Quantum Information Reaching the Black Holes

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Abstract:
In this paper, we considereted the quantum state information of qubits moving according to the quantum adiabatic dynamics around a black hole. We demonstrated that a fibre bundles geometric structure gives a quantum state description and spacetime geometry discription in a same framework. We analysed the fact that the black hole induces a decoherence on the qubit in this framework, particulary on the quantum information when it reaches the event horizon. Then we analysed Kerr, Schwarzschild and Rindler black hole cases by computing the fidelity of the teleportation.

Keywords: Quantum teleportation, Black Hole,qubit in curved spacetimes, non-Hermitian adiabatic dynamics, brebundle

I. INTRODUCTION

Black holes are such dense bodies that the speed of release in their vicinity exceeds the speed of light in a vacuum. As a result, no body too close can escape from the black hole, and not even light due to a relativistic effect. The border of the region around the black hole where no return is possible is called the event horizon. For the simplest black holes (Schwarzschild black holes) this horizon is spherical. In 1975, Stephen Hawking demonstrated that black holes must emit thermal radiation. This radiation can only be emitted from the horizon, because below it would be trapped. Or, nothing is on the horizon! Being defined only as the boundary such as vlib = c, the event horizon is not a material structure. But for thermal radiation to be emitted, atoms or particles must be agitated. From this observation, an black hole entropy was then associated with this object to compensate for the loss of information. Nevertheless, it remains to find a theoretical framework in which this macroscopic quantity is associated with the microstates of a thermodynamic system. String theory and loop theory of gravity seem to be possible frameworks at present. Physicists believe the answer to this question lies in a theory of quantum gravity. One of them, brane theory (a variant of string theory), assumes that space is not continuous but a frame made up of a network of quantum strings on the Planck scale $(10-33cm)$. Gravitation is then the manifestation of the structure of the network and its vibrations. In this theory, the event horizon would be a network of quantum strings with the topology of a sphere (what is called a fuzzy sphere), in a state of thermal equilibrium (it is the agitation resulting from vibrations of the strings constituting the structure of the space which would be at the origin of the thermal radiation). To be in a thermal state, the fuzzy sphere must be in contact with an environment, the latter is made up of emptiness ! It is the quantum fluctuations of the vacuum which would cause the thermalization of the fuzzy sphere. Note that the gravity effects on quantum systems have been demonstrated by many works such as [1],[2]. Palmer et al [3]. The work here is to rewrite the qubit quantum states and spacetime geometry in a same framework by using a fibre bundles and we showing that the black hole created quantum decoherence on the qubit, especially if the qubit reach it. In addition, the qubits reaching a black hole is describes by the quantum adiabatic dynamics; In other words, we showing that the teleportation fidelity is describes by the geometric and dynamical phases in quantum decoherence.The case of many blacks hole is analysed.

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II. METHODS

A. Adiabatic approximation in a curved space time

let $e_i^A(x)$ be the moving coordinate system associated with $g_{ij} = \eta_{AB} e_i^A e_j^B$ the metric and where η_{AB} is the Minkowski metric. With $\omega_\mu^{AB} = e_i^A \partial_\mu e^{Bi} + e_i^A \Gamma^i_{\mu j} e^{Bi}$ the Lorentz connection. The Dirac-Einstein equation is given by:

$$(\imath \gamma_A e_i^A(x) D_i - \mu)\phi(x) = 0 \tag{1}$$

with $\gamma_{A\in 0,1,2,3}$ the Dirac matrix and D_i is a spinor covariant derivate:

$$D_i = \frac{\partial}{\partial x^i} + \omega^i_{AB}\Sigma(M_{AB}) \tag{2}$$

Here Σ is (1/2, 0) $\oplus$ (0, 1/2) the $SL(2,\mathbb{C}^2)$ representation(main group of group $SO^+(3,1)$ on $\mathbb{C}^2 \oplus \mathbb{C}^2$

$$\Sigma(M_{AB}) = \frac{1}{4}[\gamma_A, \gamma_B] \tag{3}$$

let M tangent bundle be $PM \to M$ and M main bundle space $FM \to M$. Where $\psi_T : \mathbb{R}^4 \otimes M \to PM$ and $\psi_F : M \otimes SO^+(3,1) \to FM$:

$$\psi_T^i(x,v) = e_A^i v^A \tag{4}$$

and

$$\psi_F(x,\Lambda) = e_A^i \Lambda \tag{5}$$

(with e ∈ GL(4, $\mathbb{R}^4$ the element matrix e^i_A) The quadri field can be designed as the

$$FM : x \rightarrow e(x) = \varphi_F(x, id) \in \Gamma(M, FM)$$

local sections trivialization.
if $T \rightarrow M$ is a principal bundle with the spinors transformation $SL(2,\mathbb{C})$ (T is an FM extented like $FM = T/Z_2$. If $E \rightarrow TM$ and $\bar{E} \rightarrow TM$ vector bundles with the (1/2,0) and (0,1/2) representation, a trivialization can defined $E \rightarrow TM$

$$\psi_E = TM \times \mathbb{C}^2 \rightarrow E \quad (6)$$

with

$$\psi_E(v(x), \phi) = [\psi_T(\pi_P(v(x), g), g^{-1}\phi]; g \in SL(2,\mathbb{C}) \quad (7)$$

With ψ_P the $T \rightarrow PM$ a local trivialization and πP the $PM \rightarrow M$ projection;

$$\psi_E(v(x), \phi) = [\psi_{T^{-1}}(\pi_P(v(x), g), g^{-1}\phi]; g \in SL(2,\mathbb{C}) \quad (8)$$

(Let $(\frac{1}{2}, 0)$ the $g \in SL(2,\mathbb{C})$ action by $g\phi \in \mathbb{C}^2$ and (0,1/2) the g action on ϕ by $g^{-1}\phi$; $\Gamma(FM, \bar{E} \oplus E)$ be a Hilbert module $\mathbb{C}^0(TM)$ with an inner product such as

$$\forall \phi, \varphi \in \Gamma(FM, \bar{E} \oplus E)$$

,

$$\langle \phi \varphi \rangle_{\Gamma(FM, \bar{E} \oplus E}(u(x)) = \langle \phi u(x) | \gamma_A \gamma_0 | \varphi u(x) \rangle_{\mathbb{C}^4} u_A(x) \quad (9)$$

In other words $u(x) \in P_x M(u_A.u_A = 1, u_A = e^i_A u_i)$ if $\Xi \subset M$ is a spatial hypersurface of M,then

$$M^+\Xi = \{n \in PM_\Xi; \forall t \in P\Xi, g_{ij}n^i t^j = 0; g_{ij}n^i t^j > 0\} \quad (10)$$

with forward-oriented normal vectors at Ξ. The Dirac field ϕ is a Dirac space vector

$$L^2(M^+\Xi, \bar{E} \oplus E) = \{\phi \in \quad (11)$$

$$\Gamma(M^+\Xi, E^{-1} \oplus E) \int_\Xi \|\phi\|^2(n(x)) d\Xi(x) < 0\} \quad (12)$$

The living Dirac spin fields space has a moment in the instantaneous space Ξ. If l is a line of the geodesic and $\Xi_{\tau \in R}$ a M violation along l by spatial hypersurfaces (with τ, the time appropriate along l) by Wentzel Kramers Brillouin. The curvature scale is big than compton wavelength in approximation assumption association

$$\int L^2(M^+\Xi, \bar{E} \oplus E) d\tau \rightarrow^{WentzelKramersBrillouin} \Gamma(P\mathbb{C}, \bar{E} \oplus E) \quad (13)$$

the spatial delocalization of the fermion supporting the qubit is deleted by the semi-classical approximation and the ambiguity of the number of particles is also deleted by the absence of a second quantization. Note that the description of a single spin (qubit) is been due to the representation (1/2.0) ,let project on the space $\Gamma(T\mathbb{C}, E \oplus \bar{E})$. Then work is based on the composite bundle $^4\ E \rightarrow TM \rightarrow M$. Note the easierly to work with the bundle $E^+ \rightarrow M$ and the fiber structure $\mathbb{R}^4 \times \mathbb{C}^2$. The local trivialization defines E^+ by

$$\psi_{E^+} : M \times \mathbb{R}^4 \times \mathbb{C}^2 \rightarrow E^- \quad (14)$$

$$\psi_{E^+} : \varphi_E(\psi_T(x, v), \phi) = \psi_E(e(x)v, \phi) \quad (15)$$

by the action of $SL(2,\mathbb{C})$ on E^+ defined by:

$$\forall g \in SL(2,\mathbb{C})$$

;

$$D_+(g)\psi_{E^+} = \psi_{E^+}(x, \wedge(g)v), Dg\phi) \quad (16)$$

where ∧: the group $SL(2,\mathbb{C}) \rightarrow SO^+(3,1)$ associated with quotient $SO^+(3,1)/SL(2,\mathbb{C})/\mathbb{Z}_2$.
The $E^+\Gamma(M, E^+)$ locals space is identified at the $SO^+(3,1)$ E sections locales space invariant:

$$\Gamma_i(PM, E) = \{\phi \in \Gamma(PM, E); \quad (17)$$

$$\forall \wedge \in S0^+(3,1), \forall v \in PM, \phi(\wedge v(x)) = \phi(v(x))\} \quad (18)$$

The following property is obtains by $\Gamma(TM, E)$ restriction to be invariant sections :

$$\forall \phi, \varphi \in \Gamma_i(PM, E), \forall g \in SL(2,\mathbb{C}), \forall v \in PM$$

$$\langle D(g)\phi | D(g)\varphi \rangle_{\Gamma(PM,E)}(\Lambda(g)v(x)) = \langle \phi | \varphi \rangle_{\Gamma(PM,E)}(v(x)) \quad (19)$$

