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Optimal Portfolio Policy for a Multi - Period Mean Variance Investors

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Abstract: ***We study the optimal portfolio policy for a multi period mean - variance investor facing multiple risky assets subject to proportional transaction costs, market impact or quadratic transaction costs. We demonstrate analytically that, in the presence of proportional transaction costs, the optimal strategy for the multiperiod investor is to trade in the first period to the boundary of a no - trade region shaped as a parallelogram, and not to trade thereafter. For the case with market impact costs, the optimal portfolio policy is to trade to the boundary of a state dependent rebalancing region. In addition, the rebalancing region converges to the Markowitz portfolio as the investment horizon grows large. We contribute to the literature by characterizing the no - trade region for a multiperiod investor facing proportional transaction costs, and studying the analytical properties of the optimal trading strategy for the model with impact costs. Finally, our contribution is to study numerically the utility losses associated with ignoring transaction costs and investing myopically, as well as how these utility losses depend on relevant parameters. We find that the losses associated with either ignoring transaction costs or behaving myopically can be large. Moreover, the losses from ignoring transaction costs increase in the level of transaction costs, and decrease with the investment horizon, whereas the losses from behaving myopically increase with the investment horizon and are concave unimodal on the level of transaction costs. Our work is related to mean - variance utility and proportional transaction costs. For the case with multiple risky assets, the optimal portfolio policy is characterized by a no - trade region shaped as a parallelogram.***

Keywords: Optimal, Portfolio, Risky assets

1. Introduction

In this section, we study numerically the utility loss associated with ignoring transaction costs and investing myopically, as well as how these utility losses depend on the transaction cost parameter, the price - change correlation, the investment horizon, and the risk - aversion parameter. We first consider the case with proportional transaction costs, and then study how the monotonicity properties of the utility losses change when transaction costs are quadratic. We have also considered the case with market impact costs (p=1: 5), but the insights are similar to those from the case with quadratic transaction costs and thus wedo not report the results to conserve space. For each type of transaction cost (proportional or quadratic), we consider three different portfolio policies. First, we consider the target portfolio policy, which consists of trading to the target or Markowitz portfolio in the first period and not trading thereafter. This is the optimal portfolio policy for an investor in the absence of transaction costs. Second, the static portfolio policy, which consists of trading at each period to the solution to the single - period problem subject to transaction costs. This is the optimal portfolio policy for a myopic investor who takes into account transaction costs. Third, we consider the multiperiod portfolio policy, which is the optimal portfolio policy for a multi period investor who takes into account transaction costs. Finally, we evaluate the utility of each of the three portfolio policies using the appropriate multiperiod framework; that is, when considering proportional transaction costs, we evaluate the investor's utility from each portfolio with the objective function and when considering quadratic transaction costs, we evaluate the investor's utility using the objective function.

2. Procedure

2.1 General Framework

Our framework is closely related to that proposed by Garleanu and Pedersen (2012); herein G&P. Like G&P, we consider a multi period setting, where the investor tries to maximize her discounted mean - variance utility net of transaction costs by choosing the number of shares to hold of each of the Nrisky assets. There are three main differences between our model and the model by G&P. First, we consider a more general class of transaction costs that includes not only quadratic transaction costs, but also proportional and market impact costs. Second, we assume price changes in excess of the risk - free rate are independent and identically distributed with mean u and covariance matrix £, while G & P consider the more general case in which these price changes are predictable. Third, we consider both finite and infinite investment horizons, whereas G&P focus on the infinite horizon case.

Our first contribution is to use the multi - period framework proposed by Garleanu and Pedersen to characterize the optimal portfolio policy for general case with multiple risky asset and proportional transaction cost. Specifically we characterize analytically that there exist a no - trade region shaped as a parallelogram, such that if the starting portfolio is inside the no - trade region then it is optimal not to trade at any period. If, on the other hand, the starting portfolio is outside the no - trade region, then it is optimal to trade to boundary of the no - trade region in the first period and not to trade thereafter. Furthermore we show that the size of the no - trade region grows with the level of proportional transaction cost and that discount factor and srinks with the investment horizon and the risk aversion parameter.

