A Deterministic Inventory Model with Demand as a Function of Time for Items with a Static Rate of Deterioration

Pooja Soni[1], Rajender Kumar[2]

[1]Research Scholar, Department of Mathematics, Indira Gandhi University, Meerpur (Rewari)-122502 (Haryana) India
pooja.math.rs@igu.ac.in

[2]Assistant Professor, Department of Mathematics, Indira Gandhi University, Meerpur (Rewari)-122502, (Haryana) India

Abstract: ***In this paper, a Deterministic Model is evolving for the objects decline by Demand as well as Deterioration, where we take demand as a function of Time with the Static rate of Deterioration. Here, Shortage is allowing and fully backlogging, Length of every production cycle is constant. Here, Refill magnitude is static and Refill rate is unlimited. Also, lead time is taking as zero. An Organization can use this model where demand varies with Time with a Constant rate of Deterioration.***

Keywords: EOQ Model, time dependent Demand and Deterioration, shortage

1. Introduction

Inventory is the priority to run a business. But it may be a blessing or crush for a Business Owner. It is a blessing because when an owner buys a large no of goods then he gets benefit due to the lower wholesale price, which increases the quality of customer service. But it becomes a curse for an owner due to its large cost of maintenance: like its Deteriorating cost (because many of the physical goods undergo damage or chemically change with time for example product like milk, fruits, bread, butter, etc gets spoil with time), cost arises due to out of fashion of the product, storage and handling cost of goods, ordering cost, etc. So, an owner has to decide two main things; How much to order called EOQ and When to order to control Inventory. So, Inventory Management is very important to use working Capital Effectively. Because an owner's main aim is to Maximize Profit and Minimize Cost. So, in previous years various mathematical models have been created by researchers to minimize cost. Some of them are discussed here.

Datta & Pal (1998) [4], Lee & Wu (2002) [9], Sharma, Sharrma & Ramani (2012) [15] and Sharma and Preeti (2013) [14] considered Power Demand pattern for Deteriorating Items with time varying deterioration in their respective models. Wu (1999) [17], Wu (2002) [18] considered Weibull distributed Deterioration in their respective models. Giri & Chaudhuri (1998) [5] considered demand rate as a function of on hand inventory in their model. Bhowmic & Samant a(2007) [2] considered stock dependent time - varying demand rate, Mishra and singh (2011) [11], Singh & Srivastava (2017) [16] considered Linear Demand, Mishra and Singh (2013)[10] considered time dependent demand and deteriration, Bhowmic & Samanta (2011) [3] considered constant demand rate and variable production cycle, Roy (2008) [13] considered time dependent deterioration rate and assumed Demand rate as a function of the selling price, Karmakar & Choudhury (2014) [6] assumed general ramp type demand rate, Kumar & Kumar (2015) [8] assumed time-dependent demand, Rasel(2017) [12] considered power distribution deterioration, Priya & Senbagam (2018) [7] assumed two parameter Weibull deterioration with quadratic time-dependent demand, Bansal,Kumar et al. (2021) [1] took stock-dependent demand rates, in their respective model.

In this paper, I have developed a model by considering demand as a function of time with a constant rate of deterioration.

2. Assumptions and Notations

2.1 Notations

- $C1$ Inventory carrying charge per object per unit Time.
- $C2$ Cost due to deficiency of one object per unit Time.
- $C3$ Cost of one Decayed Unit.
- T Length of every Production cycle.
- $C(t)$ Average total cost.
- S Inventory at $t = 0$, where t is used for time.
- $I(t)$ Inventory at any time t.
- $D(t)$ Demand rate.
- $\theta(t)$ Deterioration rate function.

Assumptions

1) The Demand rate is assumed as $D(t)$.
2) Deterioraton rate function, $\theta(t) = \theta_0$, where $0 < \theta_0 << 1$.
3) Lead time has been taken as 0.
4) Shortages are allowed and completely reserved.
5) Refill magnitude is static and refill rate is unlimited.
6) During the time period T, there is neither substitution nor repair of decayed objects.

3. Analysis of Model

Let the no of objects in stock at any time t be $I(t)$.In time period $0 < t < t_1$, $I(t)$ lessens gradually due to requirement and decaying of objects and falls to zero at $t = t_1$. In the time period (t_1, T), deficiency of objects occurs which are wholly backlogged, where $t_1 < T$. The esquations of this system are given by:

$$\frac{dI(t)}{dt} + \theta(t)\,I(t) = -D(t) \qquad 0 \le t \le t_1 \qquad (1)$$

$$\frac{dI(t)}{dt} = -D(t) \qquad t_1 \le t \le T \qquad (2)$$

Put $\theta = \theta_0$ then (1) and (2) implies

$$\frac{dI(t)}{dt} + \theta_0\, I(t) = -D(t) \qquad (3)$$

$$\frac{dI(t)}{dt} = -D(t) \qquad (4)$$

Solution of (3),

$$I(t)\, e^{\theta_0 t} = -\int D(t)\, e^{\theta_0 t} dt + C$$
$$= -\int D(t)(1+\theta_0 t)dt + C \qquad (5)$$