In other words the Lorentz transformations allows the quantum properties to be invariant underthe inner product

$$\langle \phi_v | \varphi_w \rangle_{\Gamma(PM,E^+)}(x) = \langle \phi | \varphi \rangle_{\Gamma(PM,E)}(v(x)) \delta(u(x) - w(x)) \quad (20)$$

Let $\phi_v(x) = \phi(v(x)) = \phi_E(Pr\phi^-_E + (\phi_v))\delta$ be the distribution of dirac, $\Gamma(PM, E^+)$ be a Hilbert module $C(M)$. Let's rewrite the Dirac Einstein equation as the Van der waerden equation

$$\phi = (\varphi_A, \chi_{A'}) \quad (21)$$

$$\gamma_I = \begin{pmatrix} 0 & \sigma_A \\ \bar{\sigma_A} & 0 \end{pmatrix} \quad (22)$$

$$\imath \bar{\sigma_A} D_u \varphi_A = m \chi^{A'} \quad (23)$$

$$\imath \sigma_A D_v \chi^{A'} = m \varphi_A \quad (24)$$

With $\chi^{A'}$ pulled in the first equation and inserted in the second we got

$$e^A_u \bar{\sigma}_A e^B_u \sigma_A D_u D_v \phi_E + m^2 \phi_E = 0 \quad (25)$$

$\phi_E \in \int^\oplus_R L^2(M^+\Xi, E) d\tau$ with

$$\{\sigma_A\}_A = \{id, \sigma_x, \sigma_y, \sigma_z\}$$

;

$$\{\bar{\sigma_A}\}_A = \{id, -\sigma_x, -\sigma_y, -\sigma_z\}(\sigma_x, \sigma_y, \sigma_z)$$

The Pauli matrices. It allowed

$$e_u^A \bar{\sigma}_A e_u^B \sigma_A (D_{\{u}D_{v\}} + D_{[u}D_{v]})\phi_E + m^2\phi_E = 0 \tag{26}$$

$2D_{[u}D_{v]} = [D_u D_v]$;$2D_{\{u}D_{v\}} = \{D_u D_v\}$ et $g_{uv} = \sigma_{\{A}\bar{\sigma_{A\}}}$

$$g_{uv} D_v D_u \phi_E - \iota L_{AB}(R_{uv} - ie\delta F_{uv})\phi_E + m^2\phi_E = 0 \tag{27}$$

$F_{uv} = 2D_{[u}A_{v]}$; $\sigma_{\{u}\sigma_{v\}} = g_{uv}$; $R_{uv}\psi_E = 2D_{\{u}D_{v\}}\psi_E$ and $L_{AB} = e_A^u e_B^v L_{IJ} = \frac{1}{2}\sigma_{[u}\sigma_{v]}$ which results in $L_{IJ} = \frac{1}{2}[\sigma_I\bar{\sigma}_J - \sigma_J\bar{\sigma}_I]$ and R_{uv} the Ricci tensor; we are setting $\phi_E = \phi e^{\iota S/\varepsilon}$ with $D_u S = k^u$ (k^u is the length wave, $k_u k^u = m^2$) in the WKB approximation that assumes that:

$$g_{uv} D_v D_u \phi - \iota L_{AB} R_{uv}\phi \tag{28}$$

$$+\frac{\iota}{\varepsilon}(2k^u D_u\phi + \phi D_u k^u \phi + ie\delta F_{uv})\phi_E) \tag{29}$$

$$-\frac{1}{\varepsilon^2}k_u k^u \phi \tag{30}$$

$$+m^2\phi_E = 0 \tag{31}$$

$$2k^u D_u\phi + \phi D_u k^u \phi + ie\delta F_{uv})\phi_E = 0 \tag{32}$$

$$k_u k^u \phi - m^2\phi_E = 0 \tag{33}$$

$k^u = D_u\theta - eA_u$ and by deriving the last equation

$$D_u(k_u k^u - m^2) = 2k^u D_u k_u = 2k^u D_u(D_u\theta - eA_u) \tag{34}$$

$$= 2(k^u D_u k_v \theta + ek^u F_{uv}) = 0 \tag{35}$$

Let $u^u(x) = \frac{k^u(x)}{m}$ defined by $\frac{dx^u}{d\tau} = u^u$

$$m\frac{D^2x^u}{D\tau^2} + e\frac{dx^v}{d\tau}F_{uv} = 0 \tag{36}$$

If we multiply the first equation of the previous system by $k_u\bar{\sigma}_A^u\psi$ and by adding with the conjugate

$$2k_u D_u\phi + \phi D_u k_u = 0 \tag{37}$$

with the geodesic C

$$\ddot{x}^u + \Gamma^u_{\rho v} x^\rho x^v = 0 \tag{38}$$

with $\frac{k^u}{m} = \dot{x}^u$

$$\psi \in \Gamma_i(PM, E) \simeq \Gamma(M, E_+) \tag{39}$$

which respects the Schrödinger equation:

$$\frac{\iota d\phi}{d\tau} = -\frac{1}{2}\omega_u^{AB}(x(\tau))\dot{x}^u(\tau) L_{AB}\phi(\tau) \tag{40}$$

with natural time τ along the geodesic where the qubit reaches. In the WKB properties ψ consists of a very small wave packet localized essentially around the spatio-temporal curvature. If $\phi(\tau) \in \pi_E^{-1}(u(\tau))$ where π_E is the total space E and base PM fibration and $u(\tau) = \dot{x}^u\frac{\partial}{\partial x^u} \in T_{x(\tau)}M$, the Hilbert space for a instantaneous qubit $\pi_E^{-1}(u(\tau)) \subset \Gamma_i(PM, E)$ depends of quadri-velocity. Dirac's field theory induces the connection of the inner product

$$\langle\phi, \varphi\rangle_{\tau(TM,E)}(u(x)) = \langle\phi u(x)|\bar{\sigma}_A\varphi u(x)\rangle_{\mathbb{C}^2}(u(x)) \tag{41}$$

$$= \langle\phi^* u(x)|\varphi u(x)\rangle_{\mathbb{C}^2} \tag{42}$$

where $\phi^* = \bar{\sigma}_A u_A(x)\phi(u(x))$ is the conjugate state. The Einstein Dirac gives an interaction of the Dirac field with gravity. These terms in the localized qubit of Schrodinger's equation with the operator $-\frac{1}{2}\omega_u^{AB}(x(\tau))\dot{x}^u(\tau)L_{AB}\phi(\tau)$ are found. There is an equivalence with these two and the equations of a fermion. Where Lorentz connection is considerated as "the field of gravity" felt by qubit or fermion. The local inertial frame of reference in the neighborhood of x is defined by $\{e^A\}_A$. For best interpreting of the Hilbert space-time dependence on the speed quadrivector instantaneous spin, it is important to remind some quantum mechanics axioms. Hilbert space constitutes a quantum system states space but its dual H^*, represents the continuous linear space functions of H, is the elements amplitudes probability space : Let $l \in H^*$ be H representation in $\mathbb{C}$ with$(l(\psi))^2$, the probability of associated events to l performing the measurement for ψ state quantum system. According to Riesz's theorem $\forall l \in H^*, \exists! n \in H$ (phase factor and renormalization with $\forall\psi \in H$), we have:

$$l(\phi) = \frac{\langle n_l|\phi\rangle_H}{\|n_l\|_H\|\phi\|_H} \tag{43}$$

Let's take $\langle n_l| \in H^*$ an observable Θ co-vector associated with measure. Where $\langle n_l|$ is Θ result event measure λ_l with (λ_l the value associated with $\langle n_l|$. With ket $|\phi\rangle$ measuring the quantum system event. Geting back ours objectives is to define two linear functions $|\phi\rangle$ and $\langle\phi^*| = \langle\bar{\sigma}_A^+ u_A\phi|$ difference(bra is the domestic product part of $\mathbb{C}^2$:$\langle\phi|.\rangle_{\mathbb{C}^2}$, never $\langle\phi|.\rangle_{\Gamma(PM,E)}$. If $\phi \in \Gamma_i(PM, E)$ is a normalized state we have:

$$\langle\phi|.\rangle_{\Gamma(PM,E)} = 1 \tag{44}$$

$$\iff \langle\phi|\bar{\sigma}_A|\phi\rangle_{\mathbb{C}^2} u_A = 1 \tag{45}$$

$$\iff \|\phi^2\|u_0 - \langle\phi|\sigma_A|\phi\rangle_{\mathbb{C}^2} u_A = 1 \tag{46}$$

$$\iff \gamma(S_0 - \vec{S}\vec{v}) = S_{0*} \tag{47}$$

with quadri-magnetic moment operator is:

$$\{\acute{S_A}\}_A = \{id, \sigma_x, \sigma_y, \sigma_z\} \tag{48}$$

$S_A = \langle\phi|\acute{S_A}|\phi\rangle_{\mathbb{C}^2}$; $\gamma = u_0$,$\gamma\vec{v} = \vec{u}$ and

$$S_{0*} = \langle\phi|\frac{id}{2}|\rangle_{\Gamma(TM,E)} = \langle\phi^*|\frac{id}{2}|\rangle_{\mathbb{C}^2} = \frac{id}{2} \tag{49}$$

