

Thermal Instability of Visco-Elastic Rivlin-Ericksen Nanofluid Layer under the Effect of Magnetic Field

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Abstract: *The numerical and graphical study on magnetic field impression on thermal instability of visco-elastic Rivlin-Ericksen nanofluid layer has been carried out in this paper. The influence of physical parameters as Lewis number, modified diffusivity ratio number, nano particle Rayleigh number, and magnetic field studied analytically on the stationary deformation. A number of theorems established, which satisfying the conditions for stability or instability. The Lewis number, modified diffusivity ratio number, nano particle Rayleigh number are found to have destabilizing influence and magnetic field has stabilizing influence on stationary convection of fluid layer.*

Keywords: Rivlin-Ericksen nanofluid, Lewis number, magnetic field, thermal impermanence, nano particle Rayleigh number

1. Introduction

Nanofluid paid an important role in the various field such as food processing, geophysics, oil reservoir, automotive etc. Nanofluid is formed by adding nano particles in the base fluid. Choi [1] first time discussed the term nanofluid. Nanofluid has the property that it increases the thermal conductivity of fluids due to the presence of nano particles. Nano particles such as Al_2O_3 , CuO , Nitride ceramics are mixed in base fluids to form of nanofluids. Chandrasekhar [2] discussed in detail the thermal impermanence of a Newtonian fluid under the assumption of hydro-dynamic and hydro-magnetic. Tzou [3] and Kuznetsov and Nield [4] investigated the thermal impermanence of nanofluids with the help of conservation equation. They observed the conditions for oscillatory deformation and found an expression for thermal Rayleigh number. The impression of rotation on the impermanence of nanofluids have discussed by Bahaduria and Agarwal [5]. Buongiorno [6] explained that the sum of velocity of the base fluid and relative velocity (slip velocity) can be treated as absolute velocity of nano particles. He also explained the impression of seven slip mechanisms: Magnus effect, Inertia, Fluid drainage, Brownian diffusion, diffusiophoresis, Thermophoresis and gravity settings. The above explanations related with the study of nanofluids as Newtonian fluids. But it is realized by the researchers that non-Newtonian fluid having a great importance in technologies and industries, the discussion of such type of fluids are desirable. Sheu [7], Chand and Rana [8] and Rana et al [9] have studied about the Bénard convection problems of non-Newtonian fluids. There are many visco-elastic fluids which do not obey the Maxwell's constitutive relations. One such type of visco-elastic fluid is Rivlin-Ericksen fluid nanofluid. Rivlin-Ericksen [10] discussed the stress deformation relaxations for isotropic materials. Prakash and Chand [11], Sharma and Rana [12] have discussed the thermal impermanence problems in the Rivlin-Ericksen visco-elastic fluids under the assumption of hydrodynamic and hydromagnetic. The theory of magneto-hydrodynamics has several applications in the field of Geophysics, atmospheric science, plasma physics etc. Kapil and Kumar [13] have discussed about the hydro-magnetic instability of visco-elastic Walter's (Modal B') nanofluid

layer heated from below and found that Magnetic field has stabilize impression on the thermal deformation of nanofluid layer. Gupta et al [14] investigated the nanofluid convection under vertical magnetic field and analyzed that magnetic field has stabilizing impression on the stationary deformation of nanofluid layer. The impression of magnetic field on binary nanofluid convection was discussed by Gupta et al [15].

The present paper is devoted to the consideration of magnetic field impression on thermal impermanence of visco-elastic Rivlin-Ericksen nanofluid layer.

2. Mathematical Observation

Suppose an infinite layer of thickness d^* of Rivlin-Ericksen visco-elastic nanofluid is bounded by $z = 0$ and $z = d^*$ and heated from below. The gravitational force $(0, 0, -g)$ is working on the layer due to which the layer is acting in upward direction. Suppose T_0, φ_0 and T_1, φ_1 are temperatures and volumetric fractions at $z = 0$ and $z = d^*$ respectively. Since the layer is heated from below that means $T_0 > T_1$. The fluid layer is acting under vertical magnetic field $(0, 0, H_0)$.