Applying t = 0 and I(0) = S in (5), we get

$$C = S + [\int D(t)(1+\theta_0 t)dt_{t=0} \qquad (6)$$

Putting the value of C from (6) into (5), we get

$$I(t)\, e^{\theta_0 t} = -\int_0^t D(t)(1+\theta_0 t)dt + S \qquad (7)$$

Applying $I(t_1) = 0$ in (7) , we get

$$S = \int_0^{t_1} D(t)(1+\theta_0 t)dt \qquad (8)$$

Putting the value of S from (8) into (7) , we get

$$I(t) = (1-\theta_0 t) \int_t^{t_1} D(t)(1+\theta_0 t)dt \qquad (9)$$

Also, solution of (4) ,

$I(t) = -\int D(t)dt + A$

Where A is a constant of integration

Applying $I(t_1) = 0$ in above equation, we get

$$I(t) = \int_t^{t_1} D(u)du \qquad (10)$$

Therefore amount of Deteriorated Items = I(0) - Stock loss due to Demand

$= S - \int_0^{t_1} D(t)dt$

$= \int_0^{t_1} D(t)(1+\theta_0 t)dt - \int_0^{t_1} D(t)dt$

$= \theta_0 \int_0^{t_1} tD(t)dt$

Total amount of Inventory held during $[0,t_1]$ is

$I_1 = \int_0^{t_1} I(u)du$

$= \int_0^{t_1}$

$= \int_0^{t_1}\{\int_u^{t_1} D(t)(1+\theta_0 t)dt\}du - \theta_0 \int_0^{t_1} u\{\int_u^{t_1} D(t)(1+\theta0tdt\}du$

$= \int_{t_1}^{0}\{\int_{t_1}^{u} D(t)(1+\theta_0 t)dt\}du - \theta_0 \int_0^{t_1} u\{\int_u^{t_1} D(t)(1+\theta0tdt\}du$

$$= \int_0^{t_1} tD(t)\left(1+\frac{\theta_0 t}{2}\right)dt \qquad (11)$$

$$\text{No of Shortage units} = -\int_{t_1}^{T} I(t)dt \qquad (12)$$

$= \int_{t_1}^{T}$

$= \int_{t_1}^{T} (T-u)D(u)du$

Inventory holding cost = C_1 * Amount of Inventory held

$$= C_1 * [\int_0^{t_1} tD(t)\left(1+\frac{\theta_0 t}{2}\right)dt] \qquad (13)$$

Shortage cost = C_2 * quantity of shortage units

$= C_2 * \int_{t_1}^{T}(T-u)D(u)du$

Cost due to Deterioration = C_3* quantity of Deteriorated units

$$= C_3 * \theta_0 \int_0^{t_1} tD(t)dt \qquad (14)$$

Entire cost per unit time = Inventory carrying cost + shortage cost + cost due to Decaying objects

$$= C_1 * [\int_0^{t_1} tD(t)\left(1+\frac{\theta_0 t}{2}\right)dt] + C_2 * \int_{t_1}^{T}(T-u)D(u)du + C_3 * \theta_0 \int_0^{t_1} tD(t)dt \qquad (15)$$

Average entire cost per unit time

$C(t_1) = \frac{1}{T}$[Entire cost per unit time]

$= \frac{1}{T}[C_1 * [\int_0^{t_1} tD(t)\left(1+\frac{\theta_0 t}{2}\right)dt] + C_2 * \int_{t_1}^{T}(T-u)D(u)du + C_3 * \theta_0 \int_0^{t_1} tD(t)dt]$

For minimum average total cost put $\frac{dC(t_1)}{dt_1} = 0$

We get,

$D(t)\left[\frac{C_1\theta_0}{2T} t_1^2 + \left(\frac{C_1}{T} + \frac{C_3\theta_0}{T} + \frac{C_2}{T}\right) t_1 - C_2\right] = 0$

Which is quadratic in t_1 with last term negative so, it has atleast one positive root say $t_1^*$and $\frac{d^2C(t_1^*)}{dt_1^{*2}} > 0$. So optimum value of t_1 is t_1^*. Hence the optimum value of S is

$$S^* = \int_0^{t_1^*} D(t)(1+\theta_0 t)dt \qquad (16)$$

Minimum value of $C(t_1)$ is

$$C(t_1^*) = \frac{1}{T}[C_1 * [\int_0^{t_1^*} tD(t)\left(1+\frac{\theta_0 t}{2}\right)dt] + C_2 * \int_{t_1^*}^{T}(T-u)D(u)du + C_3 * \theta_0 \int_0^{t_1^*} tD(t)dt] \qquad (17)$$

Thus (17) gives optimal value of total average cost per unit time.