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the previous equation shows the relationship between quadri-moment magnetic S measured in inert space K and quadri-moment S_* measured in the space at quadri-velocity $\vec{u} = (\gamma, \gamma\vec{v})$ rest K^* based on K. For solving this problem, K and the black hole must co-moving, in others part K^* and qubit must co- moving. By the normalization analysis using $\langle .\|.\rangle_{\Gamma(TM,E)}$: $\langle\psi|$ must be linear function linked with amplitude of probability (not normalized) for getting ϕ state ,which an observer who co-moving with black hole measures($\langle\phi||\rangle_{\mathbb{C}^2} = 2S_0$). $\langle\phi^*|$ must be the linear function linked with the normalized amplitude of probability for getting the spin in the ψ state, which an observer who co-moving with qubit measures($\langle\phi^*||\rangle_{\mathbb{C}^2} = 2S_{0*} = 1$; $\|\phi\|_{\Gamma(TM,E)=1}$) The Schr$\ddot{o}$dinger equation:

$$\iota\frac{d\phi}{d\tau} = H\phi \tag{50}$$

$H = H_0 + H_\sharp$ the Halmitonian with $H^+ \neq H$

$$H_0 = -\omega_u^{a0}\dot{x}^u L_{a0} \tag{51}$$

$$L_{a0} = \frac{1}{2}[\sigma_A\bar{\sigma}_0 - \sigma_0\bar{\sigma}_A] \tag{52}$$

$$H_0 = \frac{\iota}{2}\begin{pmatrix} W^{03} & W^{01} - \iota W^{02} \\ W^{01} + \iota W^{02} & W^{03}\end{pmatrix} \tag{53}$$

$$\sigma_0 = id,\ \sigma_x = \begin{pmatrix} 0 & 1 \\ 0 & 1\end{pmatrix}, \sigma_x = \begin{pmatrix} 0 & \iota \\ -\iota & 0\end{pmatrix}, \sigma_x = \begin{pmatrix} 1 & 0 \\ 0 & -1\end{pmatrix}$$

$$H_\sharp = -\frac{1}{2}\omega_u^{ab}\dot{x}^u L_{ab} \tag{54}$$

$$H_\sharp = -\frac{1}{2}\begin{pmatrix} W^{12} & W^{23} - \iota W^{31} \\ W^{23} + \iota W^{31} & -W^{12}\end{pmatrix} \tag{55}$$

($W^{AB} \equiv W_u^{AB}\dot{x}^u$). We get $H_0^+ = -H_0$ (dissipation operator), $H_\sharp^+ = -H_\sharp$ (Hamiltonian of the rotating qubit):

$$H = \frac{1}{2}\begin{pmatrix} z^3 & z^1 - \iota z^2 \\ z^1 + \iota z^2 & -z^3\end{pmatrix} \tag{56}$$

Where $z^i = \iota W^{0i} - \frac{1}{2}\varepsilon^i_{jk}W^{jk}$ ($z^i_u = \iota W^{0i}_u - \frac{1}{2}\varepsilon^i_{jk}W^{jk}_u$) is the dual complex of the lorentz connection from the qubit point of view, the spin subjected to a complex magnetic field is similar to the interaction with the gravitational field. We take $z = (z^1, z^2, z^3) \in \mathbb{C}^3$, for the dynamics integration that the Schr$\ddot{o}$dinger equation induces.

$$\phi(\tau) \simeq \Sigma_{k\in\{+,-\}}\langle\varphi_k^*(z(0))|\phi(0)\rangle_{C^2} e^{-\iota\int_0^\tau \lambda_k d\tau - \int_\Gamma^{A_k}}\varphi_k(z(\tau)) \tag{57}$$

With φ_k;φ_k^*, λ_k respectively right-left co-vectors and co-values of H:

$$H(z)\varphi_k(z) = \lambda_k\varphi_k(z) \tag{58}$$

$$H(z)^+\varphi_k^*(z) = \bar{\lambda}_k\varphi_k^*(z) \tag{59}$$

(the bar for the conjugate complex); $\langle\varphi_k^*|\varphi_q\rangle_{\mathbb{C}^2} = \delta_{kq}$, A_k the non-unit geometric phase generators:

$$\langle\varphi_k^*|d_3|\varphi_q\rangle_{C^2} \tag{60}$$

Γ is the curve caractherizes by $\tau \to z(\tau)$ in C^3

$$z^i = \iota W^{0i} - \frac{1}{2}\varepsilon^i_{jk}W^{jk} \tag{61}$$

$$z^i_u = \iota W^{0i}_u - \frac{1}{2}\varepsilon^i_{jk}W^{jk}_u \tag{62}$$

According to [5 6].

$$P(\tau) = \frac{1}{2}\begin{Vmatrix} z^3 - \lambda & z^1 - \iota z^2 \\ z^1 + \iota z^2 & -z^3 - \lambda\end{Vmatrix} \tag{63}$$

$$-(z^3 - \lambda)(z^3 + \lambda) - (z^1 + \iota z^2)(z^1 - \iota z^2) = 0 \tag{64}$$

$$\lambda_\pm = \pm\sqrt{(z^1)^2 + (z^2)^2 + (z^3)^2} = \pm\xi \tag{65}$$

We get:

$$|\phi_+(z)\rangle = \frac{1}{\sqrt{2\xi(\xi + z^2)}}\begin{Vmatrix} \xi + z^3 \\ z^1 + \iota z^2\end{Vmatrix} \tag{66}$$

$$|\phi_+^*(z)\rangle = \frac{1}{\sqrt{2\bar{\xi}(\bar{\xi} + \bar{z^2})}}\begin{Vmatrix} \bar{\xi} + \bar{z^3} \\ \bar{z^1} + \iota\bar{z^2}\end{Vmatrix} \tag{67}$$

$$|\phi_-(z)\rangle = \frac{1}{\sqrt{2\xi(\xi + z^2)}}\begin{Vmatrix} -z^1 + \iota z^2 \\ \xi + z^3\end{Vmatrix} \tag{68}$$

$$|\phi_-^*(z)\rangle = \frac{1}{\sqrt{2\bar{\xi}(\bar{\xi} + \bar{z^2})}}\begin{Vmatrix} -\bar{z^1} + \iota\bar{z^2} \\ \bar{\xi} + \bar{z^3}\end{Vmatrix} \tag{69}$$

$$A_\pm(z) = \pm\frac{1}{2}\frac{z^1dz^2 - z^2dz^1}{\bar{\xi}(\bar{\xi} + \bar{z^2})} \tag{70}$$

In the non-adiabatic coupling negligible condition, the adiabatic approximation is valid

$$M_\pm = \pm\left|\frac{\langle\varphi_-^*(z(\tau))|\dot{H}(z(\tau))|\varphi_+(z(\tau))\rangle}{\lambda_+(z(\tau)) - \lambda_-(z(\tau))}\right| \tag{71}$$

$$M_\pm \ll 1 \tag{72}$$

We take A the main bundle P connection space $SL(2,\mathbb{C})$. With the eigenvectors like maps $\acute{\varphi}_\pm : A \times TM \to \mathbb{C}^2$ such as $\acute{\varphi}_\pm(w,u) = \phi_\pm(\xi i_u w)$ with i the M inner product with $\xi : sl(2,\mathbb{C}) \to \mathbb{C}^3$ which is defined by $\xi(W_{AB}L_{AB}) = (\iota W_u^{0i} - \frac{1}{2}\varepsilon^i_{jk}W_u^{jk})_{i=1,2,3}(\{L_{AB}\}_{A,B})$ constitute a $sl(2,\mathbb{C})$ set lie algebra

$SL(2,\mathbb{C})$ generators. Where eigenvectors is defined as normalized with a phase factor; The C fiber lines is defined by $\phi_{\pm} \to A \times TM$ with the trivialization $\acute{\phi}_{\pm} : A \times TM \times \mathbb{C} \to^{\simeq} \varphi_{\pm}$, $\acute{\phi}_{\pm}(w,u,\lambda) = \lambda \acute{\phi}_{\pm}(w,u)$. We obtain ϕ using the local section adiabatic transport $\varphi_{-} \oplus \varphi_{+}$ on $A \times T\mathbb{C}$. As remark the right eigenvectors fixes the normalization factors, with the left eigenvectors don't define the bundles [7 8 9]. Let's summarized it by the following commutative diagram: where $i_w(u) = (w,u) \in A \times TM$, with w $\in A$ a connection of the main bundle

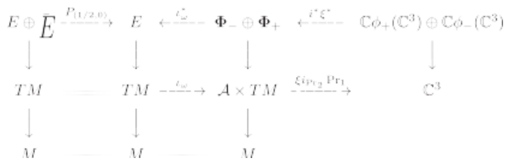

FIG. 1. Geometric structure [10]

P and $A_{\pm}$ are the main fibers $\mathbb{C}^*$ associated with $\varphi_{\pm}$. Where three gauge types changes linked with each bundle composite stage:In each stage: $\phi \in Diff(M)$ (main space-time diffeomorphism) $\tilde{W} = \varphi_{*}\omega$ and $\tilde{u} = \varphi_{*}u$

First stage: $\Lambda \in \mathbb{C}^{\,infty}(M, S0^{+}(3,1)$

$$\tilde{W}^{A}_{uB} = \Lambda^{A}_{C} W^{C}_{uD} \Lambda^{D}_{B} + \Lambda^{D}_{C} \partial_u \Lambda^{D}_{B} \tag{73}$$

with $\tilde{u} = \Lambda^{B}_{A} u_B$ Second stage: $\mu_I \in \mathbb{C}^{\infty}(A \times TM^*, \mathbb{C})$ (standardization and change local phase)

$$\tilde{A}_{\pm} = A_{\pm} + dln\mu_I \tag{74}$$

Let's summarize it:

$$\int_{R}^{\oplus} L^2(M^{+}\Xi_{\tau}, \bar{E} \oplus E) d\tau \longrightarrow^{WKB} WKB\tau(PC, \bar{E} \oplus E)(75)$$

$$\longrightarrow^{T(\frac{1}{2},0)} \Gamma_i(PC, E) \simeq \Gamma(C, E^{+})(76)$$

$$\longrightarrow^{adiab} \Gamma(A \times PC, \vartheta_{+} \oplus \vartheta_{-})(77)$$

