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The Quantum Relativity in Extra-Dimensions

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Abstract:

In this work, we demonstrated that space-time dimensions of the universe is over than four and we developed a new theory that we called Quantum Relativity. We analytically calculated the relationship between the black hole space-time dimensions and its horizon area. Then the space-time dimensions generated of black holes in their geometric frames by Black hole entropy deduced by Hawking. With these we deduce the dimensions x generated by a black hole . We finally deduce and compute the total approximately numbers of the space-time dimensions of the universe and developed the new theory.

Keywords: Entropy, Black Hole,Event horizon, quantum relativity

I. INTRODUCTION

Black holes are Astrophysics objets that developped fascinated physicists and astronomers imagination. There is growing astronomical evidence for objects like black hole properties; In the Universe there is the black Holes in abundance. The black holes properties have continued to be study both at quantum and macroscopic leves for two century. In seventies of the last century, Bekenstein, Hawking, and others suggested that black holes have properties and thermodynamic attributes like entropy and temperature. Bekenstein-Hawking developped a law of Black hole entropy that is given by a quarter of horizon area. Then the horizon quantum gravity microstates could created the entropy. In this fact the quantum theory can be tested if it reproduce these thermodynamic properties of black holes. There are many propositions for quantum gravity theory. Two of them are M-theory where the space-time has twelve(12) dimensions and quantum loop gravity where the space-time has five(5) dimensions. There are also dynamic triangulations and Sorkins causal set framework. What of them estimate the good space-time dimensions? So to answer the question, we will review some of the developments concerning the black hole entropy in a particular quantum gravity theory. This will be applied on known black holes (Schzarchild, Kerr) and also for the horizons of general black holes, i.e. black holes of 4+n where n is supplementaries unknown dimensions of based four(4) dimensions of the space-time. And finally, we will deduce the possible dimension number x=4+n of our universe by based on the constant H_0 where we are going to determine n and develop the new theory Quantum gravity; which is indeed our main goal and t.

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II. METHODS

A. Bekenstein-Hawking entropy

Hawking proved that during the process of combining two Kerr black holes into a Kerr black hole [1], [2], [3], [4], the horizon area never decreases [5]. Since area A never decreases during the black hole combining process, A is proportional to entropy S. Bekenstein envisioned massive particle capture, a process to roughly determine normalization [6]. The precise ratio is determined by the precise value of Hawking's black hole temperature [7] If $A = 4\pi R_S^2$ is the area of the horizon of the black hole, we therefore have for a Scharzchild black hole

$$\delta A = 8\pi R_S \delta R_S = \frac{32\pi G^2 M \delta M}{c^4} \tag{1}$$

When we increase the mass of δM ie when we bring an additional heat of $\delta M c^2$, the entropy increases by:

$$\delta S = \frac{\delta M c^2}{T_{TN}} \tag{2}$$

The work that is gained by letting the radiation plunge into the black hole is:

$$W = mc^2 - mg\frac{d}{2} = mc^2 - m\frac{c^4}{4GM}\frac{d}{2} \tag{3}$$

$$= mc^2(1 - \frac{\hbar c^3}{8GM\kappa T}) \tag{4}$$

with $g = \frac{GM}{R_S^2} = \frac{c^4}{4GM}$ and $d = \frac{\hbar c^3}{\kappa T}$ The efficiency of our thermal machine is therefore:

$$e \simeq 1 - \frac{\hbar c^3}{8GM\kappa T} \tag{5}$$

which gives

$$\delta S = \frac{8\kappa GM\delta M}{\hbar c} \tag{6}$$

Thus:

$$S = \frac{\kappa c^3}{4\pi\hbar G} A \tag{7}$$

and

$$T = \frac{1}{4\Pi} \frac{r_+ - r_-}{r_+^2 + a^2} \quad (8)$$

which is the Bekenstein-Hawking temperature. The temperature can be from the analytical suite of the metric to Euclidean spaces or by considering the entanglement entropy of a field in one and the parallel universe [8]. By absorbing a particle of radius b and mass μ, the minimal horizon zone increases by $\triangle A = 8\pi\mu b$. However, the radius b of the particle cannot be zero by the uncertainty principle, b $\geq \mu/2\pi$, so the minimum horizon increase is, (G = 1) $\triangle A \geq 4$. The third law is that as a system approaches absolute zero, entropy suggests a minimum value. Note that this does not mean that when $T \to 0$, the entropy must disappear like the perfect crystal. The extremal black hole has zero temperature but non-zero entropy, which means that there is degeneration on $exp(S)$ of black hole ground states. However, it is difficult to achieve this degeneration in classical physics. In a higher dimensional space, Bekenstein's entropy formula is

$$S = \frac{A}{4G_D} \quad (9)$$

where G_D is Newton's D-dimensional constant. Entropy is proportional to the area. Bekenstein showed in 1973 that indeed

$$S = \frac{\kappa c^3}{4\pi\hbar G} A \quad (10)$$

B. Dimensions number generated by a black hole

Before Enstein, the universe is knowing to have tree dimensions but Enstein added time dimension to have four(4) space-time dimensions; so after Enstein others theory proposed twelve(12) dimensions like string theory and five(5) dimensions for loop quantum gravity. To estimate the real x dimensions we have been taking x=4+n where n is supplementaries unknown dimensions of the four(4) space-time dimensions and to be determine. Let x unknown dimensions kerr black hole metric is

$$ds^2 = (1 - \frac{\mu}{\Sigma r^{n-1}})dt^2 + \frac{2a\mu sin^2\theta}{\Sigma r^{n-1}} dtd\varphi \quad (11)$$

$$-\frac{\Sigma}{\Delta}dr^2 - \Sigma d\theta^2 \quad (12)$$

$$-(r^2 + a^2 + \frac{a^2\mu sin^2\theta}{\Sigma r^{n-1}})sin^2\theta d\varphi^2 \quad (13)$$

$$-r^2 cos^2\theta d\Omega_n \quad (14)$$

Where

$$\Delta = r^2 + a^2 - \frac{\mu}{r^{n-1}} \quad (15)$$

and

$$\Sigma = r^2 + a^2 cos^2\theta \quad (16)$$

Let Ω_n be the element of the n^{th} sphere. Mass and angular momentum are given by:

$$M = \frac{(n+2)S_{n+2}}{16\pi G}\mu \quad (17)$$

$$J = \frac{2}{n+2} Ma \quad (18)$$

Note that S_{n+2} is the $n+2$ dimensional sphere area

$$S_{n+2} = \frac{2\pi^{\frac{n+3}{2}}}{\Gamma[(n+3)/2]} \quad (19)$$

Here horizon is determined by solving the equation $\Delta(r) = 0$ whose unique solution is

$$r_h^{n+1} = \frac{\mu}{1+a_*^2} \quad (20)$$

Then we get the area :

$$A = \frac{2\pi^{\frac{n+3}{2}}}{\Gamma[(n+3)/2]}(\frac{\mu}{1+a_*^2}) \quad (21)$$

This area is a section t = const of the event horizon H, therefore

$$S = \frac{\pi^{\frac{n+3}{2}}}{2\Gamma[(n+3)/2]}(\frac{\mu}{1+a_*^2}) \quad (22)$$

By taking the neperian logarithm of the entropy, we get:

$$ln(S) - ln(\frac{\mu}{1+a_*^2}) = (\frac{5n+9}{10})ln(\pi) - ln(\Gamma[(n+3)/2]) \quad (23)$$

then, we deduce the relationship between the supplementaries dimensions number n and the entropy S such as:

$$(\frac{5n+9}{10})ln(\pi) - ln(\Gamma[(n+3)/2]) = ln(S) - ln(\frac{\mu}{1+a_*^2}) \quad (24)$$