The governing equations for visco-elastic Rivlin-Ericksen nanofluid are

$$\nabla q_d = 0 \quad (1)$$

$$\rho \frac{dq_d}{dt} = -\nabla p + \rho g + \left(\delta + \delta' \frac{d}{dt} \right) \nabla^2 q_d + \frac{\mu_m}{4\pi} (\mathbf{H} \cdot \nabla) \mathbf{H} \quad (2)$$

where $\frac{d}{dt} = \frac{d}{dt} + (\mathbf{q}_d \cdot \nabla)$ stands for deformation derivative, $\mathbf{q}_d(u, v, w)$ is the velocity vector, p is the hydrostatic pressure, δ and δ' are the viscosity and kinematic visco-elasticity respectively and $g(0, 0, -g)$ is acceleration due to gravity, μ_m is the fluid magnetic permeability and \mathbf{H} is the magnetic field. The density ρ of nanofluid can be written as

$$\rho = \varphi \rho_p + (1 - \varphi) \rho_f \quad (3)$$

where φ denotes volume fraction of nano particles, ρ_p and ρ_f are the densities of nano particles and base fluid. The

equation of motion for Rivlin-Ericksen visco-elastic nanofluid is given as:

$$\rho \frac{dq_d}{dt} = -\nabla p + (\varphi \rho_p + (1 - \varphi)\{\rho(1 - \alpha(T - T_0))\})g + (\delta + \delta' \frac{d}{dt}) \nabla^2 \mathbf{q}_d + \frac{\mu_m}{4\pi} (\mathbf{H} \cdot \nabla) \mathbf{H} \quad (4)$$

where α is the coefficient of thermal expansion and μ_m is the fluid magnetic permeability.

The continuity equation for the nano particles is

$$\frac{\partial \varphi}{\partial t} + \mathbf{q}_d \cdot \nabla \varphi = D_B \nabla^2 \varphi + \frac{D_T}{T^*} \nabla^2 T \quad (5)$$

where D_B is the Brownian coefficient diffusion and D_T is the Thermoporetic coefficient diffusion of the nano particles.

The energy equation in nanofluid is

$$\rho_c \left[\frac{\partial T}{\partial t} + \mathbf{q}_d \cdot \nabla T \right] = k \nabla^2 T + (\rho_c)_p \left(D_B \nabla \varphi \cdot \nabla T + \partial T \partial t_{\nabla T \nabla T} \right) \quad (6)$$

Where ρ_c is the heat capacity of fluid, $(\rho_c)_p$ is the heat capacity of nano particles and k is the thermal conductivity.

The Maxwell equation being

$$\frac{\partial \mathbf{H}}{\partial t} + (\mathbf{q}_d \cdot \nabla) \mathbf{H} = (\mathbf{H} \cdot \nabla) \mathbf{q}_d + \eta \nabla^2 \mathbf{H} \quad (7)$$

$$\nabla \cdot \mathbf{H} = 0 \quad (8)$$

Where η is the electrical resistivity of fluid. On taking non-dimensional variables as:

$$(x', y', z') = \left(\frac{x, y, z}{d^*} \right), \mathbf{q}_d' (u', v', w') = \mathbf{q}_d \left(\frac{u, v, w}{K} \right) d^*, t' = \frac{tk}{\sigma d^{*2}}, p' = \frac{p}{\sigma k^2} d^{*2}, \varphi' = \frac{\varphi - \varphi_0}{\varphi_1 - \varphi_0}, T' = \frac{T - T_0}{T_0 - T^*}$$

where $\frac{k}{\rho_c} = k$ is the thermal diffusivity of the fluid. Equations (1), (4), (5), (6), (7) and (8), in non dimensional form can be taken as:

$$\nabla \mathbf{q}_d = 0 \quad (9)$$

$$\frac{1}{pr_1} \frac{dq_d}{dt} = -\nabla p + (1 + nF) \nabla^2 \mathbf{q}_d - R_m \hat{e}_z + R_a T \hat{e}_z - R_n \varphi \hat{e}_z + Q \frac{pr_1}{pr_2} (\mathbf{H} \cdot \nabla) \mathbf{H} \quad (10)$$

$$\frac{\partial \varphi}{\partial t} + \mathbf{q}_d \cdot \nabla \varphi = \frac{1}{L_e} \nabla^2 \varphi + \frac{N_A}{L_e} \nabla^2 T \quad (11)$$

$$\frac{\partial T}{\partial t} + \mathbf{q}_d \cdot \nabla T = \nabla^2 T + \frac{N_B}{L_e} \nabla^2 \varphi \cdot \nabla T + \frac{N_A N_B}{L_e} \nabla T \cdot \nabla T \quad (12)$$

$$\frac{\partial \mathbf{H}}{\partial t} + (\mathbf{q}_d \cdot \nabla) \mathbf{H} = (\mathbf{H} \cdot \nabla) \mathbf{q}_d + \frac{pr_1}{pr_2} \nabla^2 \mathbf{H} \quad (13)$$

$$\nabla \cdot \mathbf{H} = 0 \quad (14)$$

[The dashes (') have not been consider for simplicity] The non-dimensional parameters are:

Lewis number $L_e = \frac{K}{D_B}$, Prandtl number $pr_1 = \frac{\delta}{\rho K}$, Magnetic Prandtl number $pr_2 = \frac{\delta}{\rho \eta}$, Kinematic viscoelasticity parameter $F = \frac{\delta' k}{\delta \sigma d^{*2}}$, Thermal Rayleigh number $R_a =$

$\frac{\rho g \alpha d^{*3}}{\delta K} (T_0 - T^*)$, Density Rayleigh number $R_m = \frac{[\rho_p \varphi_0 + \rho (1 - \varphi_0) g d^{*3}]}{\delta K}$ Concentration Rayleigh number $R_n = \frac{(\rho_p - \rho) (\varphi_1 - \varphi_0) g d^{*3}}{\delta K}$

Modified particle density increment $N_B = \frac{(\rho_p - \rho) (\varphi_1 - \varphi_0)}{(\rho_c)}$, Chandrasekhar number $Q = \frac{\mu_m H_0^2 d^{*2}}{4\pi \nu \rho \eta}$

We assume that temperature and volumetric fraction of nano particles are constant on boundaries. Thus the dimensionless boundaries conditions are

$$w = 0, T = 1, \varphi = 0 \text{ at } z = 0$$

$$w = 0, T = 0, \varphi = 0 \text{ at } z = 1$$

3. Basic States and it's solution

It is supposed that the basic state of nanofluid is free of time and is described by $q_d'(u, v, w) = 0, p' = p(z), T' = T_p(z), \varphi' = \varphi_p(z), \mathbf{H}' = (0, 0, 1)$. The subscript 'p' denote the primary variable. Equations (9) to (12) using boundary conditions (15) and (16) give solution as:

$$T_p = 1 - z \text{ and } \varphi_p = z \quad (17)$$

4. Perturbation Solution

The stability of the system can be studied by introducing small perturbations to primary flow, and written as

$$q_d'(u, v, w) = 0 + q_d''(u, v, w), T' = T_p + T'', \varphi' = \varphi_p + \varphi'', p' = p_p + p'', \text{ with } T_p = 1 - z \text{ and } \varphi_p = z \quad (18)$$