Note that the holonomy of $W \in A$ along the geodesic C (between 0 and τ) is

$$Hol(W,C) = \mathbb{P}e^{-\iota \int_C z_{u^i} \sigma_i dx^u} \tag{78}$$

$$= Te^{-\iota \int_0^{\tau} H(z_\tau) d\tau} \tag{79}$$

$$= \Sigma_{k \in \{+,-\}} e^{-\iota(\int_0^{\tau} \lambda_k - \int_{\tau} A_k) d\tau} |\vartheta_k(z(\tau))\rangle \langle \vartheta_k(z(0))| \tag{80}$$

With $\mathbb{P}e$, $\mathbb{T}e$ showing exponentials ordered (Dyson series) $\varphi_{\mathbb{C},\pm} = e^{-\iota(\int_0^{\tau} \lambda_{\pm} - \int_{\tau} A_{\pm}) d\tau}$ considered as the cylindrical functions of the Lorentz connection space, $\varphi_{\mathbb{C},\pm} \in cyl(A)$, ψ as these two cylindrical functions linear combination[11]. Note that the $H = \frac{1}{2} z^i \sigma_i$ form, with Pauli matrices $\{\sigma\}_{i=1,2,3}$ and the Lorentz connection self-dual complex $\{z^i\}_{i=1,2,3}$ is taken by localized qubit Hamiltonian. There exist an effective formalism $H^{eff}_{MM} = (Z^i - z^i)\sigma_i$ in D-brane matrix models [12 13]. The non-commutative geometry emerging from membranes [12] can be associate whit the equigeneration $H^{eff}_{MM}|\Lambda\rangle = 0$ with $|\Lambda\rangle \in \kappa \otimes \mathbb{C}^2$ [13 14]. $H^{eff}_{MM}|\Lambda\rangle = 0 \Longrightarrow Z^i \otimes \sigma_i |\Lambda\rangle = z^i \otimes \sigma_i |\Lambda\rangle$ where the localized qubit is considered as matrix model noncommutative eigenvalue with super-gravity [15]. In other parts, noncommutative equations like $Z^i \otimes \sigma_i |\Lambda\rangle = z^i \otimes \sigma_i |\Lambda\rangle$ also appear in the entangled quantum systems adiabatic theory and their geometric phase operators [16 17]. the connection between D-brane matrix models and localized qubit theory on qubit reaching the black hole is discussing in[18 19]. It is very important that:

$$\langle \phi^*(\tau)|\phi(\tau)\rangle_{\mathbb{C}^2} = \langle \phi^*(0)|\psi(0)\rangle_{\mathbb{C}^2} \tag{81}$$

$$\langle Hol(W,\mathbb{C})\phi(0)|\bar{\sigma}|Hol(W,\mathbb{C})\phi(0)\rangle_{\mathbb{C}} u_A(\tau) = \langle \phi(0)|\bar{\sigma}|\phi(0)\rangle_{\mathbb{C}} u_A(0) \tag{82}$$

It is'nt necessary unitary with $\langle . \| . \rangle_{\mathbb{C}^2}$

B. Physical phenomena of non unitary evolution

The articles [20 21 22 23] studied the non-self-deputy two-level quantum system adiabatic transport. With $\vec{W}^0 = (W_{01}, W_{02}, W_{03})$ and $\vec{W}^{\sharp} = (W_{23}, W_{31}, W_{12})$

$$\xi^2 = (\vec{W}^{\sharp} - \iota \vec{W}^0)^2 = \|\vec{W}^{\sharp}\|^2 - \|\vec{W}^0\|^2 - 2\iota \vec{W}^{\sharp} \vec{W}^0 \tag{83}$$

with $\lambda_{+}(z) = \lambda_{-}(z) \Rightarrow \xi = 0$,

$$M = \{^{\|\vec{W}^{\sharp}\| = \|\vec{W}^0\|}_{\vec{W}^{\sharp}.\vec{W}^0 = 0} \tag{84}$$

is the complex magnetic monopole where $dim_{\mathbb{R}} M = 4$

In the condition $\vec{W}^{\sharp}.\vec{W}^0 = 0$ satisfied outside of $M(\|\vec{W}^{\sharp}\| > \|\vec{W}^0\|)$ $\xi \in R$, $e^{\iota \int \lambda_I d\tau} \in U(1)$ are just pure phases in $M(\|\vec{W}^{\sharp}\| < \|\vec{W}^0\|)$ $\xi \in \iota R$ and $e^{\iota \int \lambda_I d\tau} \in R$ are non-unit dynamic phases. For the last situation, the weights of the superposition is modified by the evolution of φ_I(relatively at $\langle . | . \rangle_{\mathbb{C}^2}$. With an observer and black hole co-moving. This effect is called a decoherence because:

$$\frac{|\langle \varphi_{+}^{*}\phi|\phi\rangle\langle \varphi_{-}^{*}\rangle|}{\|\phi\|^2} = \frac{|c_{+}c_{-}|e^{\frac{1}{2}\int|\xi|d\tau} e^{-\frac{1}{2}\int|\xi|d\tau}}{c_{+}^{2} e^{\int|\xi|d\tau} + c_{-}e^{-\int|\xi|d\tau}} \simeq |\frac{c_{-}}{c_{+}}| e^{-\int|\xi|d\tau} \tag{85}$$

tends towards 0 for τ big (with $c_k = \langle \varphi_{+}^{*}\phi|\phi\rangle$, the geometric phases is neglected and we assume that $Im\xi = |\xi| > 0$). The effects of non-unit geometric phases $e^{-\int_{\tau} A_{\pm}}$ have not been considered which induces a geometric decoherence if $A_{\pm} \in R$ (let us use the geometric decoherence in case the geometric phase never depends on proper time).

We take $M_W = I_W^{-1} \xi^{-1}(M)$ a pre-image of M on PM. In this case the monopole complex magnetic for a fixed geometric space time M_W can'nt be a time M sub-space but for the tangent bundle PM. These four velocity afects the complex magnetic monopole around the hole black in a qubit point of view . In sum the set complex magnetic monopoles can be $M_A = \{(W, M_W), \omega \in A\} \subset A \times TM$, to conclude a prime integrals $\{I_\alpha\}_\alpha$ in $i_u(W)(u \in T \times C, C \in g_{\{I_\alpha\}_\alpha}$ α can be used to define a class $g_{\{I_\alpha\}_\alpha}$ of geodesic on $\{I_\alpha\}_\alpha$ and x. It appends $M_{W,\{I_\alpha\}_\alpha} = \pi_T(M_W \cap \pi_T^{-1}(_{\{I_\alpha\}_\alpha})$ a sub-space of M that could be an image for the following qubits co-moving with the geodesics of $_{\{I_\alpha\}_\alpha}$, a complex magnetic monopoles in space-time. To be more clear the non-physical come from the non-unitarity relative to $\langle .\|.\rangle_{C^2}$. We first take a simpler model get up of a system of levels wrote $|d\rangle, |0\rangle, |1\rangle$; with a $|0\rangle$ spontaneous emission with a ratio γ_{-1} in $|d\rangle$ state. In second time we took system restricted to $(|0\rangle, |1\rangle)$ as qubit[18].

$$\frac{d\rho}{dt} = -\imath[H,\rho] - \frac{\gamma_-}{2}\{\sigma^+_{d0}\sigma^-_{d0}, \rho\} + \gamma_-\sigma^+_{d0}\sigma^-_{d0} \tag{86}$$

With ρ the system matrix density, $\{.,.\}$ shows an anti-switch, $\sigma^-_{d0} = |d\rangle\langle 0|$ and $\sigma^-_{d0} = |d\rangle\langle 0|$, we rewritten it as:

$$\frac{d\rho}{dt} = -\imath(H^{eff}\rho - \rho H^{eff}) + \gamma_-\rho_{00}|d\rangle\langle d| \tag{87}$$

here $H^{eff} = H - \frac{\gamma_-}{2}\imath|0\rangle\langle 0|$. This Hamiltonian anti-self-adjoint part $-\frac{\gamma_-}{2}\imath|0\rangle\langle 0|$ models the loss population of $|0\rangle$ to "black state" i.e this black state gains population gain which is modeled by $\gamma_-\rho_{00}|d\rangle\langle d|$. Let's forget the black state in this modeling, the Schrodinger equation is obtained by the qubit governed by a non-self-adjoint Hamiltonian $H^{eff}_{|0\rangle,|1\rangle} = H_{|0\rangle,|1\rangle} - \frac{\gamma_-}{2}\imath|0\rangle\langle 0|$, this non-self-adjoint part of H^{eff}, $-\frac{\gamma_-}{2}\imath|0\rangle\langle 0|$, could be renamed by the loss operator since it has been modeling the wave function dissipation induced by the $|0\rangle$ loss population at the black state [24] [25]. i.e If $\langle 0|H|\psi\rangle = \langle d|H|\psi\rangle = 0$, then :

$$\frac{d\rho}{dt} = -\imath(H^{eff}_{i0}\rho_{0j} + H^{eff}_{i1}\rho_{1j} - \bar{H}^{eff}_{j0}\rho_{i0} - \bar{H}^{eff}_{j1}\rho_{i1}) \tag{88}$$

$\forall i,j \in \{0,1\}$. We take ψ as solution of $\imath\ \dot{psi} = H^{eff}_{|0\rangle,|1\rangle}$, where $P = |\psi\rangle\langle\psi|$ obeys $\dot{P} = -\imath(H^{eff}_{|0\rangle,|1\rangle}P - PH^{eff}_{|0\rangle,|1\rangle})$, the population and the consistency $P_{ij} = \langle i|P|j\rangle$ obey the same above equation. So $\rho_{ij} = P_{ij}$ ($P^2 \neq P$ for ψ is'nt normalized due to the $H^{eff}_{|0\rangle,|1\rangle}$ non-Hermitian character. A single qubit is had, the latter is forgotten by a semi-classical WKB approximations using in this thesis. In this case two qubit states is had, $|1_0\rangle$ and $|1_1\rangle$ (C^2 canonical basic formation is used in the different fibrations construction) and a black state $|\oslash\rangle$ of the void. In many articles of physical systems the modeling for a spontaneous evolution decreasing is described [20], also for a finite vacuum state [21] [22].

