis needed to develop the expressions for the Short Time Higher-Order Frequency Response Function (STHFR) and the Short time HigherOrder Fourier Transform (STHFT).

We need to reconsiderthe Duffing oscillator and its second-order operator. From the definition given in equation (14), a second-order kernel is obtained fromequation (16).

From equation (5), the bilinear Fourier transform (FT) is,

$$\mathcal{F}\ f(t_1,t_2)=\int_{-\infty}^{\infty}(\int_{-\infty}^{\infty}f(t_1,t_2)\,e^{-i\omega_1 t_1}\,dt_1)\,e^{-i\omega_2 t_2}\,dt_2 \ (18)$$

Let's divide the transform in two independent integrals, i.e. two linear FT. If the function is now the second-order kernel, one has,

$$\mathcal{F}\ y_2(a_1,a_2,a_3,b_1,b_2,b_3,t_1,t_2)$$
$$=\int_{-\infty}^{\infty}\left(\int_{-\infty}^{\infty}\eta_2(a_1,a_2,a_3,b_1,b_2,b_3,\tau_1,\tau_2)\delta(t_1-\tau_1)\delta(t_2-\tau_2)\,e^{-i\omega_1 t_1}\,dt_1\right)e^{-i\omega_2 t_2}\,dt_2$$

Developing the internal integral for the bidimensional unit impulse response,

$$\mathcal{F}\ y_2(a_1,a_2,a_3,b_1,b_2,b_3,t_1,t_2)=\int_{-\infty}^{\infty}\left(\int_{-\infty}^{\infty}N_2(a_1,a_2,a_3,b_1,b_2,b_3,\omega_1,\tau_2)\,e^{i\omega_1 t_1}\delta(t_2-\tau 2 e{-}i\omega 2t2\,dt2 \qquad (19)$$

The second unit impulseresponse$\delta(t_2-\tau_2)$ is a constant resulting from the integral. Integrating again (4.22),

$$\mathcal{F}\ y_2(a_1,a_2,a_3,b_1,b_2,b_3,t_1,t_2)=N_2(a_1,a_2,a_3,b_1,b_2,b_3,\omega_1,\omega_2)e^{i\omega_1 t_1}e^{i\omega_2 t_2} \qquad (20)$$

it reproduces the well-known result that is the base of the harmonic probing technique. The FT of the n-impulse response is the response in time of the input of nphasors; both time parameters must be equal to obtain a non-zero result,

$$y_2(a_1,a_2,a_3,b_1,b_2,b_3,t)=N_2(a_1,a_2,a_3,b_1,b_2,b_3,\omega_1,\omega_2)e^{-i(\omega_{1+}\,\omega_2)t} \qquad (21)$$

This isthe response of the system under the effect of three different windows.

Here, $N_2(a_1,a_2,a_3,b_1,b_2,b_3,\omega_1,\omega_2)$is the Short Time Higher Order Frequency Response Function (STFRF) of second order.

Because of equation (16), the STHFRF may be obtained by transforming,

$$N_a(a_1,a_2,a_3,b_1,b_2,b_3,\omega_1,\omega_2) = \iiint_{-\infty}^{\infty} \eta_{1(2)}\,(a_3,b_3,\tau)\eta_1(a_1,b_1,\tau-\tau_2)\eta_1(a_2,b_2,\tau-\tau 1 e - i\omega 1 \tau 1 e - i\omega 2 \tau 2\ \ d\tau 1\, d\tau 2\, d\tau \quad (22)$$

Substitutingthe following parameters

$$\tau_a = \tau_2 - \tau$$
y
$$\tau_b = \tau_1 - \tau$$

one has

$$N_a(a_1,a_2,a_3,b_1,b_2,b_3,\omega_1,\omega_2) = \iiint_{-\infty}^{\infty} \eta_{1(2)}\,(a_3,b_3,\tau)\eta_1(a_1,b_1,\tau_a)\eta_1(a_2,b_2,\tau_b)e^{-i\omega_1(\tau_b+\tau)}e^{-i\omega_2(\tau_b+\tau)}\ \ d\tau\, d\tau_a\, d\tau_b$$