Using equation (18) in equation (9) to (14) and the product of the prime quantities can be neglect for linearizing the system we obtain the following equations:

$$\nabla \mathbf{q}_d = 0 \quad (19)$$

$$\frac{1}{pr_1} \frac{\partial q_d}{\partial t} = -\nabla p + (1 + nF) \nabla^2 \mathbf{q}_d + R_a T \hat{e}_z - R_n \varphi \hat{e}_z + Q \frac{pr_1}{pr_2} \frac{\partial \mathbf{H}}{\partial z} \quad (20)$$

$$\frac{\partial \varphi}{\partial t} + w = \frac{1}{L_e} \nabla^2 \varphi + \frac{N_A}{L_e} \nabla^2 T \quad (21)$$

$$\frac{\partial T}{\partial t} - w = \nabla^2 T + \frac{N_B}{L_e} \left(\frac{\partial T}{\partial z} - \frac{\partial \varphi}{\partial z} \right) - \frac{2N_A N_B}{L_e} \frac{\partial T}{\partial z} \quad (22)$$

$$\frac{\partial \mathbf{H}}{\partial t} = \frac{\partial w}{\partial z} \hat{e}_z + \frac{pr_1}{pr_2} \nabla^2 \mathbf{H} \quad (23)$$

$$\nabla \cdot \mathbf{H} = 0 \quad (24)$$

The dashes (') have not been considered for simplicity. It is noted that R_m is not involved in the equation (20) to (24). Since it is a measure of basic static pressure gradient. The unknowns u, v, w, p, T, φ can be reduced by operating equation (20) $\hat{e}_z \cdot \text{curl} \cdot \text{curl}$, we get

$$\frac{1}{pr_1} \frac{\partial}{\partial t} \nabla^2 w - (1 + nF) \nabla^4 w = R_a \nabla^2_H T - R_n \nabla^2_H \varphi - Q \frac{\partial^2 w}{\partial z^2} \quad (25)$$

where $\nabla_H^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is the Laplacian operator on horizontal plane.

5. Normal Mode Observation

On analyzing the disturbances in to normal modes and supposed that the perturbed quantities are of the form:

$$[w, T, \varphi] = [(z), (z), \varphi(z)] e^{(ik_x x + ik_y y + nt)} \quad (26)$$

Where k_x and k_y represent wave numbers in x and y directions respectively, while n is growth rate of disturbances. Using eq. (26), eq. (21), (22), and (25) become

$$[(D^2 - a^2) \frac{n}{pr_1} - (1 + nF)(D^2 - a^2)^2 + QD^2] w + a^2 R_a T - a^2 R_n \varphi = 0 \quad (27)$$

$$w - \frac{N_A}{L_e} (D^2 - a^2) T - [\frac{1}{L_e} (D^2 - a^2) - n] \varphi = 0 \quad (28)$$

$$w + [(D^2 - a^2) - n + \frac{N_B D}{L_e} - \frac{2N_A N_B}{L_e} D] T - \frac{N_B}{L_e} D \varphi = 0 \quad (29)$$

Where $D = \frac{d}{dz}$ and $a = \sqrt{k_x^2 + k_y^2}$ is the resultant wave number. The boundary conditions are:

$$\begin{aligned} w = 0, D^2 w = 0, T = 0, \varphi = 0 \text{ at } z = 0 \\ w = 0, D^2 w = 0, T = 0, \varphi = 0 \text{ at } z = 0 \end{aligned} \quad (30)$$

6. Linear Stability Observation

Consider the solution in the form w, T, φ is given as:

$$w = w_0 \sin \pi z, T = T_0 \sin \pi z, \varphi = \varphi_0 \sin \pi z \quad (31)$$

$$\left[\left\{ \frac{n}{pr_1} + (1 + nF) \right\} J + Q(J - a^2) \right] w_0 - a^2 R_a T_0 + a^2 R_n \varphi_0 = 0 \quad (32)$$

$$w_0 + \frac{N_A}{L_e} J T_0 + \left(\frac{1}{L_e} J + n \right) \varphi_0 = 0 \quad (33)$$

$$w_0 - (J + n) T_0 = 0 \quad (34)$$

From equation (33) & (34), we get

$$\left[(J + n) + \frac{N_A J}{L_e} \right] T_0 + \left(\frac{1}{L_e} J + n \right) \varphi_0 = 0 \quad (35)$$

