RGη-Closed Sets in Topological Spaces

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Abstract: In this paper, a new class of sets called regular generalized η-closed (briefly rgη-closed) sets is introduced and its properties are studied. The relationships among closed, α-closed, s-closed, η-closed, rgη-closed and their generalized closed sets are investigated. Several examples are provided to illustrate the behavior of these new class of sets.

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1. Introduction


2. Preliminaries

Throughout this paper, spaces (X, DidChange to Θ), (Y, σ), and (Z, γ) (or simply X, Y and Z) always mean topological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a space X. The closure of A and interior of A are denoted by cl(A) and int(A) respectively. A subset A is said to be regular open [12] (resp. regular closed [12]) if A = cl(int(A)) (resp. A = int(cl(A)). The finite union of regular open sets is said to be π-open [16]. The complement of a π-open set is said to be π-closed [16].

Definition 2.1. A subset A of a topological space (X, DidChange to Θ) is said to be

(i) s-open [7] if A ⊆ cl(int(A)).
(ii) α-open [11] if A ⊆ int(cl(int(A))).
(iii) η-open [14] if A ⊆ cl(int(int(A))) ∪ cl(int(A)).
(iv) η-closed [14] if A ⊇ cl(int(cl(A))) ∪ int(cl(A)).

The complement of an s-open (resp. α-open, η-open) set is called s-closed (resp. α-closed, η-closed). The intersection of all s-closed (resp. α-closed, η-closed) sets containing A, is called s-closure (resp. α-closure, η-closure) of A, and is denoted by s-cl(A) (resp. α-cl(A), η-cl(A)). The η-interior of A, denoted by η-int(A) is defined as union of all η-open sets contained in A. We denote the family of all η-open (resp. η-closed) sets of a topological space by η-O(X) (resp. η-C(X)).

Definition 2.2. A subset A of a space (X, DidChange to Θ) is said to be

(1) generalized closed (briefly g-closed) [8] if cl(A) ⊆ U whenever A ⊆ U and U ∈ DidChange to Θ.
(2) πg-closed [4] if cl(A) ⊆ U whenever A ⊆ U and U is π-open in X.
(3) rg-closed [4] if cl(A) ⊆ U whenever A ⊆ U and U is regular open in X.
(4) α-generalized closed (briefly αg-closed) [9, 10] if α-cl(A) ⊆ U whenever A ⊆ U and U ∈ DidChange to Θ.
(5) πgα-closed [2] if α-cl(A) ⊆ U whenever A ⊆ U and U is π-open in X.
(6) gur-closed [13] if α-cl(A) ⊆ U whenever A ⊆ U and U is regular open in X.
(7) generalized semi-closed (briefly gs-closed) [1] if s-cl(A) ⊆ U whenever A ⊆ U and U ∈ DidChange to Θ.
(8) πgs-closed [3] if s-cl(A) ⊆ U whenever A ⊆ U and U is π-open in X.
(9) rgs-closed if s-cl(A) ⊆ U whenever A ⊆ U and U is regular open in X.
(10) generalized η-closed (briefly gη-closed) [15] if η-cl(A) ⊆ U whenever A ⊆ U and U ∈ DidChange to Θ.
(11) πηg-closed (briefly gη-closed) [6] if η-cl(A) ⊆ U whenever A ⊆ U and is π-open in X.
(12) g-closed (resp. πg-open, rg-open, αg-open, πgs-open, gur-open, gs-open, rgs-open, gη-open, πηg-open) set if the complement of A is g-closed (resp. πg-closed, rg-closed, αg-closed, πgs-closed, gur-closed, gs-closed, rgs-closed, gη-closed, πηg-closed).

3. Regular Generalized η-closed Sets

Definition 3.1. A subset A of a space (X, DidChange to Θ) is said to be regular generalized η-closed (briefly rgη-closed) if η-cl(A) ⊆ U whenever A ⊆ U and U is regular open in X. The family of all rgη-closed subsets of X will be denoted by
set

Theorem 3.2. Every closed set is \( \operatorname{rgn} \)-closed.