This allows us to separate the integral as follows;

$$N_a(a_1,a_2,a_3,b_1,b_2,b_3,\omega_1,\omega_2) = \int_{-\infty}^{\infty} \eta_{1(2)}\,(a_3,b_3,\tau)e^{-i(\omega_1+\omega_2)\tau} \int_{-\infty}^{\infty} \eta_1(a_1,b_1,\tau_a)\, e^{-i\omega_1\tau_a} \int_{-\infty}^{\infty} \eta_1(a_2,b_2,\tau_b)\, e^{-i\omega_2\tau_b} \quad (23)$$

Developing the integrals,

$$N_a(a_1,a_2,a_3,b_1,b_2,b_3,\omega_1,\omega_2) = N_{1(2)}(a_3,b_3,\omega_1+\omega_2)\ N_1(a_1,b_1,\omega_1)\ N_1(a_2,b_2,\omega_2) \quad (24)$$

The FT of equation (14) gives

$$N_1(a,b,\omega) = \int h_1(t-\tau)\emptyset(a,t-b)\,e^{-i\omega t}\,dt \quad (25)$$

Since b is a constant, this equation represents the FT of a convolution; then,

$$N_1(a,b,\omega) = H_1(\omega)\emptyset(a,b,\omega) \quad (26)$$

$\emptyset(a,b,\omega)$is the FT of the wavelet function. Substituting in (24) one has

$$N_2(a_1,a_2,a_3,b_1,b_2,b_3,\omega_1,\omega_2) = H_{1(2)}(\omega_1+\omega_2)\emptyset(a_3,b_3,\omega_1+\omega_2)\ H_1(\omega_1)\emptyset(a_1,b_1,\omega_1)\ H_1(\omega_2)\emptyset(a_2,b_2,\omega_2)$$

which can be rearranged as

$$N_2(a_1,a_2,a_3,b_1,b_2,b_3,\omega_1,\omega_2) = H_{1(2)}(\omega_1+\omega_2)H_1(\omega_1)H_1(\omega_2)\emptyset(a_3,b_3,\omega_1+\omega_2)\emptyset(a_1,b_1,\omega_1)\emptyset(a_2,b_2,\omega_2) \quad (27)$$

or as a function of the second-order kernel,

$$N_2(a_1,a_2,a_3,b_1,b_2,b_3,\omega_1,\omega_2) = H_2(\omega_1,\omega_2)\emptyset(a_3,b_3,\omega_1+\omega_2)\emptyset(a_1,b_1,\omega_1)\emptyset(a_2,b_2,\omega_2) \quad (28)$$

This is the relationship between the second-order FRF and the second-order STHFRF; for any order, one has

$$N_n(a_1,b_1,a_2,b_2,\dots a_{n+1},b_{n+1},\omega_1,\omega_2\dots\omega_n) = H_n(\omega_1,\omega_2\dots\omega_n)\emptyset(a_{n+1},b_{n+1},\omega_1+\omega_2+\dots+\omega_n)\sum_{i=1}^{n}\emptyset(a_i,b_i,\omega_i) \quad (29)$$

It is well known that $H_n(\omega_1,\omega_2\dots\omega_n)$ is the FT of the kernel of the same order (equation (5)). The STHFRF can be obtained as

$$N_n(a_1,b_1,a_2,b_2,\dots a_{n+1},b_{n+1},\omega_1,\omega_2\dots\omega_n) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h_n(\tau_1,\tau_2,\dots\tau_n)\,e^{-i\omega_1 t_1}e^{-i\omega_2 t_2}\dots e^{-i\omega_n t_n}dt_1\ dt_2\dots\ dt_n\emptyset(a_{n+1},b_{n+1},\omega_1+\omega_2+\dots+\omega_n)\sum_{i=1}^{n}\emptyset(a_i,b_i,\omega_i) \quad (30)$$