From equation (31), (34) & (35), we get

$$R_a = \frac{1}{a^2} \left[\left\{ \frac{n}{pr_1} + (1 + nF) \right\} J + Q(J - a^2) \right] (J + n) - \left\{ \frac{(J+n) + \frac{N_A J}{L_e}}{\left(\frac{1}{L_e} J + n \right)} \right\} R_n \quad (36)$$

For neutral stability, the real part of n is zero. Hence, on putting $n = i$, (ω is the real and frequency of oscillation) in eq.(36), we get:

$$R_a = \Delta_1 + i \omega \Delta_2$$

where

$$\Delta_1 = \frac{1}{a^2} \left[J^3 + Q(J - a^2)J - \frac{\omega^2}{pr_1} J - \omega^2 F J^2 \right] - \frac{1}{\left(\frac{J^2}{L_e^2} + \omega^2 \right)}$$

$$\left[\frac{J^2}{L_e} \left(1 + \frac{N_A}{L_e} \right) + \omega^2 \right] R_n \quad (38)$$

Δ_1 is the real part. The imaginary part Δ_2 is given as

$$\Delta_2 = \frac{1}{a^2} \left[\left\{ 1 + JF + \frac{1}{pr_1} \right\} J^2 + Q(J - a^2) \right] - \frac{1}{\left(\frac{J^2}{L_e^2} + \omega^2 \right)}$$

$$\left[\frac{J}{L_e} - J \left(1 + \frac{N_A}{L_e} \right) \right] R_n \quad (39)$$

Now for exchange of stability $\omega = 0$ and Δ_2 must be zero for over stability.

7. Stationary Deportation

When the stability occurs in as stationary deportation, the marginal state will be characterized by $\omega = 0$ ($n = 0$). Then equation (37) reduced as

$$(R_a)_s = \left(\frac{\pi^2 + a^2}{a^2} \right) [(\pi^2 + a^2)^2 + Q\pi^2] - (L_e + N_A) R_n \quad (40)$$

It is observed that R_a is independent of both Prandtl number and the parameters containing the Brownian effects and the thermophoresis effects. Visco-elastic parameter F is not present in eq.(40) it means in stationary deportation Rivlin-Ericksen visco-elastic fluid behaves like an ordinary Newtonian fluid. Take $x = \frac{a^2}{\pi^2}$

$$(R_a)_s = \frac{(1+x)\pi^2}{x} [\pi^2(1+x)^2 + Q] - (L_e + N_A) R_n \quad (41)$$

To study the effect of Lewis number modified diffusivity ratio and nano particle Rayleigh number and magnetic field on stationary deportation. We examine the nature of

Observation	Stabilize Effect on Fluid Layer	Destabilize Effect on Fluid Layer
$\frac{\partial(R_a)_s}{\partial N_a} < 0$	No	Yes
$\frac{\partial(R_a)_s}{\partial L_e} < 0$	No	Yes
$\frac{\partial(R_a)_s}{\partial R_n} < 0$	No	Yes
$\frac{\partial(R_a)_s}{\partial Q} > 0$	Yes	No

8. Some important theorems

From equation (36), we get a cubic equation in n as

$$\begin{aligned} n^3 \left[J \left(\frac{1}{pr_1} + F \right) \right] + n^2 \left[J^2 \left(1 + \frac{1}{L_e} \right) \left(\frac{1}{pr_1} + F \right) + \right. \\ \left. QJ - a^2 J + n J 3 L_e 1 Pr 1 + F + 1 + 1 L_e J^2 + Q J^2 - Q J a^2 - \right. \\ \left. a^2 R_a - R_n + J 2 L_e J + Q J L_e - Q a^2 L_e - J 1 + \right. \\ \left. N A L_e R_n + a^2 R_a L_e \right] = 0 \end{aligned} \quad (37)$$