Our second contribution is to study analytically the optimal portfolio policy in the presence of market impact posts which arises when the investor makes large trades that distort market prices. Traditionally, researchers have assumed that the market price impact is linear on the amount trade and thus the market impact costs are quadratic. Under this assumption, Garleanu and Pedersen derive closed - form expressions for the optimal portfolio policy within their multiperiod setting. However Torre and Ferrari, Grinold and Kahn show that the square root function is more appropriate for modeling market price impact, thus suggesting market impact costs grow at a rate of slower than quadratic. Our contribution is to extend the analysis by Garleanu and Pedersen to a general case where we are able to capture the distortion on market price through a power function with an exponent between one and two. For this general formulation, we show that there exist an analytical rebalancing region for every time period, such that the optimal policy at each period is to trade to the boundary of corresponding rebalancing region. Moreover and that the rebalancing regions shrink throughout the investment horizon, which means that unlike the proportional transaction cost, it is optimal for the investor to trade at every period when faced market impact costs.

Finally, our third contribution is to study numerically the utility losses associated with ignoring transaction costs and investing myopically as well as how these utility losses depend on relevant parameters. We find that the losses associated with either ignoring transaction costs or behaving myopically can be large. Moreover the losses from ignoring transaction costs increase in level of transaction costs and decrease with the investment horizon, whereas the losses from behaving myopically increase with the investment horizone and are concave unimodal on the label of transaction cost.

2.2 Proportional Transaction Costs:

In this study we consider the case where transaction costs are proportional to the amount traded ($p = 1$). These so – called proportional transaction costs are appropriate to model the cost associated with trades that are small, and thus the transaction cost originates from the bid - ask spread and other brokerage commissions. For exposition purposes, we first study the single - period case, and show that for this case the optimal portfolio policy is analytically characterized by a no - trade region shaped as a parallelogram. We then study the general multiperiod case, and again show that there is a no - trade region shaped as a parallelogram. Moreover, if the starting portfolio is inside the no - trade region, then it is optimal not to trade at any period. If, on the other hand, the starting portfolio is outside the no - trade region, then it is optimal to trade to the boundary of the no - trade region in the first period, and not to trade thereafter. Furthermore, we study how the no - trade region depends on the level of proportional transaction costs, the correlation in asset returns, the discount factor, the investment horizon, and the risk - aversion parameter.

2.3 Quadratic Transaction Costs

In this section we study whether and how the presence of quadratic transaction costs (as opposed to proportional transaction costs) affects the utility losses of the static and target portfolios. The Base Case. We consider the same parameters as in the base case with proportional transaction costs, plus we assume the transaction cost matrix A, and the transaction cost parameter $k= 1.5/10^4$. Similar to the case with proportional transaction costs, we find that the losses associated with either ignoring transaction costs or behaving myopically are substantial. For instance, for the base case with find that the utility loss associated with investing myopically is 30.81%, whereas the utility loss associated with ignoring transaction costs is 116.20%. Moreover, we find that the utility losses associated with the target portfolio are relatively larger, compared to those of the static portfolio, for the case with quadratic transaction costs. The explanation for this is that the target portfolio requires large trades in the first period, which are penalized heavily in the context of quadratic transaction costs. The static portfolio, on the other hand, results in smaller trades over successive periods and this will result in overall smaller quadratic transaction costs.

2.4 Market Impact Costs

In this section we consider market impact costs, which arise when the investor makes large trades that distort market prices. Traditionally, researchers have assumed that the market price impact is linear on the amount traded and thus that market impact costs are quadratic. Under this assumption, Garleanu and Pedersen (2012) derive closed - form expressions for the optimal portfolio policy within their multiperiod setting. However, Torre and Ferrari (1997), Grinold and Kahn (2000), Almgren et al. (2005), and Gatheral (2010) show that the square root function may be more appropriate for modelling market price impact, thus suggesting market impact costs grow at a rate slower than quadratic. Therefore in this section we consider a general case, where the transaction costs are given by the p - norm with p € (1, 2), and where we capture the distortions on market prices through the transaction cost matrix.

3. Result

Finally, we find that the utility loss associated with investing myopically and ignoring transaction costs is monotonically decreasing for values of the risk - aversion level ranging from $5 /10^6$ to $5 / 10^5$. The explanation for this is that when the risk - aversion parameter is large, the mean - variance utility is relatively more important compared to the quadratic transaction costs, and thus the target and static portfolios are more similar to the multi period portfolio. This is in contrast to the case with the risk - aversion parameter. The reason for this difference is that with quadratic transaction costs, the mean - variance utility term and the transaction cost term are both quadratic, and thus the risk - aversion parameter does have an impact on the overall utility loss. Finally, we have repeated our analysis for a case with market impact costs ($p = 1.5$) and we find that the monotonicity properties of the utility losses are roughly in the middle of those for the case with proportional transaction costs ($p = 1$) and quadratic transaction costs (p = 2), and thus we do not report the results to conserve space.

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