Equation (30) containsn+1 integrals, i.e. n+1FTs. Thiscan be manipulated to form a single argument for all the integrals, as follows,

$$N_n(a_1,b_1,a_2,b_2,\dots a_{n+1},b_{n+1},\omega_1,\omega_2\dots\omega_n) = \int_{-\infty}^{\infty}\int \dots\int_{-\infty}^{\infty}\emptyset(a_{n+1},\tau_1-b_{n+1},\tau_2-b_{n+1},\dots\tau_n - bn+1 hn\tau 1,\tau 2,\dots\tau n\emptyset a1,\tau 1-b1e-i\omega 1\tau 1\emptyset a2,\tau 2-b2e-i\omega 2\tau 2\dots\dots\emptyset an,\tau n-bne-i\omega 2\tau nd\tau 1d\tau 2\dots d\tau n \quad (31)$$

As any function can be the argument, the Short Time Higher Order Fourier Transform STHFRF can be defined as

$$\mathcal{F}_n(a_1,a_2,\dots\dots a_{n+1},b_1,b_2,\dots\dots b_{n+1},\omega_1,\omega_2,\dots\dots\omega_n) = \int_{-\infty}^{\infty}\int \dots\int_{-\infty}^{\infty}\emptyset(a_{n+1},t_1-b_{n+1},t_2-b_{n+1},\dots t_n-b_{n+1})f_2(t_1,t_2,\dots t_n)\emptyset(a_1,t_1-b_1)e^{-i\omega_1(t_1-\tau_1)}\emptyset(a_2,t_2-b_2)e^{-i\omega_2(t_2-\tau_2)}\dots\dots\emptyset(a_n,t_n-b_n)e^{-i\omega_2(t_n-\tau_n)}dt_1\,dt_2\dots dt_n \quad (32)$$

5. A Second Order Signal

In this section, a simulated Duffing oscillator of the second degree is used to obtain a second-order signal, then the Short Time Higher-order Frequency Response Function (STHFRF) and the Short Time Higher Fourier Transform (STHFT) of the second harmonic order output is obtained. The model is,

$$\ddot{y} + A(t)\dot{y}(t) + B(t)y(t) + C(t)y(t)^2 = D(t)x(t) \quad (33)$$

Observe that the system parameters time-varying. This system is simulated and the response is obtained by the addition of the exact solution of each Volterra operator,

$$y_n(t) = \sum_{\substack{combination\ \ of \\ elements\ \ taken\ \ n\ in\ n}} \int_{-\infty}^{\infty} H_{1(n)}\Big(\sum\nolimits_{nth-frequencies}\ \omega\Big)e^{-i(\sum_{nth-frequencies}\ \omega)t}dt \quad (34)$$

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The first four operators were found significant. The corresponding Associated Linear Equations (ALEs) are,

$$\ddot{y}_1 + A(t)\dot{y}_1(t) + B(t)y_1(t) = D(t)x(t) \quad (34)$$

$$\ddot{y}_2 + A(t)\dot{y}_2(t) + B(t)y_2(t) = C(t)y_1^2(t) \quad (35)$$

$$\ddot{y}_3 + A(t)\dot{y}_3(t) + B(t)y_3(t) = 2C(t)y_2(t)y_1(t) \quad (36)$$

$$\ddot{y}_4 + A(t)\dot{y}_4(t) + B(t)y_4(t) = C(t)y_2^2(t) + C(t)y_3(t)y_1(t) \quad (37)$$

The coefficients of equation (33) vary as a function of the system natural frequencies (500, 750, 1000 y 300rd/sec), though damping ratio can be handled also, changing natural frequencies is easier to detect time changes.

The input is a band-limited white noise (Figure (1)). The output in Figure (2) shows the system changes along the time.