Proof. Let \( A \) be a closed set in \( X \). Let \( U \) be a regular open set in \( X \) such that \( A \subset U \). Since \( A \) is closed, that is, \( \operatorname{cl}(A) = A \). \( \operatorname{cl}(A) \subset U \). But we have \( \eta \operatorname{cl}(A) \subset \alpha \operatorname{cl}(A) \subset U \). Therefore \( \eta \operatorname{cl}(A) \subset U \). Hence \( A \) is \( \operatorname{rgn} \)-closed in \( X \).

Theorem 3.3. For a topological space \( X \) the followings hold:

1. Every \( g \)-closed set is \( \operatorname{rgn} \)-closed.
2. Every \( \operatorname{rg} \)-closed set is \( \operatorname{rgn} \)-closed.
3. Every \( \alpha \)-closed set is \( \operatorname{rgn} \)-closed.
4. Every \( \alpha \)-open set is \( \operatorname{rgn} \)-closed.
5. Every \( \eta \)-closed set is \( \operatorname{rgn} \)-closed.
6. Every \( g\alpha - \)closed set is \( \operatorname{rgn} \)-closed.
7. Every \( g\alpha \)-closed set is \( \operatorname{rgn} \)-closed.
8. Every \( \eta \)-open set is \( \operatorname{rgn} \)-closed.
9. Every \( g\eta \)-closed set is \( \operatorname{rgn} \)-closed.
10. Every \( \eta \alpha \)-closed set is \( \operatorname{rgn} \)-closed.
11. Every \( \eta \alpha \)-open set is \( \operatorname{rgn} \)-closed.
12. Every \( \eta \)-open set is \( \operatorname{rgn} \)-closed.
13. Every \( g\eta \)-closed set is \( \operatorname{rgn} \)-closed.
14. Every \( \eta \alpha \)-open set is \( \operatorname{rgn} \)-closed.

Proof. Let \( A \) be a \( g \)-closed set in \( X \). Let \( U \) be a regular open set in \( X \) such that \( A \subset U \). Since every regular open set is open and since \( A \) is \( g \)-closed, that is, \( \operatorname{cl}(A) \subset U \). We then have \( \eta \operatorname{cl}(A) \subset \alpha \operatorname{cl}(A) \subset U \). Therefore \( \eta \operatorname{cl}(A) \subset U \). Hence \( A \) is \( \operatorname{rgn} \)-closed in \( X \).

Remark 3.4: From the above definitions, theorems and known results the relationship between \( \operatorname{rgn} \)-closed sets and some other existing generalized closed sets are implemented in the following figure:

\[
\begin{array}{c}
g \rightarrow \\
\alpha \rightarrow \\
s \rightarrow \\
\eta \rightarrow \\
g \alpha \rightarrow \\
g \eta \rightarrow \\
\alpha \eta \rightarrow \\
g \eta \alpha \rightarrow \\
g \operatorname{closed} \\
\alpha \operatorname{closed} \\
s \operatorname{closed} \\
\eta \operatorname{closed} \\
g \operatorname{closed} \\
\alpha \operatorname{closed} \\
s \operatorname{closed} \\
\eta \operatorname{closed} \\
\end{array}
\]

Where none of the implications is reversible as can be seen from the following examples:
Example 3.5: Let $X = \{a, b, c, d\}$ and $\mathcal{I} = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}, X\}$. Then $A = \{a, b, c\}$ and $B = \{a, b, d\}$ are $\eta$-closed as well as $\eta$-$\alpha$ closed sets. A and B are also $\eta$-$\alpha$ closed but not closed.

Example 3.6: Let $X = \{a, b, c, d\}$ and $\mathcal{I} = \{\emptyset, \{a\}, \{d\}, \{a, d\}, \{c, d\}, \{a, c, d\}, X\}$. Then $A = \{c\}$ is $\eta$-$\alpha$ closed as well as $\eta$-$\alpha$ closed. It is also $\eta$-$\alpha$ closed set. But it is neither closed nor g-$\alpha$ closed. It is not $\eta$-$\alpha$ closed set.

Example 3.7: Let $X = \{a, b, c, d\}$ and $\mathcal{I} = \{\emptyset, \{c\}, \{d\}, \{c, d\}, \{b, c, d\}, X\}$. Then $A = \{b\}$ is g-$\alpha$ closed, $\alpha$-$\alpha$ closed, $\eta$-$\alpha$ closed, $\eta$-$\alpha$ closed. It is also $\eta$-$\alpha$ closed set. But it is not closed.

