Analysis of Turbulent Natural Convection with Localized Heating on the Ceiling and on the Floor and Cooling on Opposite Vertical Walls in a Rectangular Enclosure

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Abstract: In this study, turbulent natural convection was investigated in a three-dimensional rectangular enclosure in the form of a room which has convectional heaters positioned at the center of the ceiling and center of the floor and two convective windows on the opposite vertical walls. The study aimed at examining the velocity profile and temperature distribution. Numerical solution aimed at governing the momentum and energy equations are attained using Taylor’s central finite difference estimates with suitable set of boundary conditions. The method chosen above alternative solutions because it converges quickly, is stable, inexpensive, and adaptable when using boundary conditions. The central difference approach and the forward difference method are used to solve the differential equations. To generate results, the resulting equations are solved in MATLAB computer software. The results are graphed at various Reynold numbers, Froude numbers, Euler numbers, and a fixed Prandtl number of 0.71 to demonstrate the velocity profile and temperature distribution in the enclosure. The 2D graphs are used to examine the behavior of fluid flow fields. When Froude number is increased, the velocity decreases with increase in room height, an increase in Euler number leads to an increase in fluid velocity and when the Reynolds number is increased, the temperature decreases. The temperature similarly decreases as the height of the room is increased.

Keywords: Turbulent, natural convection; The flow of heat, a three-dimensional rectangular enclosure

1. Introduction

The experimental and numerical investigation of natural convection in enclosure cavities has been significant. Solar energy systems, air conditioners, heat transfer in buildings, thermal efficiency in microelectronics, and nuclear reactor coolant are just a few of the engineering applications that rely on it. Electronic components must be kept cool in order to function properly.

The flow of heat and temperature is referred to as heat transfer. The transfer of thermal energy from one point to another caused by temperature differences is known as heat flow. Temperature is used to determine the quantity of thermal energy available. Any substance that flows is referred to as a fluid. Convection is the transmission of heat from one point to another through the movement of fluids. In most fluids, convection is the primary mode of heat transmission. Free convection, natural convection, and forced convection are the three types of convection. Natural convection occurs when buoyancy causes a fluid motion. Buoyancy forces are caused by temperature gradients in the fluid region near the heat transfer surface, which cause density changes. The fluid's molecules divide and scatter, making the fluid less dense. Hotter fluids ascend, while cooler fluids get denser and sink. Forced convection occurs when an external source, such as fans or pumps, forces a fluid to flow over the surface, resulting in an artificially generated convection current. The cooling and heating impacts of a fluid alter as a result of temperature differences. Convection currents, for example, are responsible for the boiling of water. The rate of heat transmission increases as the velocity of the moving fluid increases.

The convection of heat is determined by specific heat, density, thermal conductivity, and viscosity. The viscosity of a fluid flow influences its velocity profile. Temperature affects the viscosity of all fluids. Thus, the rate of heat transfer is proportional to the velocity of the fluid flow. The type of convection that dominates is determined by the size of the thermal buoyancy force and the acceleration force. Forceful forced convection may well be omitted if thermal buoyancy is significantly stronger than acceleration, while free convection may well be neglected if acceleration force is so much stronger than thermal buoyancy. Both free and forced convection are dominating when the magnitude is of order one.

The specific objectives of the study are:
- To determine the effect of Froude number on fluid velocity profile
- To determine the effect of Euler number on fluid velocity profile
- To investigate the effects of Reynolds number on fluid temperature distribution.
2. Literature Review

Andima et al (2012) investigated internal heat transfer by natural convection through a cubic enclosure. The heater and the window were situated on opposing sides of the room. The mat lab's findings revealed that areas near the heater had high temperatures while those close to the window had low temperatures.

Ochaga, C.O. et al (2012) studied turbulent convection of heat with localized heating and cooling and on adjacent vertical wall enclosed on a cubic enclosure. The temperature of a convectional heater was regulated by placing it on the bottom surface of the center with two windows on nearby vertical walls. Three-point central and forward difference approximations were used to represent the governing equations with boundary conditions. From the graphs of velocity profile and temperature distribution, it was discovered that the heater records high temperatures, but that these temperatures diminish as the heater moves upwards. This occurred owing to the warm air gaining energy and rising as a result. Temperature was recorded at the window's lowest point, so the fluid drained in the direction of the heater.

Mairura, O.E. et al. (2013) investigated natural convection through heating and cooling two vertical walls inside of an enclosure. In a rectangular enclosure, a turbulent flow was investigated in free convection. The enclosure took the form of a room, with heaters on two opposite vertical walls and the windows positioned on the other two opposite vertical walls. Three-point central and forward difference approximations were used to discretize the governing equations using boundary conditions. The results of the mat lab demonstrated that turbulent natural convection has a considerable impact on temperature distribution in a closed enclosure. The room was partitioned into many regions, with warmer temperatures near the heaters and cooler temperatures near the windows.