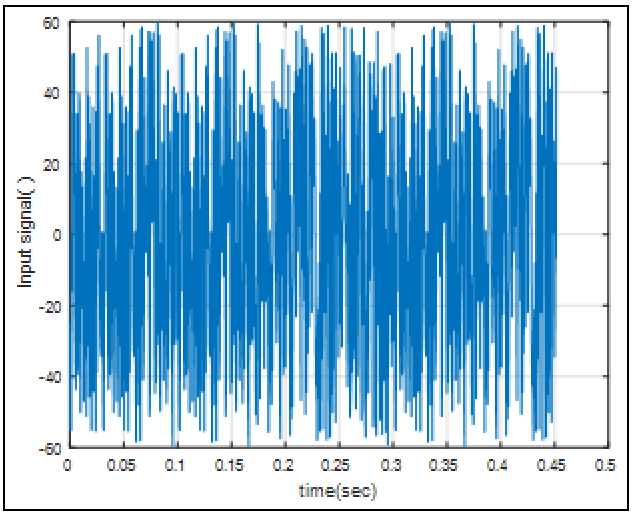


Figure 1: Duffing oscillator input system

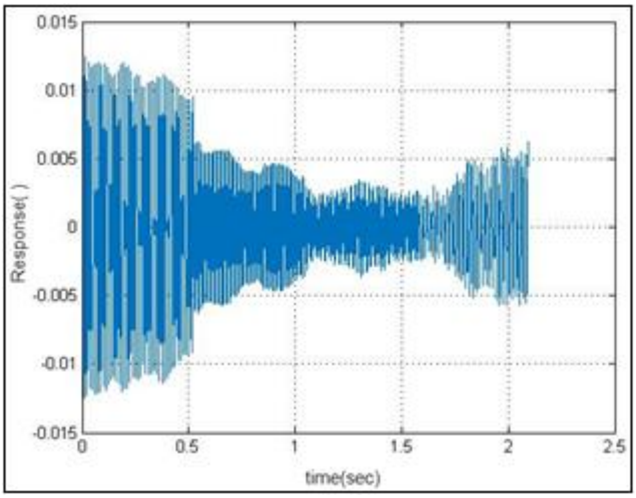


Figure 2: System response showing its change in time

Small input power allows detecting the first-order signal in the response. The Mexican hat wavelet that is used for this work is shown in Figure 3.

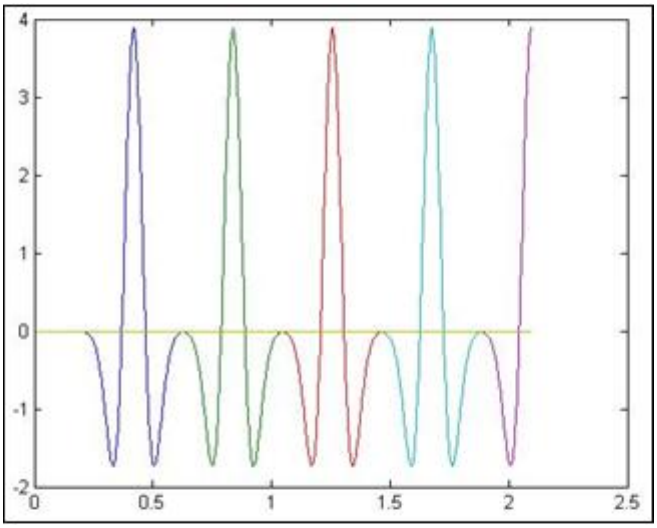


Figure 3: The time wavelet distribution

The Short-Time Fourier Transform of the first-order signal shows the clear four región of four different frequencies. Figure 4.

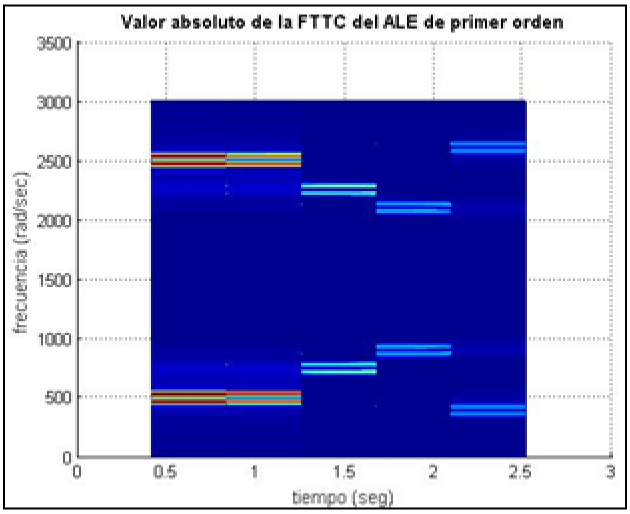


Figure 4: The STFT of the first-order signal

The first order Frequency Response Function (FRF) is shown if Figure 5.