C. Teleportation fidelity

For starting Bob and Alice are at initially point x_B of M supposedly so far of black hole for setting M flat in x_B neighborhood. Let black hole moving with $|0\rangle = \begin{pmatrix}0\\1\end{pmatrix}$ $|1\rangle = \begin{pmatrix}1\\0\end{pmatrix}$ but the geodesic is following by Alice reaching the horizon at x_A point. Alice teleports informations. We take $\tau_B = \tau_A = 0$ for Alice leaving Bob; a tangled qubit pair in a bell state has had by them

$$|\psi^0_{AB}\rangle\rangle = \frac{1}{\sqrt{2}}(|0_A 0_B\rangle\rangle + |1_A 1_B\rangle\rangle) \in \Pi^{-1}(u^0_A) \otimes \Pi^{-1}(u^0_A) \tag{89}$$

$(u^0_A) \in P_{x_B}M$ and $u^0_B = (1,0,0,0) \in P_{x_B}M$ the Bob and Alice quadri-verctor velocity. Then Bob and Alice belong are in differents total own qubit states spaces definitions. Firstly Bob's linear functions are in particular state $\langle 0|$ and $\langle 1|$, but Alice's linear functions are $\langle 0^*|$ and $\langle 1^*|$ because the black hole is not co-moved by her. The Alice's qubit state is $\langle a^*|b_A\rangle = \langle a|\bar{\sigma}_A|b_A\rangle u^0_{AA} = \delta_{ab}$
$(\forall a,b \in \{0,1\})$. It happens $|0_A\rangle = \sigma_A u^0_{AA}|0\rangle$ with $|1_A\rangle = \sigma_A u^0_{AA}|0\rangle(\sigma_A u^0_{AA} = (\bar{\sigma_A} u^0_{AA})^{-1}$

$$|\rangle\rangle = \frac{1}{\sqrt{2}}\langle a|\sigma_A|b_A\rangle u^0_{AA}\Sigma^{ab}|ab\rangle\rangle \tag{90}$$

for $z_B = 0$, $|\varphi_+(z_B)\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$ and $|\varphi_-(z_B)\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}-1\\1\end{pmatrix}$

$$|\psi^0_{AB}\rangle\rangle = \frac{1}{2}\Sigma^{ab}\Sigma_{i=\pm}\langle a|\sigma_A|b_A\rangle u^0_{AA} i^a|\phi_-(z_B)\rangle \otimes |b\rangle \tag{91}$$

We take τ_A^{-1} the proper time when Alice travels to x_A. According adiabatic approximation; For $\tau_A = \tau_A^{-1}$ and $\tau_B > 0$

$$|\psi^1_{AB}\rangle\rangle = \frac{1}{2}\Sigma^{abi}\langle a|\sigma_A|b_A\rangle u^0_{AA} i^a e^{\imath\varphi_i}|\phi_-(z_B)\rangle \otimes |b\rangle \tag{92}$$

$\varphi_i = -\int_0^{\tau^1_A}\lambda_i d\tau + \imath\int_\Gamma A_i$). $|\psi^1_{AB}\rangle\rangle$ is defined by two pproper times, one for Bob and other for Alice, with asynchronous clocks. Because evolution is inertial in the flat M part, Bob's qubits is trivial.

$$|\phi_i(z_B)\rangle = \Sigma_c\langle c^*|\phi_i(z_B)|c_A\rangle = \langle c|\bar{\sigma}_C\phi_i(z_B)u^1_{AC}|c_A\rangle \tag{93}$$

where the four velocity for Alice at τ^1_A is $u^1_A \in T_{x_A}M$ is:

$$|\psi^1_{AB}\rangle\rangle = \frac{1}{\sqrt{2}}\Sigma_{bc}\chi_{bc}|c_A b\rangle\rangle \tag{94}$$

Alice's quantum information is encoded by

$$\chi_{bc} = \frac{1}{\sqrt{2}}\Sigma_{ai}\langle a|\sigma_A|b_A\rangle u^0_{AA} i^a e^{\imath\varphi_i}\langle c|\bar{\sigma}_C|\rangle u^1_{AC} \tag{95}$$

in the qubit $|\phi_I\rangle = \alpha|0_A\rangle + \beta|1_A\rangle$ with $(|\alpha|^2 + |\beta|^2 = 1)$, $|\psi^1_{AAB}\rangle\rangle\rangle = |\psi_I\rangle \otimes |\psi^1_{AB}\rangle\rangle$ in usual teleportation protocol operations is performed by Alice:

$$|\phi^2_{AAB}\rangle\rangle\rangle = (h_A \otimes id \otimes id)(cnot_A \otimes id)|\phi^1_{ABI}\rangle\rangle\rangle \tag{96}$$

$cnot_A$ and h_A are the CNOT and Hadamard for Alice's part. It happens

$$|\psi^2_{AAB}\rangle\rangle = |0_A 0_A\rangle\rangle \otimes (\frac{\alpha\chi_{00}+\beta\chi_{10}}{2}|0\rangle \quad (97)$$

$$+\frac{\alpha\chi_{01}+\beta\chi_{11}}{2}|1\rangle) \quad (98)$$

$$|1_A 0_A\rangle\rangle \otimes (\frac{\alpha\chi_{00}-\beta\chi_{10}}{2}|0\rangle \quad (99)$$

$$+\frac{\alpha\chi_{01}-\beta\chi_{11}}{2}|1\rangle) \quad (100)$$

$$|0_A 1_A\rangle\rangle \otimes (\frac{\alpha\chi_{10}+\beta\chi_{00}}{2}|0\rangle \quad (101)$$

$$+\frac{\alpha\chi_{11}+\beta\chi_{10}}{2}|1\rangle) \quad (102)$$

$$|1_A 1_A\rangle\rangle \otimes (\frac{\alpha\chi_{10}-\beta\chi_{00}}{2}|0\rangle \quad (103)$$

$$+\frac{\alpha\chi_{11}-\beta\chi_{10}}{2}|1\rangle) \quad (104)$$

Alice takes a qubit measurement as $0_A 1_A$. Alice communicates it to Bob that she used the identity operation to performe her qubit. The message is received by Bob at τ^3_B. The state is for $\tau_A > \tau^{-1}_A$ and $\tau_B = \tau^3_B$

$$|\psi^3_{AAB}\rangle\rangle = (U_A \otimes U_A|0_A 1_A\rangle\rangle) \otimes ((\alpha\chi_{10}+\beta\chi_{00})|0\rangle \quad (105)$$

$$+(\alpha\chi_{11}+\beta\chi_{10})|1\rangle) \quad (106)$$

With U_A the operator evolution for Alice's qubits after τ^{-1}_A.

$$F(\alpha,\beta) = |\frac{(\bar{\alpha}\langle 0| + \bar{\beta}\langle 1|)((\alpha\chi_{10}+\beta\chi_{00})|0\rangle + (\alpha\chi_{11}+\beta\chi_{10})|1\rangle)}{\|((\alpha\chi_{10}+\beta\chi_{00})|0\rangle + (\alpha\chi_{11}+\beta\chi_{10})|1\rangle)\|}| \quad (107)$$

$$F(\alpha,\beta) = \frac{||\alpha|^2\chi_{10} + \bar{\alpha}\beta\chi_{00} + \alpha\bar{\beta}\chi_{11} + |\beta|^2\chi_{10}|}{\sqrt{|\alpha\chi_{10}+\beta\chi_{00}|^2 + |\alpha\chi_{11}+\beta\chi_{10}|^2}} \quad (108)$$

the black hole induces a decoherence which degrades the fidelity of teleportation,encoding in χ_{bc}. With a Alice's constant four velocity in a flat space-time:

$$\chi_{bc} = \Sigma_a \langle c|\sigma_A|a\rangle\langle a|\bar{\sigma}_C|b\rangle u^0_{AA} u^1_{AC} = \delta_{cb} \quad (109)$$

It happens F = 1.