III. RESULTS AND DISCUSSION

In the last section we obtained we equation which have been given a relationship between the supplementaries dimensions number n and the entropy S such as:

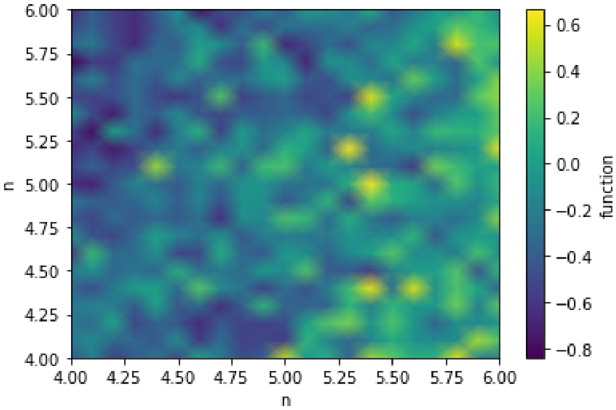
$$(\frac{5n+9}{10})ln(\pi) - ln(\Gamma[(n+3)/2]) = ln(S) - ln(\frac{\mu}{1+a_*^2}) \quad (25)$$

In this section we are going to suppose that the universe entropy is given by $S = H_0$ which is the difference between collapsing entropies which are negative and those of positive radiations,then we get:

$$(\frac{5n+9}{10})ln(\pi) - ln(\Gamma[(n+3)/2]) - ln(H_0) + ln(\frac{\mu}{1+a_*^2}) = 0 \quad (26)$$

Let the supplementaries dimensions n of the Universe be the equation solution, then by simulation as shown in the next figure, we got the supplementaries dimensions n=5 as the equation zero.

FIG. 1. The results gave five(5) supplementaries dimensions for the Universe



To concluded, every black hole generate a supplementaries dimensions in the universe, and so is themselves others universe. But on based of the characteristics of the universe, we found that the universe have five(5) supplementaries dimensions and adding to the four(4) known dimensions, we got x= 9 dimensions. The dimensions of the Universe is minimun nine(9) dimensions. So we lived in a black hole and there are others five(5) dimensions to find experimentally.

A. Quantum relativity theory

In the last section, we demonstrated that the Universe has nine(9) dimensions and therefore greater than four(4) dimensions. it is therefore necessary to introduce a new theory that we call " Quantum Relativity theory". Let's take the Einstein Hilbert action [9], [10]:

$$S_{HE}[g,\varphi] = \int_Q \sqrt{-det[g_{ij}]}\left(\frac{1}{2k}R + L_m\right)d^4x \tag{27}$$

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$$S_{HE}[g,\varphi] = \int_Q \sqrt{-det[g_{ij}]}\frac{1}{2k}R\left(1+\frac{2kL_m}{R}\right)d^4x \tag{28}$$

we deduce for a space time constitute of three spatial dimensions and a temporal dimension:

$$S_{HE}[g,\varphi] = \int_Q \sqrt{-det[g_{ij}]}\frac{1}{2k}R^1\left(1+\frac{2kL_m}{R}\right)^1 d^{3+1}x \tag{29}$$

That we are generalizing to α unknow dimension:

$$S_{HE}[g,\varphi] = \int_Q \sqrt{-det[g_{ij}]}\frac{1}{2k}R^{\alpha-3}\left(1+\frac{2kL_m}{R}\right)^{\alpha-3} d^{\alpha}x \tag{30}$$

We end up with a new theory of which in the next section we will establish the equations of motion in this one

B. Tougma's field equation

We have demonstrated that the Quantum relativity action is:

$$S_{HE}[g,\varphi] = \int_Q \sqrt{-det[g_{ij}]}\frac{1}{2k}R^{\alpha-3}\left(1+\frac{2kL_m}{R}\right)^{\alpha-3} d^{\alpha}x \tag{31}$$