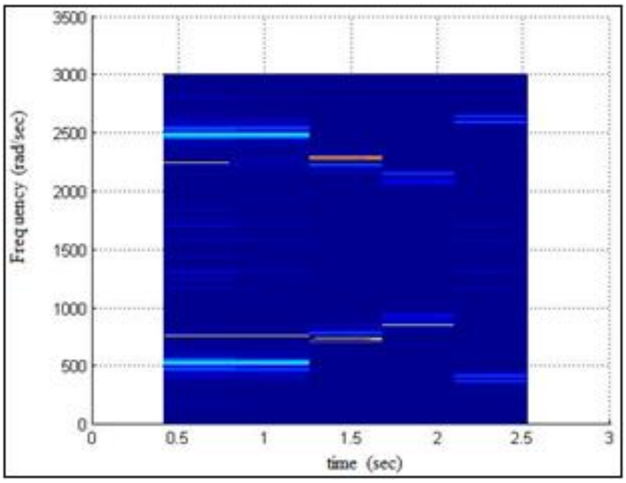
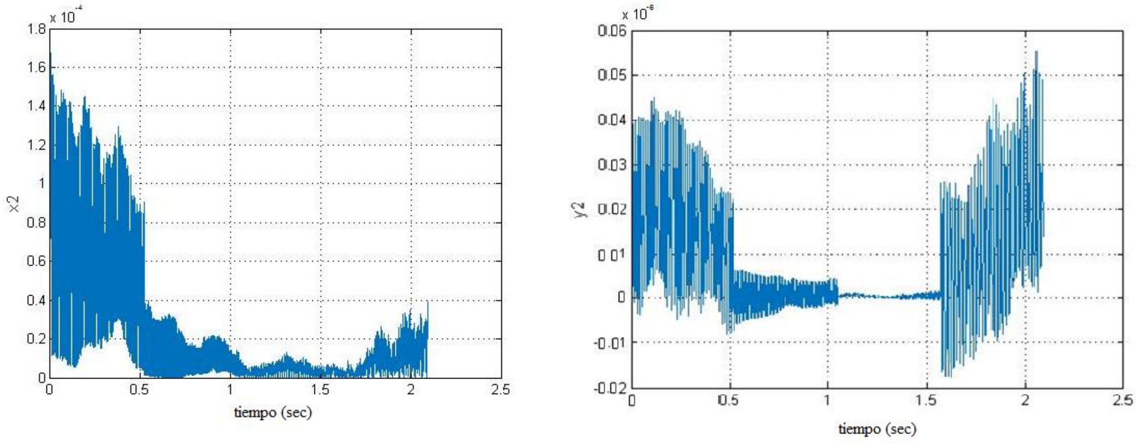


Figure 5: First-order FRF

The input into the second-order ALE is according with equation (36),

$$x_2 = {y_1}^2$$

The second-order input x_2 and output y_2 can be seen in figure 6. A linear FRF ($H_{1(2)}$) can be obtained for this second-order ALE, and it is displayed in Figure 7.



a) Entrada a el ALE de segundo order

b)Salida del ALE de segundo order

Figure 6: Second-order ALEs signals

Just as in the case of the FRF, the STFRF of second-order depends on a sum of two frequencies that are the vertical axis of the surface graph shown in Figure 8.