Example 3.8: Let $X = \{a, b, c, d\}$ and $\mathcal{I} = \{\emptyset, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, d\}, \{a, c, d\}, X\}$. Then $A = \{a, b\}$ is $\eta$-$\alpha$ closed as well as $\eta$-$\alpha$ closed. It is also $\eta$-$\alpha$ closed set. But it is neither closed nor g-$\alpha$-$\alpha$ closed set.

Example 3.9: Let $X = \{a, b, c\}$ and $\mathcal{I} = \{\emptyset, \{a\}, \{b, c\}, \{a, c\}, X\}$. Then $A = \{c\}$ is $\eta$-$\alpha$ closed as well as $\eta$-$\alpha$ closed. It is also $\eta$-$\alpha$ closed set. But it is not $\alpha$-$\alpha$ closed.

Example 3.10: Let $X = \{a, b, c\}$ and $\mathcal{I} = \{\emptyset, \{a\}, \{b\}, \{c\}, X\}$. Then $A = \{a, b\}$ is $\eta$-$\alpha$ closed as well as $\eta$-$\alpha$ closed. It is also $\eta$-$\alpha$ closed set. But it is not $\eta$-$\alpha$ closed.

Example 3.11: Let $X = \{0, 1\}$ and $\mathcal{I} = \{\emptyset, \{0\}, X\}$. The topological space $(X, \mathcal{I})$ is called the Sierpinski space. Then the set $A = \{0\}$ is $\eta$-$\alpha$ closed as well as $\eta$-$\alpha$ closed sets but not closed.

Example 3.12: Let $X = \{a, b, c, d, e\}$ and $\mathcal{I} = \{\emptyset, \{a, b\}, \{c, d\}, \{a, b, c, d\}, X\}$. Then

1) $\eta$-$\alpha$ closed sets are $\emptyset, \{a, b\}, \{c, d\}, \{a, b, c, d\}, X$.

2) $\eta$-$\alpha$ closed sets are $\emptyset, \{a, b\}, \{c, d\}, \{a, b, c, d\}, X$.

3) $\eta$-$\alpha$ closed sets are $\emptyset, \{a, b\}, \{c, d\}, \{a, b, c, d\}, X$.

4) $\eta$-$\alpha$ closed sets are $\emptyset, \{a, b\}, \{c, d\}, \{a, b, c, d\}, X$.

4. Characteristics of $\eta$-$\alpha$ Closed Sets

Theorem 4.1: If $A$ is regular open and $\eta$-$\alpha$ closed, then $A$ is $\eta$-$\alpha$ closed and hence clopen.

Proof: If $A$ is regular open and $\eta$-$\alpha$ closed, then $\eta$-$\alpha$-$\alpha$ closed $\subseteq A$. This implies $A$ is $\eta$-$\alpha$ closed. Hence $A$ is clopen, since every $\eta$-$\alpha$-$\alpha$ closed (regular open) set is (regular) closed.

Theorem 4.2: If $A$ and $B$ are $\eta$-$\alpha$ closed sets in $X$ then $A \cup B$ is an $\eta$-$\alpha$ closed set in $X$.

Proof: Let $A$ and $B$ be $\eta$-$\alpha$ closed sets in $X$ and $U$ be any regular open set containing $A$ and $B$. Therefore $\eta$-$\alpha$-$\alpha$ closed $\subseteq U$. $\eta$-$\alpha$-$\alpha$ closed $\subseteq U$. Since $A \subseteq U$, $B \subseteq U$ then $A \cup B \subseteq U$. Hence $\eta$-$\alpha$-$\alpha$ closed $\subseteq U$. Therefore $A \cup B$ is $\eta$-$\alpha$ closed set in $X$.

Theorem 4.3: A set $A$ is $\eta$-$\alpha$ closed set iff $\eta$-$\alpha$-$\alpha$ closed $\subseteq A$ contains no non-empty regular closed set.