Karanja et al. (2017) studied Turbulent Natural Convection in an Enclosure with different Rayleigh Numbers. A Boussinesq buoyancy-driven turbulent airflows of fluid flow distribution was investigated for $10^9 < Ra < 10^{14}$ and $Pr = 0.71$ and the impact of the Rayleigh number was determined. The flow domain consisted of a rectangular enclosure with a constant aspect ratio, heated on one sidewall and cooled on the other. To ensure that the conservation principles are followed at both the local and global levels of the flow domain, the equations were discretized using the robust finite volume approach. The solution was reached using a segregated pressure-based iterative technique because the equations were coupled. The collected results revealed that flow fields were distributed non-uniformly throughout the flow domain, with the Rayleigh number of the flow having a significant impact on the distribution.

Karanja et al (2017) investigated turbulent natural convection in an enclosure with varied aspect ratios. The SST k-w turbulence model was used in conjunction with the Boussinesq approximation to solve the turbulent quantities created by this process. The non-dimensionalized equations are discretized using the robust finite volume approach to verify that the conservation rules are satisfied at the discrete level and over the whole solution domain. The results demonstrated that the velocity and temperature fields in the enclosure are non-uniformly distributed, and their magnitude and distribution are considerably influenced by the enclosure's aspect ratio.

Miroshnichenko, I. et al (2018) investigated turbulent natural convection paired along with surface thermal radiation in a square enclosure with local heater. They considered to use cold vertical walls, horizontal walls that were adiabatic, as well as an isothermal heater just on base. Surface thermal radiation was taken to consider for more accurate analysis of complex heat transfer. The results demonstrated that in the presence of surface radiation, the average Nusselt number increases, and the system cools rapidly. An increase in Rayleigh number resulted in a significant increase in convective flow.

Jephtier, E.N et al (2019), studied how to improve natural convection heat transmission in a square enclosure whereby they did heating from above. The top of the enclosed cavity was heated, while the two opposite vertical walls were cooled. In a fixed place, the heat source was kept constant. According to the findings, the temperature dropped away from the source, the top corner was relatively warmer, and hot fluids migrated away from the source, Cold fluid ascended towards the source and heat transfer was high when the Reynolds number was high than at low Reynold number.

Gikundi, A et al (2020) Performed a numerical study of a turbulent natural convection in a rectangular enclosure created by gradually heating the floor with a convection heater. The enclosure floor was horizontal and air was locally heated from below by a heater placed at the Centre of the floor. The enclosure widths had two cold windows, each on opposite vertical wall near the top of the ceiling. The results showed that the upper region near the roof was cool, the region between the two windows was generally warm, but the region above the heater was hot. In addition, it was noted that high velocities characterizing the natural convections were more pronounced in the central region of the enclosure.

El-kady M.S and Araid F.F (2021) studied natural convection from a single surface heater in a vertical rectangular enclosure. A constant heat flux single heater was placed at one adiabatic insulated vertical wall, the opposite vertical wall was isothermally cooled and the horizontal walls were insulated. The results showed that the local heater minimum temperature and the maximum value of the local Nusselt number occur at the lower edge and the maximum heater surface temperature occurred at the upper edge.

El-Kady, M.S (2021) studied natural convection from dual surface heat sources in a vertical rectangular enclosure. Laminar flow of air in the enclosure was considered with constant heat flux dual heaters were placed on one adiabatic insulated vertical wall. The other vertical wall was isothermally cooled and horizontal walls insulated. The results showed that the average and maximum surface
temperature of upper heater were higher than those of lower heater. The Nusselt number for the upper heater was smaller compared to the lower heater.

Mayoyo, R. et al (2015) studied heat transmission in a cylindrical enclosure. Investigation of the effects of changing Reynolds Number and Prandtl number were done. In addition, they considered Froude number and Euler number on the temperature distribution and velocity profile. A vertical cylindrical container's top surface was chilled, the bottom surface was heated, and the vertical cylinder walls were considered adiabatic. To understand the fluid flow and temperature profiles at many points in the closed cylindrical enclosure, the results were thoroughly examined and displayed in graphs. Results showed the buoyancy forces caused by the temperature differential between the bottom and top of the cylinder were found to be essential in determining the air velocity in the enclosure.

3. Mathematical Formulation

Natural convection in an enclosure caused by heating and cooling can be found in a variety of situations, for instance using convectional heaters in various enclosures. The velocity profile and temperature distribution in a room are mainly affected by the temperature from heat source and window and also the rate of flow of air ventilation. Newtonian motion comprises the flow of heat. In this case, we consider a three-dimensional rectangular inclusion in the form of a room which has a convectional heater positioned at the center of the ceiling and center of the floor and two convectional windows on the opposite vertical wall.