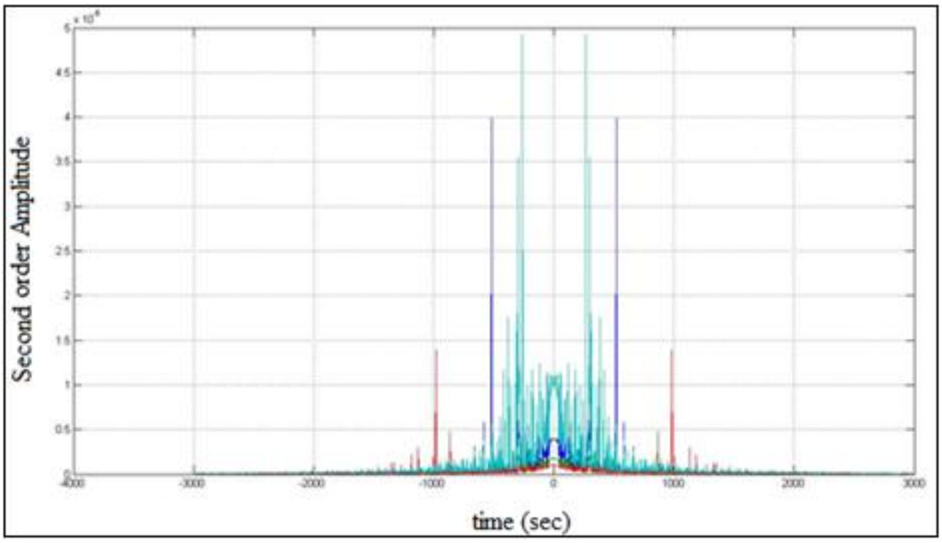


Figure 7: The absolute value of the second-order FRF $H_{1(2)}$

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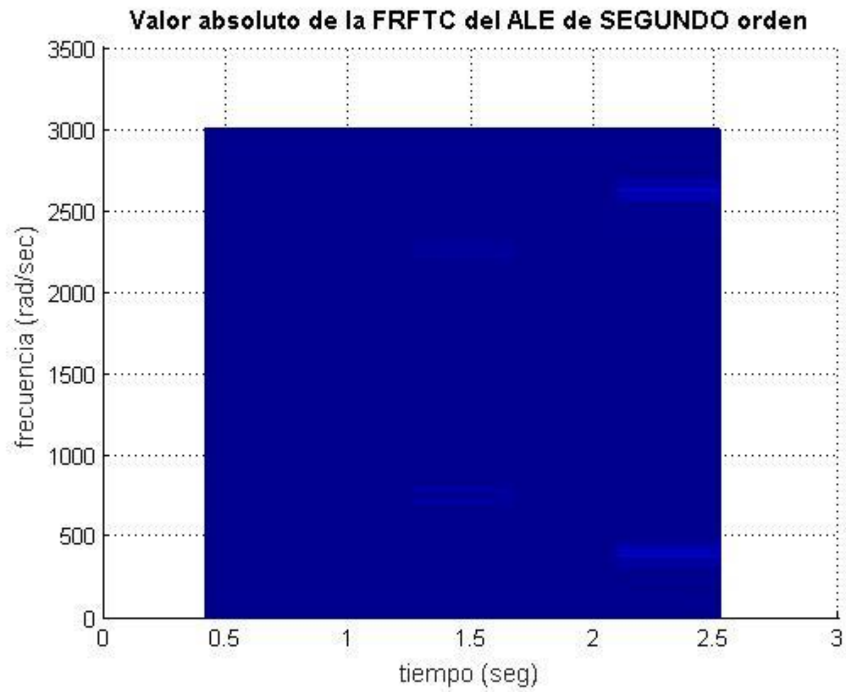
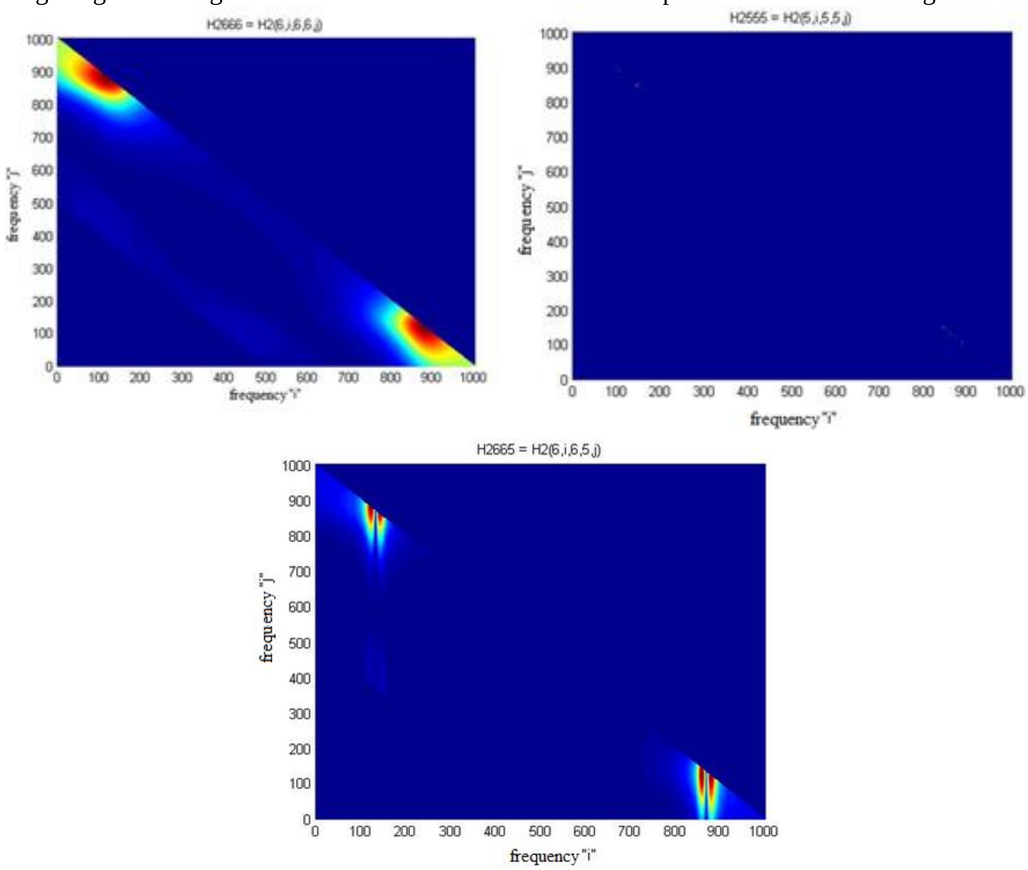