4. Conclusion

Here, an Inventory model has been created for items depleted due to demand as well as Deterioration by taking demand as a function of time and constant deterioration rate and I have obtained minimum total average cost. This model can be extended further for various value of demand.

References

[1] Bansal,K.K;Kumar,Vijesh et al.,*"Study of Deterministic Inventory Model for Deteriorating Items With Stock-Dependent Demand Rate under Inflation"*,European Journal of Molecular and Clinical Medicine,Vol.08,Issue 03,2021.

[2] Bhowmick Jhuma and Samanta,G.P.,*"A Continuous Deterministic Inventory System for Deteriorating Items with Inventory-Level-Dependent Time Varying*

Demand Rate",Tamsui Oxford Journal of Mathematical Sciences 23(2)(2007) 173-184.

[3] Bhowmick Jhuma and Samanta,G.P.,*"A Deterministic Inventory Model of Deteriorating Items with two Rate of Production,Shortages and Variable Production Cycle"*,International Scholarly Research Network,Vol.2011,Article ID 657464,16 pages.

[4] Datta,T.K. and Pal,A.K.,*"Order level Inventory system with power demand pattern for items with variable rate of deterioration"*,Indian J.Pure Appl.Math.,Vol.19,No 11,pp.1043-1053, November 1998.

[5] Giri, B.C. and Chaudhuri,K.S.,*"Deterministic Models of Perishable Inventory with Stock-Dependent Demand Rate and nonlinear Holding Cost"*,European Journal of Operational Research 105(1998)467- 474.

[6] KARMAKAR Biplab and CHOUDHURY,K.D.,*"Inventory Models with Ramp-Type Demand for Deterioration Items with Partial Backlogging and Time-Varying Holding Cost"*,Yugoslav Journal of Operations Research,24(2014)No.2,249-266.

[7] KavithaPriya, R. and Senbagam, K.,*"An EOQ Inventory Model for Two Parameter Weibull Deterioration with Quadratic Time Dependent Demand and Shortages"*,International Journal of Pure and Applied Mathematics,vol.119,No.7,467-478,2018.

[8] Kumar Sushil and Kumar Ravendra,*"A Deterministic Inventory Model for Perishable Items with Time Dependent Demand and Shortages"*, International Journal of Mathematics and its Applications,Vol.3,Issue 4-F(2015),105-111.

[9] Lee,W.C. and Wu,J.W.,*" An EOQ Model for items with Weibull distributed deterioration,shortages and power demand pattern"*, Information and Management Sciences,Vol.13,No.2,19-34,2002.

[10] Mishra,V.K.;Singh, L.S. et al.*"An Inventory Model for Deteriorating Items with Time Dependent Demand and Time Varying Holding cost under Partial Backlogging"*.Journal of Industrial Engineering International 2013,9:4.

[11] Mishra,V.K.;Singh, L.S.,*"Deteriorating Inventory Model for Time Dependent Demand and holding cost with partial backlogging"*,International Journal of Management Science and Engineering Management,6(4):267-271,2011.

[12] RASEL,S.K.,*"A Deterministic Inventory System with power Distribution Deterioration and fully backlogged shortage"*,Asian Journal of Current Research,2(3):81-88,2017.

[13] Roy Ajanta,*"An Inventory Model for Deteriorating Items with price Dependent Demand and Time-varying holding cost"*,AMO-Advanced Modeling and Optimization,Vol.10,No.1,2008.

[14] *"Sharma,A.K. and Preeti,An Inventory model for deteriorating items with power pattern demand and partial backlogging with time-dependent holding cost"*,IJLTEMAS,Vol.2(3),pp.92-104,2013.

[15] *"Sharma,A.K.,Sharma,M.K. and Ramani,N.,An Inventory model with Weibull distribution deteriorating item with power pattern demand with shortages and time dependent holding cost"*,American Journal of Applied Mathematical Sciences,Vol.1,No.1-2,pp.17-22,2012.

[16] Srivastava Saurabh and Singh Harendra.*"Deterministic Inventory Model for Items with Linear Demand,variable Deterioration and partial backlogging"*.Int.J.Inventory Research,Vol.4,No.4, 2017.

[17] Wu Jong-Wuu ,Lin Chinho et al.,*"An EOQ Inventory Model with Ramp Type Demand Rate for Items with Weibull Deterioration"*,Information and Management Sciences , vol.10,No.3,pp.41-51,1999.

[18] WU ,Kun-Shan,*"Deterministic Inventory Model for Items with Time Varying Demand,Weibull Distribution Deterioration and Shortages"*,Yugoslav Journal of Operations Research,12(2002),No.1,61-71.