Proof: Necessity: Let $F$ be a regular closed set in $X$ such that $F \subseteq \eta$-$\alpha$-$\alpha$ closed $\subseteq A$. Then $A \subseteq X$ – $F$. Since $A$ is a $\eta$-$\alpha$-$\alpha$ closed set and $X$ – $F$ is regular open then $\eta$-$\alpha$-$\alpha$ closed $\subseteq X$ – $F$.

Sufficiency: Let us assume that $\eta$-$\alpha$-$\alpha$ closed $\subseteq A$ contains no non-empty regular closed set. Let $A \subseteq U$ and $U$ be regular-open. Suppose that $\eta$-$\alpha$-$\alpha$ closed $\subseteq U$, then $\eta$-$\alpha$-$\alpha$ closed $\subseteq U$. Therefore $\eta$-$\alpha$-$\alpha$ closed $\subseteq U$. Hence $A$ is a $\eta$-$\alpha$-$\alpha$ closed set.

Theorem 4.4: The intersection of any two subsets of $\eta$-$\alpha$ closed sets in $X$ is a $\eta$-$\alpha$ closed set in $X$.

Proof: Let $A$ and $B$ be two subsets of a $\eta$-$\alpha$ closed set. Assume $A, B \subseteq U$, where $U$ is regular-open. Then $\eta$-$\alpha$-$\alpha$ closed $\subseteq U$. Since $A$ and $B$ are $\eta$-$\alpha$-$\alpha$ closed sets, $A \cap B$ is a $\eta$-$\alpha$-$\alpha$ closed set.

Theorem 4.5: If $A$ is a $\eta$-$\alpha$-$\alpha$ closed set in $X$ and $AC B \subseteq \eta$-$\alpha$-$\alpha$ closed, then $B$ is a $\eta$-$\alpha$-$\alpha$ closed set in $X$.

Proof: Since $B \subseteq \eta$-$\alpha$-$\alpha$ closed, we have $\eta$-$\alpha$-$\alpha$ closed $\subseteq \eta$-$\alpha$-$\alpha$ closed $\subseteq B$. By Theorem 3.2, $\eta$-$\alpha$-$\alpha$ closed $\subseteq B$ contains no non-empty regular closed set. Hence $\eta$-$\alpha$-$\alpha$ closed $\subseteq B$ contains no non-empty regular closed set. Therefore $B$ is a $\eta$-$\alpha$-$\alpha$ closed set in $X$.

Theorem 4.6: Let $A \subseteq Y \subseteq X$ be a $\eta$-$\alpha$-$\alpha$ closed set in $X$. Then $A$ is a $\eta$-$\alpha$-$\alpha$ closed set relative to $Y$.

Proof: Given that $A \subseteq Y \subseteq X$ and $A$ is a $\eta$-$\alpha$-$\alpha$ closed set in $X$. To prove that $A$ is a $\eta$-$\alpha$-$\alpha$ closed set relative to $Y$, let us assume that $A \subseteq Y \subseteq U$, where $U$ is regular open in $X$. Since $A$ is an $\eta$-$\alpha$-$\alpha$ closed set, $A \subseteq U$ implies $\eta$-$\alpha$-$\alpha$ closed $\subseteq U$. It follows that $Y \subseteq \eta$-$\alpha$-$\alpha$ closed $\subseteq X \subseteq U$. That is $A$ is a $\eta$-$\alpha$-$\alpha$ closed set relative to $Y$.

Theorem 4.7: If $A$ is both regular open and $\eta$-$\alpha$-$\alpha$ closed set in $X$ then $A$ is a $\eta$-$\alpha$-$\alpha$ closed set.

Proof: Since $A$ is a regular open and $\eta$-$\alpha$-$\alpha$ closed set in $X$, $\eta$-$\alpha$-$\alpha$ closed $\subseteq U$. But $A \subseteq \eta$-$\alpha$-$\alpha$ closed. Therefore $A = \eta$-$\alpha$-$\alpha$ closed. Therefore $A$ is a regular closed set.
Theorem 4.8: For \( x \in X \), the set \( X \setminus \{x\} \) is \( \eta \)-closed or regular open.

Proof: Suppose that \( X \setminus \{x\} \) is not regular open, then \( X \) is the only regular open set containing \( X \setminus \{x\} \) (i.e. \( \eta \)-cl\( (X \setminus \{x\}) \subset X \)). Then \( X \setminus \{x\} \) is a \( \eta \)-closed set in \( X \).