Figura 8: The STFRF de el ALE de Segundo orden

The changes in the natural frequency are easily observed.

The graphics for the Short Time Higher-order Frequency Response Functions STHFRFis more complex than theseSTFRF of the second-order ALE as it depends on several windows. The input into the second ALE is a product of first-order signals affected by two different windows. This signal goes through the second-order ALE that is also affected by a third window. The response beside to be a function of two frequency and time, it is also a function of the windows that affect the signal.

Figure 9, for example, shows the graphic corresponding to the signal obtained from the first-order Volterra operator affected by the sixth and fifth window on the first-order part and the sixth window in the second-order signal H2665. Some examples are shown in this Figure.



v

Figure 9: The STHFRF of second-order

6. Conclusions

In this work, the Short-time Fourier Transform (STFT) is adapted for the analysis of nonlinear continuous time-varying systemsby the use of the Associated Linear Equations (ALEs). Two expressions are obtained, one is the Short Time Higher order Fourier Transform (STHFT) that is a formal definition, and the Short Time Higher-order Frequency response Function (STHFRF) that allow us to see how the frequency domain response change in the time of a nonlinear continuous system.

The main problem to deal with this kind of analysis is that the STHFT and the STHFRF are of more dimensions than its harmonic order. Visualizing the continuous nonlinear system under the point of view of the Volterra series, if the system has as feedback powers of the output signal, the input into the higher-order ALEs is a product of lower-order signals that comes out from operators that change over time, this changes are isolated by the use of windows (windowing). Several signals of the same order of different windows interact as products of themselves and between them, producing the input into an operator of order "n" that also varies in time. Therefore, several different windows are acting together giving the result of combinations of at least "n" deferent system conditions related to the same numbers of t_p's (central pints of the windows). This ends in a parameter that depends on 2n+1 dimensions at least.

Here, is also presented a simulation of a Duffing oscillator system of the second degree. Its second-order response is then analyzed in the frequency domain. The second-order signal is the output of the second-order ALE, The input into this ALE is the product of two lineal signals producing that the input depends on two different t_p's. As the second-order operator also varies along the time, there are three different times (t_p's) involved plus two frequencies. This means that the second-order STHFRF depends on 5 independent variables.

To be able to produce a graphic display of the frequency domain behavior it is necessary to specify which windows or times t_p's are involved in the input and the window for the nth order transform. Therefore, a lot of different graphs can be produced for each combination of the system states.

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