The variation of the action with respect to the inverse of the metric must be zero for the solutions,giving the equation:

$$0 = \delta S \tag{32}$$

$$0 = \int_Q \frac{1}{2k}\left[\frac{\delta(\sqrt{-det[g_{ij}]}R^{\alpha-3})}{\delta g^{uv}}\left(1+\frac{2kL_m}{R}\right)^{\alpha-3} \right. \tag{33}$$

$$\left. + \sqrt{-det[g_{ij}]}R^{\alpha-3}\frac{\delta\left(1+\frac{2kL_m}{R}\right)^{\alpha-3}}{\delta g^{uv}}\delta\right]g^{uv}d^{\alpha}x \tag{34}$$

Since this equation holds for all variation δg^{uv}, that implies that

$$0 = \int_Q \frac{1}{2k}\left[\left(\frac{\delta R(\alpha-3)R^{\alpha-4}}{\delta g^{uv}} + \frac{R^{\alpha-3}\frac{1}{\sqrt{-g}}\delta\sqrt{-det[g_{ij}]}}{\delta g^{uv}}\right)\right. \tag{35}$$

$$\left(1+\frac{2kL_m}{R}\right)^{\alpha-3} \tag{36}$$

$$\left. R^{\alpha-3}\frac{(\alpha-3)\delta\frac{2kL_m}{R}\left(1+\frac{2kL_m}{R}\right)^{\alpha-4}}{\delta g^{uv}}\right]g^{uv}\sqrt{-det[g_{ij}]}d^{\alpha}x \tag{37}$$

$$0 = \int_Q \frac{1}{2k}\left[\left(\frac{\delta R(\alpha-3)R^{-1}}{\delta g^{uv}} + \frac{1}{\sqrt{-g}}\frac{\delta\sqrt{-det[g_{ij}]}}{\delta g^{uv}}\right)\right. * \tag{38}$$

$$\left(1+\frac{2kL_m}{R}\right)^{\alpha-3} + \tag{39}$$

$$\left. \frac{(\alpha-3)\delta\frac{2kL_m}{R}\left(1+\frac{2kL_m}{R}\right)^{\alpha-4}}{\delta g^{uv}}\right]g^{uv}\sqrt{-det[g_{ij}]}R^{\alpha-3}d^{\alpha}x \tag{40}$$

this give:

$$\left(\frac{\delta R(\alpha-3)R^{-1}}{\delta g^{uv}} + \frac{1}{\sqrt{-g}}\frac{\delta\sqrt{-det[g_{ij}]}}{\delta g^{uv}}\right)\left(1+\frac{2kL_m}{R}\right)^{\alpha-3} + \tag{41}$$

$$\frac{(\alpha-3)\delta\frac{2kL_m}{R}\left(1+\frac{2kL_m}{R}\right)^{\alpha-4}}{\delta g^{uv}} = 0 \tag{42}$$

$$\left(\frac{\delta R(\alpha-3)R^{-1}}{\delta g^{uv}} + \frac{1}{\sqrt{-g}}\frac{\delta\sqrt{-det[g_{ij}]}}{\delta g^{uv}}\right) + \tag{43}$$

$$\frac{(\alpha-3)\delta\frac{2kL_m}{R}\left(1+\frac{2kL_m}{R}\right)^{-1}}{\delta g^{uv}} = 0 \tag{44}$$

$$\frac{\delta R(\alpha-3)R^{-1}}{\delta g^{uv}} + \frac{1}{\sqrt{-g}}\frac{\delta\sqrt{-det[g_{ij}]}}{\delta g^{uv}} \tag{45}$$

$$+\frac{(\alpha-3)\frac{2k\delta L_m}{R}\left(1+\frac{2kL_m}{R}\right)^{-1}}{\delta g^{uv}} \tag{46}$$

$$-\frac{2kL_m}{R^2}(\alpha-3)\left(1+\frac{2kL_m}{R}\right)^{-1}\frac{\delta R}{\delta g^{uv}} = 0 \tag{47}$$