![Figure 1: Model of a room](image)

Fig 1 Shows the position of the convectional heaters and convectional windows in the enclosure. Convectional heaters $H_1$ and $H_2$ are placed on the floor and on the ceiling respectively (X-Z planes) convectional windows $W_1$ and $W_2$, on (Y-Z) planes. The significance of the windows is to cool specific point of the enclosure.

4. Governing Equations

The governing equations are based on universal conservation rules, which include continuity equation, momentum equation, and energy equation. Considering a moving fluid where $\rho$ is a function of the position $x_i$ (i=1,2,3) and $u_j$ (j=1,2,3) signifies velocity components. According to Currie (1974), the equations can be written as.

$$
\frac{\partial \rho}{\partial t} + \frac{\partial }{\partial x_j} \left( \rho u_j \right) = 0 \tag{1}
$$

$$
\frac{\partial \rho u_i}{\partial t} + \frac{\partial }{\partial x_j} \left( \rho u_i u_j \right) = \frac{\partial }{\partial x_j} \left( \mu \frac{\partial u_i}{\partial x_j} \right) + \mu \frac{\partial}{\partial x_j} \frac{\partial u_i}{\partial x_j} \tag{2}
$$

$$
\frac{\partial (\rho c_p T)}{\partial t} + \frac{\partial }{\partial x_j} \left( \rho c_p u_j T \right) = \frac{\partial }{\partial x_j} \left( \lambda \frac{\partial T}{\partial x_j} \right) + \beta T \tag{3}
$$

Whereby (1) is continuity, (2) is momentum, and (3) energy equations in most general form.

$$
\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = - M_1 \frac{\partial \rho}{\partial x_j} - M_2 \Theta g_1 + M_3 \frac{\partial^2 u_j}{\partial x_j^2} \tag{4}
$$

$$
\frac{\partial \Theta}{\partial t} + 2u_j \frac{\partial \Theta}{\partial x_j} = T_2 \frac{\partial^2 \Theta}{\partial x_j^2} \tag{5}
$$

The three-point central difference technique is used to discretize the momentum equation (equation (4)) and the energy equation (equation (5)), as shown below.

5. Method of Solution

Substituting $M_1$, $M_2$ and $M_i$ in equation (4) we get;

$$
\frac{\partial u_i}{\partial t} = \frac{1}{\text{Re} \left( \frac{\partial^2 u_i}{\partial x_j^2} + \frac{\partial^2 u_i}{\partial y^2} \right)} - \nu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_i}{\partial y} \right) - \frac{E_u + g_1}{\text{Pr} \left( Fr \right)} \tag{6}
$$

![Equation](image)

$$
\frac{U_{i,j+1}^{k+1} - U_{i,j}^{k}}{\Delta t} = \frac{1}{\text{Re} \text{Pr} \left( \Delta x \right)^2} \left\{ \left( U_{i+1,j}^{k+1} - 2U_{i,j}^{k} + U_{i-1,j}^{k} \right) \right\} - \frac{1}{\Delta x} \left\{ \left( U_{i+1,j}^{k} - U_{i,j}^{k} \right) + \left( U_{i,j+1}^{k} - U_{i,j-1}^{k} \right) \right\} - \frac{E_u + g_1}{\text{Pr} \left( Fr \right)} \tag{7}
$$

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Equation (5) with substitution of $T_2$ is written as

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr \, Re} \left( \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) - 2v \left( \frac{\partial \theta}{\partial x} + \frac{\partial \theta}{\partial y} \right)$$

Applying Taylor’s central difference approximation. (8)

$$\frac{\theta_{i+1,j}^{k+1} - \theta_{i,j}^{k}}{\Delta t} = \frac{1}{Re \, Pr (\Delta x)^2} \left( \{\theta_{i+1,j}^{k} - 2\theta_{i,j}^{k} + \theta_{i-1,j}^{k}\} + \{\theta_{i,j+1}^{k} - 2\theta_{i,j}^{k} + \theta_{i,j-1}^{k}\} \right) - v \left( \{\theta_{i+1,j}^{k} - \theta_{i,j}^{k}\} + \{\theta_{i,j+1}^{k} - \theta_{i,j-1}^{k}\} \right)$$

Whereby $T_2 = \frac{\lambda_R}{C_{pr} \rho_R U_L L_R}$, $M_1 = \frac{P_R}{\rho_k U_*^2}$,

$$M_2 = \frac{g L_R}{U_*^2}, \quad M_3 = \frac{\mu_R}{\rho_k U_* L_R}, \quad \text{Eu} = \frac{P_R}{\rho_k U_*^2},$$

$$Fr = \frac{U_*}{\sqrt{gL_R}}, \quad P_r = 0.71, \quad Re = \frac{\rho_k U_* L_R}{\mu_R}.$$  

Equations 7 and 9 are combined with boundary conditions in a MATLAB program to simulate the expected results.