III. RESULTS

A. Rindler's Black Hole

Now we are going to apply the models that we developped in the previous section. For comparing with the Fuentes-Schuller-Mann model[26], the Rindler's space-time case is considered

$$d\tau^2 = (Ax)^2 dt^2 - dx^2 \quad (110)$$

$d\tau^2 = \frac{-ds}{c^2}$ that corresponds with the acceleration parameter $\frac{1}{A}$ to a flat space-time seen in a uniformly accelerated non-inertial frame $A = \frac{1}{2R_s}$, $R_s = 2GM$. For $e^0 = Axdt$ and $e^1 = dx$ the tetrad fields, we obtain the components of the Lorentz connection from:

$$W^{AB}_u = e^A_u \partial_\rho e^{uB} + e^A_u \Gamma^u_{\rho v} e^{Bv} \quad (111)$$

$$W^{AB} = \omega^{AB}_u e^u \quad (112)$$

$W^{01} = -g_{00,1}dt = -Adt$ is only nonzero ; It happens that $z^1 = -\imath A\dot{t}$ and $z^2 = z^2 = 0$

$$H = \frac{-\imath A\dot{t}}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad (113)$$

with $\lambda_\pm = \pm\frac{-\imath A\dot{t}}{2}$ the eigvalues and

$$|\varphi_-\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} -1 \\ 1 \end{pmatrix}; |\varphi_+\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad (114)$$

are the eigvectors, $A_\pm = N_\pm = 0$ for flat space-time. If the qubit is moving in the x direction with $\sigma_x\phi_\pm = \pm\phi_\pm$: φ_+ as positive helicity state of spin parallel to the linear moment. φ_- as negative helicity state of anti-parallel spin at linear moment

$$\ddot{x} + \Gamma^\beta_{uv}\dot{x}^u\dot{x}^v = 0 \quad (115)$$

$$\ddot{t} + 2\Gamma^0_{01}\dot{t}\dot{x} = 0 \quad (116)$$

$$\ddot{x} + \Gamma^1_{00}\dot{t}^2 + \Gamma^1_{11}\dot{x}^2 = 0 \quad (117)$$

$$\ddot{t} + \frac{2}{x}\dot{t}\dot{x} = 0 \quad (118)$$

$$\ddot{x} + A^2 x\dot{t}^2 = 0 \quad (119)$$

The first integral $\ddot{t} + \frac{2}{x}\dot{t}\dot{x} = 0$ is defined by the first equation

$$\Rightarrow x^2\dot{t} = K \quad (120)$$

In this situation automatically the second becomes

$$\ddot{x} + \frac{A^2K^2}{x^3} = 0 \quad (121)$$

The state qubit adiabatic transport $\phi(\tau_0) = c_+\varphi_+ + c_-\varphi_-$ is:

$$\psi(\tau) = \frac{c_-}{\sqrt{2}}e^{\frac{A}{2}(t(\tau)-t(0)}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \frac{c_+}{\sqrt{2}}e^{-\frac{A}{2}(t(\tau)-t(0)}\begin{pmatrix} -1 \\ 1 \end{pmatrix} \quad (122)$$

$$\psi(\tau) = \frac{c_-}{\sqrt{2}}e^{\frac{AK}{2}\int_0^\tau \frac{d\tau}{x^2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \frac{c_+}{\sqrt{2}}e^{-\frac{AK}{2}\int_0^\tau \frac{d\tau}{x^2}}\begin{pmatrix} -1 \\ 1 \end{pmatrix} \quad (123)$$

the positive helicity state is destroyed by the dynamic decoherence[19]. The positive helicity mode couples the fermion to this thermal bath for the matrix density ρ_+ (for $|1_{k,s}\rangle = |\phi_+\rangle$ or $|\emptyset\rangle$ obeys the main equation [23]

$$\frac{d\rho_+}{d\tau} = -\frac{\gamma}{2}(1-\bar{n})\{c_+^+ c_+, \rho_+\} + \gamma(1-\bar{n})c_+\rho_+ c_+^+ \quad (124)$$
$$-\frac{\gamma}{2}(1-\bar{n})\{c_+ c_+^+, \rho_+\} \quad (125)$$
$$+\gamma\bar{n}c_+^+ c_+ \rho_+ \quad (126)$$

the operators fermionic annihilation and creation on the positive helicity mode $c_+^{\pm}$ ($c_+ = |\emptyset\rangle\langle 1_{k,s}|$,$\gamma$ as the bath spectal density, $c_+^+ = |1_{k,s}\rangle\langle\emptyset|$) $\bar{n} = \frac{1}{e^{\frac{\omega}{k_B T}}+1}$. With assumptions $\gamma(1-\bar{n}) \simeq \frac{\gamma_0\omega}{4k_B T}$ is constant [27]. Due to $\bar{n}$ very low (the Unruh temperature is very low), the $|1_{k,s}\rangle = |\emptyset\rangle$ dissipation dominates the main equation matching the above equation;projecting on $|1_{k,+}\rangle\langle 1_{k,+}|$ and for neglecting the quantum jumps and the $\bar{n}$ terms [1019 28], we have:

$$H_+^{eff} = -\frac{\iota\gamma_0\omega}{4k_B T}|\phi_+\rangle\langle\phi_+| \quad (127)$$

As solution

$$x(\tau) = \sqrt{5A^2K^2(\beta+\tau)^2 - 1} \quad (128)$$

where $\beta = -\frac{\sqrt{x(0)^2+1}}{AK\sqrt{5}}$

$$e^{\frac{AK}{2}\int_0^\tau \frac{d\tau}{x^2}} \quad (129)$$

$$C = \int_0^\tau \frac{d\tau}{5A^2K^2(\beta+\tau)^2-1} = \frac{1}{2}\ln[(\frac{\sqrt{5}AK(\beta+\tau)-1}{\sqrt{5}AK(\beta+\tau)+1})(\frac{1+\sqrt{5}AK\beta}{(1-\sqrt{5}AK\beta)}] \quad (130)$$

where $e^{\frac{AK}{4}C} \simeq C^{\frac{AK}{4}} = C^{\frac{1}{4}}$ with AK=1

$$e^{\frac{AK}{2}\int_0^\tau \frac{d\tau}{x^2}} = [(\frac{\sqrt{5}AK(\beta+\tau)-1}{\sqrt{5}AK(\beta+\tau)+1})(\frac{1+\sqrt{5}AK\beta}{(1-\sqrt{5}AK\beta)}] \quad (131)$$

With $\tau_H = -\frac{1}{\sqrt{5}AK} - \beta$ a qubit reaching the horizon time; It happens $lim_{\tau\to\tau_H} e^{\frac{AK}{2}\int_0^\tau \frac{d\tau}{x^2}} = 0$.

B. Symmetry Spherical Black Hole

The Schwarschild is defined as

$$d\tau = T^2(r)dt^2 - R^{-2}dr^2 - r^2(d\theta^2 + sin^2\theta)d\varphi^2 \quad (132)$$

where $T = R = \sqrt{1-\frac{R_s}{r}}$,with R_s the radius. The contravariant vectors are $e^0 = Tdt$, $e^1 = R^{-1}dt$, $e^2 = rd\theta$, $e^3 = rsin\theta d\varphi$. It happens for lorentz connection

$$W_u^{AB} = e_u^A \partial_\rho e^{Bu} + e_v^A \Gamma_{\rho v}^u e^{Bv} \quad (133)$$

$$W^{AB} = W_u^{AB} e^u \quad (134)$$

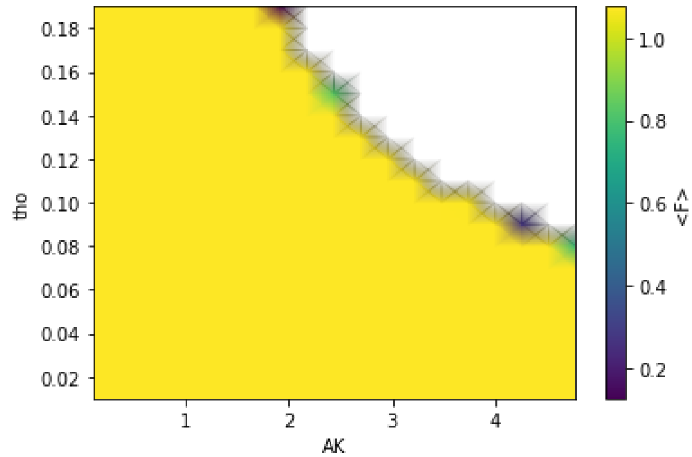

FIG. 2. Rindler's average time-space teleportation fidelity

$$\Gamma_{01}^0 = \frac{1}{2}\frac{1}{T^2dt^2}2T'Tdtdr \quad (135)$$

$W^{01} = T'Rdt$ $W^{12} = Rd\theta$, $W^{13} = Rsin\theta d\varphi$ and $W^{23} = cos\theta d\varphi$ are the non-zero components. Because of the spherical symmetry we must restrain to $\theta = \pi/2$ plan

$$\ddot{t} + 2\Gamma_{01}^0\dot{t}\dot{r} = 0 \quad (136)$$

$$\ddot{r} + \Gamma_{00}^1\dot{t}^2 + \Gamma_{11}^1\dot{r}^2 + \Gamma_{22}^1\dot{\theta}^2 + \Gamma_{33}^1\dot{\varphi}^2 = 0 \quad (137)$$

$$\ddot{\theta} + \Gamma_{22}^2\dot{\theta}\dot{r} + \Gamma_{21}^2\dot{\theta}\dot{r} + \Gamma_{33}^2\dot{\varphi}^2 = 0 \quad (138)$$

$$\ddot{\varphi} + 2\Gamma_{13}^3\dot{\varphi}\dot{r} + 2\Gamma_{23}^3\dot{\theta}\dot{\varphi} = 0 \quad (139)$$

$$\ddot{t} + 2\frac{T'}{T}\dot{t}\dot{r} = 0 \quad (140)$$

$$\ddot{r} + T'R^2T\dot{t}^2 + \frac{R'}{R}\dot{r}^2 + R^2r\dot{\varphi}^2 = 0 \quad (141)$$

$$\ddot{\varphi} + \frac{2}{r}\dot{\varphi}\dot{r} = 0 \quad (142)$$

The prime integrals are defined by the first and last equations

$$T^2\dot{t} = E \quad (143)$$

$$r^2\dot{\varphi} = L \quad (144)$$

E: energy and L:angular momentum per unit of mass; It happens $z^1 = \iota\omega^{01} = -\iota\frac{T'R}{T^2}E$, $z^2 = -\frac{\iota}{2}2\omega^{13} = \iota R\dot{\varphi} = \frac{\iota R}{r^2}L$ and $z^3 = 0$

$$H = \frac{1}{2}\begin{pmatrix} 0 & -\iota\frac{T'R}{T^2}E - \frac{\iota R}{r^2}L \\ -\iota\frac{T'R}{T^2}E + \frac{\iota R}{r^2}L & 0 \end{pmatrix} \quad (145)$$