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$$\frac{\delta R(\alpha-3)}{\delta g^{uv}}+\frac{1}{\sqrt{-g}}\frac{\delta\sqrt{-det[g_{ij}]}}{\delta g^{uv}}R \tag{48}$$
$$+(\alpha-3)2k(1+\frac{2kL_m}{R})^{-1}\frac{\delta L_m}{\delta g^{uv}} \tag{49}$$
$$-\frac{2kL_m}{R}(\alpha-3)(1+\frac{2kL_m}{R})^{-1}\frac{\delta R}{\delta g^{uv}}=0 \tag{50}$$

$$[\frac{\delta R(\alpha-3)}{\delta g^{uv}}+\frac{1}{\sqrt{-g}}\frac{\delta\sqrt{-det[g_{ij}]}}{\delta g^{uv}}R](1+\frac{2kL_m}{R}) \tag{51}$$
$$+(\alpha-3)2k\frac{\delta L_m}{\delta g^{uv}} \tag{52}$$
$$-\frac{2kL_m}{R}(\alpha-3)\frac{\delta R}{\delta g^{uv}}=0 \tag{53}$$

And the motion is given by

$$[\frac{\delta R(\alpha-3)}{\delta g^{uv}}+\frac{1}{\sqrt{-g}}\frac{\delta\sqrt{-det[g_{ij}]}}{\delta g^{uv}}R](1+\frac{2kL_m}{R}) \tag{54}$$
$$-\frac{2kL_m}{R}(\alpha-3)\frac{\delta R}{\delta g^{uv}}=(3-\alpha)2k\frac{\delta L_m}{\delta g^{uv}} \tag{55}$$

is the equation of motion for metric. The right-hand side of the equation is (by definition) proportional to the energy-momentum tensor,

$$T_{uv}=(3-\alpha)2k\frac{\delta L_m}{\delta g^{uv}} \tag{56}$$

To calculate the variation of the Ricci curvature, we start by calculating the variation of the tensor of Riemann, then of the tensor of Ricci. Recall that the Riemann tensor is locally defined through:

$$R^{\rho}_{\sigma uv}=\partial_u\Gamma^{\rho}_{v\sigma}-\partial_v\Gamma^{\rho}_{u\sigma}+\Gamma^{\rho}_{u\lambda}\Gamma^{\lambda}_{v\sigma}-\Gamma^{\rho}_{v\lambda}\Gamma^{\lambda}_{u\sigma} \tag{57}$$

Since Riemann's tensor depends only on the symbols of Christoffel Γ^{λ}_{uv}, its variation may be calculated as

$$\delta R^{\rho}_{\sigma uv}=\partial_u\delta\Gamma^{\rho}_{v\sigma}-\partial_v\delta\Gamma^{\rho}_{u\sigma}+\delta\Gamma^{\rho}_{u\lambda}\Gamma^{\lambda}_{v\sigma}+\Gamma^{\rho}_{u\lambda}\delta\Gamma^{\lambda}_{v\sigma} \tag{58}$$
$$-\delta\Gamma^{\rho}_{v\lambda}\Gamma^{\lambda}_{u\sigma}-\Gamma^{\rho}_{v\lambda}\delta\Gamma^{\lambda}_{u\sigma} \tag{59}$$

Now, since $\Gamma^{\rho}_{v\sigma}$ is the difference of two connections, it is about a tensor, of which we can calculate the covariant derivative,

$$\nabla_u(\delta\Gamma^{\rho}_{v\sigma})=\partial_u\delta\Gamma^{\rho}_{v\sigma}+\Gamma^{\rho}_{u\lambda}\delta\Gamma^{\lambda}_{v\sigma}-\Gamma^{\lambda}_{uv}\delta\Gamma^{\rho}_{\lambda\sigma}-\Gamma^{\lambda}_{u\lambda}\delta\Gamma^{\rho}_{v\sigma} \tag{60}$$