Definition 4.9: Let \((X, \mathcal{I})\) be a topological space, \( A \subset X \) and \( x \in X \). Then \( x \) is said to be a \( \eta \)-limit point of \( A \) iff every \( \eta \)-open set containing \( x \) contains a point of \( A \) different from \( x \), and the set of all \( \eta \)-limit points of \( A \) is said to be the \( \eta \)-derived set of \( A \) and is denoted by \( D_\eta(A) \).

Usual derived set of \( A \) is denoted by \( D(A) \).

The proof of the following result is analogous to the well known ones.

Lemma 4.10: Let \((X, \mathcal{I})\) be a topological space and \( A \subset X \). Then \( \eta \text{-cl}(A) = A \cup D_\eta(A) \).

Theorem 4.11: Let \( A \) and \( B \) be \( \eta \)-closed sets in \((X, \mathcal{I})\) such that \( D(A) \subset D_\eta(A) \) and \( D(B) \subset D_\eta(B) \). Then \( A \cup B \) is \( \eta \)-closed.

Proof: For any set \( E \subset (X, \mathcal{I}) \), \( D_\eta(E) \subset D(E) \). Therefore \( D_\eta(A) = D(A) \) and \( D_\eta(B) = D(B) \). That is \( \text{cl}(A) = \eta \text{-cl}(A) \) and \( \text{cl}(B) = \eta \text{-cl}(B) \).

Let \( A \cup B \subset U \) where \( U \) is regular open. Then \( A \subset U \) and \( B \subset U \). Since \( A \) and \( B \) are \( \eta \)-closed, \( \eta \text{-cl}(A) \subset U \) and \( \eta \text{-cl}(B) \subset U \). Now, \( \text{cl}(A \cup B) = \text{cl}(A) \cup \text{cl}(B) = \eta \text{-cl}(A) \cup \eta \text{-cl}(B) \subset U \). But \( \eta \text{-cl}(A \cup B) \subset \text{cl}(A \cup B) \). So \( \eta \text{-cl}(A \cup B) \subset U \) and hence \( A \cup B \) is \( \eta \)-closed.

Theorem 4.12: If \( A \) is \( \eta \)-closed and \( B \) is any set \( A \subset B \subset \eta \text{-cl}(A) \), then \( B \) is \( \eta \)-closed.

Proof: Let \( B \subset U \) where \( U \) is regular open. Then \( A \subset B \) implies \( A \subset U \). Since \( A \) is \( \eta \)-closed, \( \eta \text{-cl}(A) \subset U \). \( B \subset \eta \text{-cl}(A) \) implies \( \eta \text{-cl}(B) \subset \eta \text{-cl}(A) \). Thus \( \eta \text{-cl}(B) \subset U \) and shows that \( B \) is \( \eta \)-closed.

Regular Generalized \( \eta \)-open Sets and Regular Generalized \( \eta \)-Neighbourhoods

In this section new class of sets called regular generalized \( \eta \)-open (briefly \( \eta \)-open) sets and regular generalized \( \eta \)-neighbourhoods (briefly \( \eta \)-nhd) in topological spaces are introduced and we study some of their properties.

Definition 5.1: Let \((X, \mathcal{I})\) be a topological space. A subset \( A \) of \( X \) is called regular generalized \( \eta \)-open (briefly \( \eta \)-open) iff its complement is \( \eta \)-closed set. We denote the family of all \( \eta \)-open sets of a topological space by \( \text{rgn-} \text{O}(X) \).

Theorem 5.2: If \( A \) and \( B \) are \( \eta \)-open sets in a space \( X \), then \( A \cup B \) is also a \( \eta \)-open set in \( X \).

Proof: If \( A \) and \( B \) are \( \eta \)-open sets in a space \( X \), then \( A^c \) and \( B^c \) are \( \eta \)-closed sets in \( X \). By Theorem 3.1 \( A^c \cup B^c \) is a \( \eta \)-closed set in \( X \). Therefore \( A \cup B \) is a \( \eta \)-open set in \( X \).

Remark 5.3: The union of two \( \eta \)-open sets in \( X \) is generally not a \( \eta \)-open set in \( X \).