6. Results and Discussions

A. Effects of Vertical velocity for varying Froude number near the ground between 0-1M

![Figure 1: Vertical velocity for varying Froude number near the ground between 0-1M](image1)

Fig.1 shows at the base when Y=0 to Y=1, X=0 to X=5 and Z=2, there is much heat from the heater which warms up the fluid and increases the kinetic energy of the particles thus increase in the fluid velocity. As the Froude number decrease, fluid velocity increase.

B. Effects of Vertical velocity for varying Froude number at the middle between 2-3M

![Figure 2: Vertical velocity for varying Froude number at the middle between 2-3M](image2)

Fig.2 shows at the middle, when Z=2, X=0 to X=5 and Y=2 to Y=3, the fluid velocity is lower due to the effect of the cold air flowing into the room from the windows making the fluid to be cold hence reducing the velocity. The kinetic energy of the fluid is lower due to lower temperatures. As Froude number increases, fluid velocity decreases

C. Vertical velocity for varying Froude number at the top between 4-5M

![Figure 3: Vertical velocity for varying Froude number at the top between 4-5M](image3)
Fig.3 shows at the top of the enclosure at Y=4 to Y=5, X=0 to X=5 and Z=2, there is much heat from the heater which warms up the fluid and increases the kinetic energy of the particles thus increase in the fluid velocity with decrease in Froude’s number.

D. Effects of Euler number on Fluid velocity near the ground between 0-1M

From fig. 4, the general observation made is that; as Euler number increases fluid velocity increases, the graphs display same trend for all Euler number and fluid velocity decrease with increase in room height from 0 to 1 meter.

At the base when Y=0 to Y=1, X=0 to X=5 and Z=2, there is much heat from the heater which warms up the fluid and increases the kinetic energy of the particles thus increase in the fluid velocity. As the Euler number increases, fluid velocity increases.

E. Effects of Euler number on Fluid velocity at the middle between 2-3M

From fig. 5, the general observation made is that; as Euler number increases fluid velocity increases, fluid velocity is lower at the middle height of the room and the graphs display same trend for all Euler number.

At the middle, when Z=2, X=0 to X=5 and Y=2 to Y=3, the fluid velocity is lower due to the effect of the cold air flowing into the room from the windows making the fluid to be cold hence reducing the velocity. The kinetic energy of the fluid is lower due to lower temperatures. As Euler number increases, fluid velocity increases.

F. Effects of Euler number on Fluid velocity at the top between 4-5M

From fig. 6, the general observation made is that; as Euler number increases fluid velocity increases, fluid velocity is high at the top of the room and the graphs display same trend for all Euler number.

At the top when Y=4 to Y=5, X=0 to X=5 and Z=2, there is much heat from the heater which warms up the fluid and increases the kinetic energy of the particles thus increase in the fluid velocity. As the Euler number increases the fluid velocity increase.

G. Effects of Reynolds number on Fluid Temperature
General observations made is that Graphs are oscillating and overlapping each other, as Reynolds number increases the temperature of the room increases and temperature decreases with increase in room height.

After the volume of fluid has gone through the boundary layer, the temperature decreases as the distance from the heat source increases, causing a decrease in heat transfer rate and decay, as illustrated in fig 7. The temperature decreases with decrease in Reynolds number. In addition, temperature drops as the height of the room increases. The temperature is lower at the middle of the room because the effect of temperature is reduced by cold air from the windows. This is due to the fact that as we travel away from the heat source, the buoyancy forces that were essential for fluid motion become weaker, and viscous forces take over, slowing fluid motion as seen in fig. 7. The viscous forces tend to hinder fluid motion, and when the temp decreases, the viscosity of the fluid increases.

7. Conclusions

The findings reveal that turbulent natural convection has a significant impact on the velocity profile and temperature distribution. It was noted that, when Froude number is increased, the velocity decreases with increase in room height. An increase in Euler number leads to an increase in fluid velocity. The temperature decreases with decrease in Reynolds number. In addition, temperature drops as the room height increases.

8. Recommendations

1) Further studies may comprise experimental investigations of three-dimensional turbulent problems
2) Investigations of velocity field and temperature distributions in non-rectangular enclosure and at varying positions of a heater and the window.
3) Investigations of the effects of forced convection on the environment.

References


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