where $\lambda_{\pm} = \pm\sqrt{(rR^2L) - (\frac{T'R}{T^2}E)^2}$.
With $\lambda_{\pm} \in R$ if $rR^2L) - (\frac{T'R}{T^2}E)^2 \geq 0 \Rightarrow L \geq \frac{T'r^2}{T^2}E(M_{W,L,E} = \{(rLE,\phi);\phi \in [0,2\pi]$ and $\frac{T'_{(r^2_{LE})}{}^{r_{LE}}}{T^2_{(r_{LE})}} = \frac{L}{E}\})$. Note that dynamic decoherence disappears for Schwarzchild's case

$$T' = \frac{R_s}{2\sqrt{1-\frac{R_s}{r}}} \tag{146}$$

$(1-\frac{R_s}{r})^{\frac{3}{2}}L \geq \frac{R_s}{2}E$ i.e if $\frac{R_s}{2L}E < 1$ so $r > r_{LE} = \frac{R_s}{[1-(\frac{R_s}{2L}E)^{\frac{3}{2}}]}$.It happens that the qubit following the strongly rotating geodesic (L wide) is subjected to dynamic decoherence and far from the complex magnetic monopole(i.e radius sphere is $R_{LE} \geq R_s$). $A_{\pm} = \frac{1}{2}\frac{z^1dz^2-z^2dz^1}{\xi(\xi+z^3)}$ is defined as the geometric phase generators

$$A_{\pm} = \frac{1}{2}\frac{[(\frac{T'RE}{T^2}(\frac{RdrL}{r^2} - 2R\frac{dr}{r^3}) - \frac{RL}{2}(\frac{T''REdr}{T^2} - 2\frac{(T')^2REdr}{T^3} + \frac{T'R'Edr}{T^2})]}{\frac{(T')^2R^2E^2}{T^4} - \frac{R^2L^3}{r^4}} \tag{147}$$

$A_{\pm} = \pm\frac{1}{2}\frac{uvw'-u'vw-uv'w}{w^2L^2-u^2v^2E^2}LEdr \in \Omega^1(M,R)($ with Schwarzschild case $u = \frac{T'}{T}, v = \frac{R}{T}, w = \frac{R}{r^2})$, $A_{\pm} = \pm\frac{EL}{2}\frac{R_s^2}{(1-\frac{R_s}{r})^3r^2L^2-\frac{R_s^2rE^2}{4(1-\frac{R_s}{r})}}$. Always the Geometric decoherence appears except for a circular orbits(r = cst) and radial geodesic(L = 0). In other ways the adiabatic non-coupling $N_{\pm}$

$$M_{\pm} = |\frac{\langle\varphi^*_-(z(\tau))|\dot{H}(z(\tau))|\varphi_+(z(\tau))\rangle}{\lambda_+ - \lambda_-}| = \pm\frac{\dot{\lambda}}{\lambda} \tag{148}$$

$$N_{\pm} = |\frac{(wL-uE)(u'w-uw')}{4(w^2L^2-u^2E^2)^{\frac{3}{2}}}|LE|\dot{r}| \tag{149}$$

Where the adiabatic approximation validity for circular orbits and radial geodesics is ensured without any speed indication.

1. Radial geodesic (Schwarzschild metric)

The radial geodesics equation is

$$\ddot{r} + \frac{R'}{R}\dot{r} = 0 \tag{150}$$

$$\Longrightarrow \ddot{r} + \frac{R_s}{r} = 0 \tag{151}$$

$$U_1 = \dot{r} \tag{152}$$

$$dU_1 = U_2 \tag{153}$$

$$\rightarrow \int(\int rdr)dr = \int(\int \frac{-R_s}{2}d\tau)d\tau \tag{154}$$

It happens:$r^3 = -\frac{3}{2}\sqrt{R_s}(\tau)^2 + (\sqrt{r_0^3})^2$ i.e $r = (-\frac{3}{2}\sqrt{R_s}(\tau)^2 + (\sqrt{r_0})^{\frac{3}{2}})\frac{2}{3}$, by taking $A\tau\sqrt{r_0} \simeq 0$; the event horizon is reached at $\tau_H = \frac{r_0^{\frac{3}{2}} - R_s^{\frac{3}{2}}}{\frac{3}{2}R_s}$.Due to the decoherence induced by the gravitational field as well as quantum teleportation fidelity drops if Alice closes to the event horizon like Rindler's space-time ($M_{W,L=0,E} = \{(+\infty,\phi);\phi \in [0,2\pi]\}$) contains radial geodesics

2. Schwarzschild Circular orbits

Which is defined by $r = r_0 = constante$; with $\phi = \frac{L}{r_0}\tau + \phi_0$ we obtain:

$$R^2r\dot{\varphi}^2 = -T'R^2T(dt)^2 \tag{155}$$

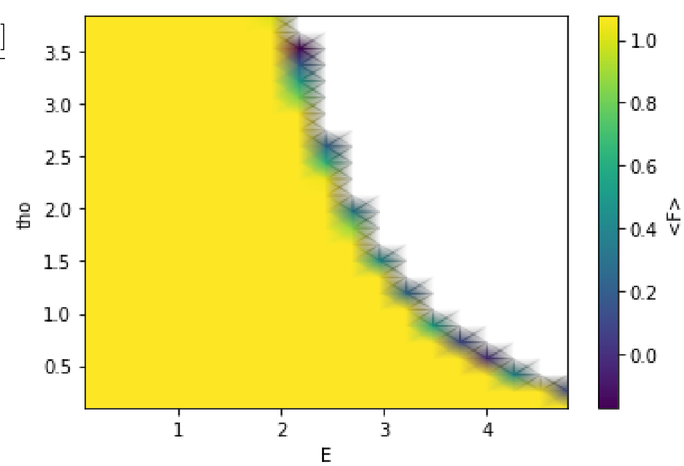

FIG. 3. Representation of the Fidelity of teleportation

$$\Longrightarrow R^2r\frac{L^2}{r_0^2}(dt)^2 = -T'R^2T(dt)^2 \tag{156}$$

$$\Longrightarrow L^2 = -\frac{R_sr_0^2}{r} \tag{157}$$

and the metric allows

$$d\tau = T^2\frac{E^2}{T^4} - r^2\frac{L^2}{r^4} \tag{158}$$

$d\tau = 1$ so $E^2 = T(r_0)^2(1+\frac{L^2}{r_0^2})$. The difference in quadrivector velocity between Alice and Bob causes the effect in circular orbits (that shows the fidelity almost uniform with respect to r_0 and τ_A^1),there is'nt decoherence for each r_0 , $\xi \in R(M_{W,r_0} = \{(3R_s,\phi) \in [0,2\pi]\})$,the photon sphere is created by the circular orbits outsiding the complex magnetic monopole). In this situation a phase difference φ_+ and φ_- is generated by the adiabatic transport inducing few interference in the quantum teleportations which explains the short oscillations of fidelity.

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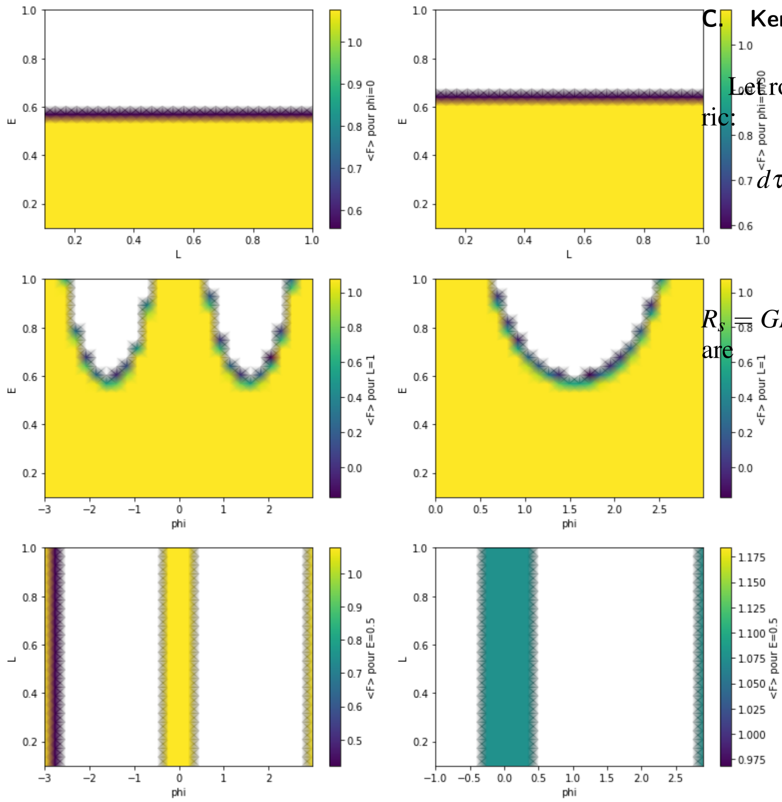

FIG. 4. Teleport Fidelity for reaching geodesics almost the event horizon

3. Geodesic reaching the event horizon

Let take the geodesic starting far enough horizon and traveling there by adiabatic approximation. Here $\xi \rightarrow^{r\rightarrow R_s} +\iota\infty$, $e^{\frac{\iota}{2}\int H^{-\varepsilon}\xi d\tau} \simeq 0$ τ_H is taken to being the necessary time reaching event horizon with $\xi << 1$). Foreover $\varphi_+ \longrightarrow^{r\rightarrow R_s} \frac{1}{2}\begin{pmatrix}1\\1\end{pmatrix}$,$\varphi_- \longrightarrow^{r\rightarrow R_s} \frac{1}{2}\begin{pmatrix}-1\\1\end{pmatrix}$ and $(u^A)_{A\in\{t,r,\theta,\varphi\}} = (\frac{E}{T}, -\sqrt{\frac{E^2}{T^2}-1-\frac{L^2}{r^2}}, 0, \frac{L}{r})$, where teleportation fidelity for geodesics reaching almost the event horizon is estimated, as well as the above figure shows Teleportation fidelity oscillates with Alice's relative angular position as soon as she arrived to the event horizon (Schr$\ddot{o}$dinger's cat Teleportation , do'nt occurs oscillation for $|0\rangle$ or $|1\rangle$ teleportation . There is small dependence of E and L on l at exception for the few values of these first integrals.