Example 5.4: Let \( X = \{a, b, c\} \) with \( \mathcal{I} = \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}\} \). The set \( A = \{a\} \) and \( B = \{b\} \) are \( \eta \)-open sets but \( A \cup B = \{a, b\} \) is not a \( \eta \)-open set in \( X \).

Remark 5.5: If \( A \) and \( B \) are \( \eta \)-open sets in \( X \), then \( A \cup B \) is not a \( \eta \)-open set in \( X \).

Example 5.6: Let \( X = \{a, b, c\} \) with \( \mathcal{I} = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}, \{b, c\}\} \). The set \( A = \{a, c\} \) and \( B = \{b, c\} \) are \( \eta \)-open sets but \( A \cup B = \{a, b, c\} \) is not a \( \eta \)-open set in \( X \).

Theorem 5.7: If int(\( B \)) \( \subset B \subset A \) and \( A \) is \( \eta \)-open set in \( X \), then \( B \) is \( \eta \)-open in \( X \).

Proof: Suppose that int(\( B \)) \( \subset B \subset A \) and \( A \) is \( \eta \)-open in \( X \). Then \( A^c \subset B^c \subset A^c \). Since \( A^c \) is a \( \eta \)-closed set in \( X \) by Theorem 5.2, \( B \) is a \( \eta \)-open sets in \( X \).

Definition 5.8: Let \( x \) be a point in a topological space \( X \). A subset \( N \) of \( X \) is said to be a \( \eta \)-nhd of \( x \) iff there exists a \( \eta \)-open set \( G \) such that \( x \in G \subset N \).

Definition 5.9: A subset \( N \) of space \( X \) is called a \( \eta \)-nhd of \( A \subset X \) iff there exists a \( \eta \)-open set \( G \) such that \( A \subset G \subset N \).

Theorem 5.10: Every nhd of \( x \in X \) is a \( \eta \)-nhd of \( x \).

Proof: Let \( N \) be a nhd point of \( x \in X \). To prove that \( N \) is a \( \eta \)-nhd of \( x \), by definition of nhd, there exists an open set \( G \) such that \( x \in G \subset N \). Hence \( N \) is a \( \eta \)-nhd of \( x \).

Remark 5.11: In general, a \( \eta \)-nhd of \( x \in X \) need not be a nhd of \( x \in X \) as seen from the following example.

Example 5.12: Let \( X = \{a, b, c\} \) with \( \mathcal{I} = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}, \{b, c\}\} \). Then \( \eta \)-O(X) = \( \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}\} \). The set \( \{a, c\} \) is \( \eta \)-nhd of point \( b \), since the \( \eta \)-open set \( \{b\} \) is such that \( b \in \{b\} \subset \{a, b\} \). However, the set \( \{a, b\} \) is not a nhd of the point \( b \), since no open set \( G \) exists such that \( b \in G \subset \{a, c\} \).

Remark 5.13: The \( \eta \)-nhd \( N \) of \( x \in X \) need not be a \( \eta \)-open in \( X \).

Example 5.14: Let \( X = \{a, b, c\} \) with \( \mathcal{I} = \{\emptyset, X, \{a\}, \{c\}, \{a, c\}, \{b, c\}\} \). Then \( \eta \)-O(X) = \( \{\emptyset, X, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}\} \). The set \( \{c\} \) is a \( \eta \)-open set, but it is a \( \eta \)-nhd of \( \{c\} \). Since \( \{c\} \) is a \( \eta \)-open set such that \( c \in \{c\} \subset \{b, c\} \).

Theorem 5.15: If a subset \( N \) of a space \( X \) is \( \eta \)-open, then \( N \) is \( \eta \)-nhd of each of all its points.
The regular generalized η-closed set is defined and proved that the class forms a topology. The rgη-closed set can be used to derive a new decomposition of unity, closed map and open map, homeomorphism, closure and interior and new separation axioms. This idea can be extended to bitopological and fuzzy topological spaces.

5. Conclusion

The regular generalized η-closed set is defined and proved that the class forms a topology. The rgη-closed set can be used to derive a new decomposition of unity, closed map and open map, homeomorphism, closure and interior and new separation axioms. This idea can be extended to bitopological and fuzzy topological spaces.

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