We can then observe that the variation of the Riemann tensor above is exactly equal to the difference of two such terms,

$$\delta R^{\rho}_{\sigma uv}=\nabla_u(\delta\Gamma^{\rho}_{v\sigma})-\nabla_v(\delta\Gamma^{\rho}_{u\sigma}) \tag{61}$$

We can now obtain the variation of the Ricci tensor simply by contacting two indices in the expression of the variation of the Riemann tensor, and we then obtain the identity of Palatini:

$$\delta R_{\sigma v}=\delta R^{\rho}_{\sigma\rho v}=\nabla_\rho(\delta\Gamma^{\rho}_{v\sigma})-\nabla_v(\delta\Gamma^{\rho}_{\rho\sigma}) \tag{62}$$

The Ricci curvature is then defined as:

$$R=g^{\sigma v}R_{\sigma v} \tag{63}$$

Therefore, its variation from the inverse of the metric $g^{\sigma v}$ is given by:

$$\delta R=\delta g^{\sigma v}R_{\sigma v}+g^{\sigma v}\delta R_{\sigma v} \tag{64}$$

In the second line, we used the compatibility of the metric with the connection ∇g^{uv}, and the result was obtained previously on the variat ion of the Ricci tensor. The last term,

$$\delta R=\delta g^{\sigma v}R_{\sigma v}+\nabla_\rho(g^{\sigma v}\delta\Gamma^{\rho}_{v\sigma}-g^{\sigma\rho}\delta\Gamma^{u}_{u\sigma}) \tag{65}$$

However, when the variation of the metric varies in the vicinity of edge or when there are no edges, this term does not contribute to the variation of the action. Thereby, we obtain outside the edges.

$$\frac{\delta R}{\delta g^{uv}}=R_{uv} \tag{66}$$

We recall the differential of the determining

$$\delta g=\delta det(g_{uv})=gg^{uv}\delta g_{uv} \tag{67}$$

that we can calculate for example via the explicit formula of the determinant and of an expansion limited. Thanks to this result, we obtain

$$\delta\sqrt{-g}=\frac{1}{2}\sqrt{-g}\delta g=\frac{1}{2}\sqrt{-g}(g^{uv}\delta g_{uv})=-\frac{1}{2}\sqrt{-g}(g_{uv}\delta g^{uv}) \tag{68}$$

In the last tie, we used the fact that

$$\delta g^{uv}=-g^{u\alpha}(\delta g_{\alpha\beta})g^{\beta v} \tag{69}$$

which follows from the different from the inverse of a matrix

$$\frac{1}{\sqrt{-g}}\frac{\delta\sqrt{-g}}{\delta g^{uv}}=-\frac{1}{2}g_{uv} \tag{70}$$

Now, we have all the variation needed to get the equation of motion. We insert the calculated equations into the equation of motion for the metric to obtain

$$[\frac{\delta R(\alpha-3)}{\delta g^{uv}}+\frac{1}{\sqrt{-g}}\frac{\delta\sqrt{-det[g_{ij}]}}{\delta g^{uv}}R](1+\frac{2kL_m}{R}) \tag{71}$$
$$-\frac{2kL_m}{R}(\alpha-3)\frac{\delta R}{\delta g^{uv}}=T_{uv} \tag{72}$$

$$[(\alpha-3)R_{uv}-\frac{1}{2}g_{uv}R](1+\frac{2kL_m}{R})-\frac{2kL_m}{R}(\alpha-3)R_{uv}=T_{uv} \tag{73}$$

$$[(\alpha-3)R_{uv}-\frac{1}{2}g_{uv}R](1+\frac{2kL_m}{R})-2k(\alpha-3)g_{uv}L_m=T_{uv} \tag{74}$$

which is the TOUGMA's equation, and
$k=\frac{8\pi G}{c^4}$
was chosen so as to obtain the desired non-relativistic limit: the universal law of gravitation of Newton, where is the gravitational constant; if we are in four dimensions

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