C. Kerr's Black Hole

Let rotating black hole in the form of Boyer -Lindquist metric:

$$d\tau = -(1-\frac{R_s r}{\rho^2})dt^2 - \frac{2R_s a r sin^2\theta}{c\rho^2}dtd\varphi + \frac{\rho^2}{c^2\Delta}dr^2 \quad (159)$$

$$+\frac{\rho^2}{c^2}d\theta^2 + (r^2+a^2+\frac{2R_s a r sin^2\theta}{\rho^2})\frac{sin^2\theta}{c^2}d\varphi^2 \quad (160)$$

$R_s = GM$ the Schwarzschild radius, the contravariant vectors are

$$e^0 = \iota(1-\frac{R_s r}{\rho^2})^{\frac{1}{2}}dt \quad (161)$$

$$e^0 e^3 = \frac{2R_s a r sin^2\theta}{c\rho^2}dtd\varphi \quad (162)$$

$$e^1 = \frac{\rho^2}{c\sqrt{\Delta}}dr \quad (163)$$

$$e^2 = \frac{\rho}{c}d\theta \quad (164)$$

$$e^3 = (r^2+a^2+\frac{2R_s a r sin^2\theta}{\rho^2})^{\frac{1}{2}}\frac{sin\theta}{c}d\varphi \quad (165)$$

$$e_0 = \frac{\iota\Sigma}{c\rho\Delta^{\frac{1}{2}}}\frac{d}{dt} \quad (166)$$

$$e_{03} = \sqrt{\frac{R_s r a}{c\Delta\rho^2}}\frac{d^2}{dtd\varphi} \quad (167)$$

$$e_1 = \iota\frac{\Delta^{\frac{1}{2}}}{\rho}\frac{d}{dr} \quad (168)$$

$$e_2 = \frac{\iota}{\rho}\frac{d}{d\theta} \quad (169)$$

$$e_3 = \sqrt{\frac{a^2 sin^2\theta - \Delta}{\rho^2\Delta sin^2\theta}}\frac{d}{d\varphi} \quad (170)$$

The non-zero Lorentz connections are as follows

$$\omega^{01} = \frac{\Sigma^2 G' G}{c^2\rho^3\Delta^{\frac{1}{2}}}dt \quad (171)$$

where $G = \sqrt{1-\frac{R_s r}{\rho^2}}$

$$\omega^{12} = \frac{\iota\rho'}{\rho c^2}d\theta \quad (172)$$

$$\omega^{13} = \frac{1}{2}\frac{K'}{\rho c^2}\sqrt{\frac{a^2 sin^2\theta - \Delta}{\rho^2 \Delta sin\theta}}d\varphi \quad (173)$$

with $K = (r^2 + a^2 + \frac{2R_s a r sin^2\theta}{\rho^2})\frac{sin^2\theta}{c^2}$

$$\omega^{23} = \frac{l^2 l'}{c}d\varphi \quad (174)$$

with $l = \sqrt{\frac{a^2 sin^2\theta - \Delta}{\rho^2 \Delta sin\theta}}$ in the equatorial plan $\theta = \frac{\Pi}{2}$

$$d\tau = -(1-\frac{R_s}{r})dt^2 - \frac{2R_s a}{cr}dtd\varphi + \frac{r^2}{c^2\Delta}dr^2 + (r^2+a^2+\frac{2R_s a}{r})\frac{1}{c^2}d\varphi^2 \quad (175)$$

$$e^0 = \iota(1-\frac{R_s}{r})^{\frac{1}{2}}dt \quad (176)$$

$$e^0 e^3 = \frac{2R_s a}{cr}dtd\varphi \quad (177)$$

$$e^1 = \frac{r^2}{c\sqrt{\Delta}}dr \quad (178)$$

$$e^2 = (r^2+a^2+\frac{2R_s a}{r})^{\frac{1}{2}}\frac{1}{c}d\varphi \quad (179)$$

$$e_0 = \frac{\iota\Sigma}{cr\Delta^{\frac{1}{2}}}\frac{d}{dt} \quad (180)$$

$$e_{02} = \sqrt{\frac{R_s a}{c\Delta r}}\frac{d^2}{dtd\varphi} \quad (181)$$

$$e_1 = \iota\frac{\Delta^{\frac{1}{2}}}{r}\frac{d}{dr} \quad (182)$$

$$e_2 = \sqrt{\frac{a^2-\Delta}{r^2\Delta}}\frac{d}{d\varphi} \quad (183)$$

like $g_{ij} = n_{ij}e^i e^j$ are only r and θ ,$\frac{\partial L}{\partial t} = \frac{\partial L}{\partial \varphi}$ and so

$$P_0 = \frac{\partial L}{\partial \dot{t}} = Ec^2 \quad (184)$$

$$P_2 = \frac{\partial L}{\partial \dot{\varphi}} = -L \quad (185)$$

en effet

$$P_0 = g_{tt}\dot{t} + g_{t\varphi}\dot{\varphi} = -c^2(1-\frac{R_s}{r})\dot{t} + \frac{R_s ac}{r}\dot{\varphi} = Ec^2 \quad (186)$$

$$P_2 = g_{t\varphi}\dot{t} + g_{\varphi\varphi}\dot{\varphi} = \frac{R_s ac}{r}\dot{t} + (r^2+a^2+\frac{2R_s a}{r})\dot{\varphi} = -L \quad (187)$$

we get the following system

$$\begin{pmatrix} g_{tt} & g_{t\varphi} \\ g_{t\varphi} & g_{\varphi\varphi} \end{pmatrix}\begin{pmatrix} \dot{t} \\ \dot{\varphi} \end{pmatrix} = \begin{pmatrix} Ec^2 \\ -L \end{pmatrix} \quad (188)$$

Noticing that

$$\Delta c^2 = -(g_{tt}g_{\varphi\varphi} - g_{t\varphi}^2) \quad (189)$$

we obtain

$$\dot{t} = -\frac{1}{\Delta}[(r^2+a^2+\frac{2R_s a}{r})E + \frac{R_s a^2 c}{r}L] \quad (190)$$

$$\dot{\varphi} = \frac{1}{\Delta}[\frac{R_s a^2 c}{r}E - (1-\frac{R_s}{r})L] \quad (191)$$

Therefore

$$z^1 = -\nu\omega^{01} \quad (192)$$

$$z^2 = -\nu\omega^{13} \quad (193)$$

$$z^1 = \iota\frac{\Sigma^2 G' G}{c^2\rho^3\Delta^{\frac{3}{2}}}[(r^2+a^2+\frac{2R_s a}{r})E + \frac{R_s a^2 c}{r}L] \quad (194)$$

$$z^2 = \iota\frac{1}{2}\frac{K'}{\rho\Delta c^2}\sqrt{\frac{a^2 sin^2\theta - \Delta}{\rho^2\Delta sin\theta}}[\frac{R_s a^2 c}{r}E - (1-\frac{R_s}{r})L] \quad (195)$$

$$z^3 = 0 \quad (196)$$

that is $z^1 = AE + BL$ with

$$A = \iota\frac{\Sigma^2 G' G}{c^2\rho^3\Delta^{\frac{3}{2}}}(r^2+a^2+\frac{2R_s a}{r})E \quad (197)$$

$$B = \iota\frac{\Sigma^2 G' G}{c^2\rho^3\Delta^{\frac{3}{2}}}\frac{R_s a^2 c}{r}L \quad (198)$$

and $z^2 = CE - DL$ with

$$C = \iota\frac{1}{2}\frac{K'}{\rho\Delta c^2}\sqrt{\frac{a^2 sin^2\theta - \Delta}{\rho^2\Delta sin\theta}}\frac{R_s a^2 c}{r}E \quad (199)$$

$$D = \iota\frac{1}{2}\frac{K'}{\rho\Delta c^2}\sqrt{\frac{a^2 sin^2\theta - \Delta}{\rho^2\Delta sin\theta}}(1-\frac{R_s}{r})L \quad (200)$$

$$H = \frac{1}{2}\begin{pmatrix} 0 & (A-C)E+(B-D)L \\ (A-C)E+(B-D)L & 0 \end{pmatrix} \quad (201)$$

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let $W=(A-C)^2E^2$, $Q=(A-C)(B-D)EL$, $S=(B-D)^2L^2$, $Z=(A-C)(B-D)EL$

$$\lambda_{\pm}=\pm\frac{1}{2}\sqrt{W-Q+S-Z} \tag{202}$$

$\lambda\in R$ if $(A-C)^2E^2+(B-D)^2L^2\geq(A-C)(B-D)EL$ This condition being verified

$$\Rightarrow L\geq-\frac{E(A-C)}{(B-D)} \tag{203}$$

$(M_{W,L,E}=\{((r_{LE},\phi),\phi\in[0,2\pi]\})$, the geodesics are strongly rotating in the Kerr metric from where the qubit is subjected to a dynamic decoherence. The geometric phase generators are

$$A_{\pm}=\pm\frac{1}{2}\frac{z^1dz^2-z^2dz^1}{(z^1)^2+(z^2)^2} \tag{204}$$

$$A_{\pm}=\pm\frac{1}{2}\frac{[(AE+BL)(C'E-D'L)-(CE-DL)(A'E+B'L)}{(AE+BL)^2+(CE-DL)^2} \tag{205}$$

Geometric coherence is always present. Moreover it comes that $N_{\pm}<<1$, which induces the validity of the adiabatic approximation in the Kerr metric.

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