Theorem 1- The stability exists in the system if $\frac{J^3}{L_e} \left[\left\{ \frac{1}{Pr_1} + F \right\} + \left(1 + \frac{1}{L_e} \right) (J^2 + QJ^2 - QJa^2) \right] > (a^2 R_a + R_n)$ and $\left(\frac{J}{L_e} \right) \left(J + \frac{QJ}{L_e} - \frac{Qa^2}{L_e} \right) > \left\{ \left(1 + \frac{N_A}{L_e} \right) \right\} R_n + \frac{a^2 R_a}{L_e}$

Proof- When the above conditions satisfy in equation (42), then there is no any change in the sign that means there does not exist any positive root. Hence stability exists in the system under these conditions.

Theorem-2- The system undergoes destabilize if $\frac{J^3}{L_e} \left[\left\{ \frac{1}{Pr_1} + F \right\} + \left(1 + \frac{1}{L_e} \right) (J^2 + QJ^2 - QJa^2) \right] < (a^2 R_a + R_n)$ and $\left(\frac{J}{L_e} \right) \left(J + \frac{QJ}{L_e} - \frac{Qa^2}{L_e} \right) > \left\{ \left(1 + \frac{N_A}{L_e} \right) \right\} R_n + \frac{a^2 R_a}{L_e}$

Proof- When the above conditions satisfy in equation (42), Then the coefficient of n in equation (40) is negative, That means one change of sign and so has at most one positive root. The occurrence of a positive root implies that the system is unstable.

Theorem-3- The system is unstable under the conditions $\frac{J^3}{L_e} \left[\left\{ \frac{1}{Pr_1} + F \right\} + \left(1 + \frac{1}{L_e} \right) (J^2 + QJ^2 - QJa^2) \right] < (a^2 R_a + R_n)$ and $\left(\frac{J}{L_e} \right) \left(J + \frac{QJ}{L_e} - \frac{Qa^2}{L_e} \right) < \left\{ \left(1 + \frac{N_A}{L_e} \right) \right\} R_n + \frac{a^2 R_a}{L_e}$

Proof- On applying these conditions in equation (42), the coefficient of n and constant term are negative. That means the product of roots is also positive. Which sure that one positive root exist. Hence the system is unstable under the above conditions.

Theorem 4- The sufficient condition for non-existence oscillatory deportation is $\frac{J^2}{Le^2} \left[\left\{ 1 + JF + \frac{1}{Pr_1} \right\} J^2 + Q(J - a^2) \right] > a^2 \left[\frac{J}{Le} - \frac{J}{1 + NALeR_n} \right]$

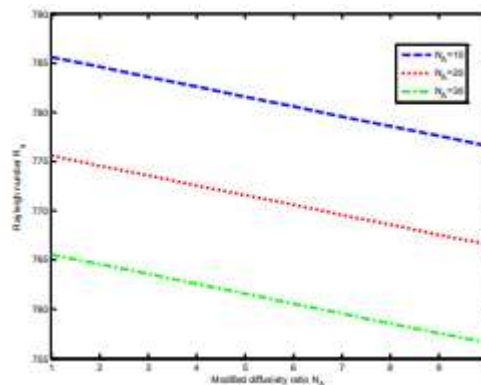
Proof: For Oscillatory deportation we have $\Delta_2 = 0$, from eq.(39), which give $0 = \frac{1}{a^2} \left[\left\{ 1 + JF + \frac{1}{Pr_1} \right\} J^2 + Q(J - a^2) \right] - \frac{1}{Le^2 + \omega^2} \left[\frac{J}{Le} - \frac{J}{1 + NALeR_n} \right]$

$$\omega^2 = \frac{a^2 \left[\frac{J}{Le} - \frac{J(1 + \frac{N_A}{Le})}{1 + NALeR_n} \right] R_n}{\left[\left\{ 1 + JF + \frac{1}{Pr_1} \right\} J^2 + Q(J - a^2) \right]} - \frac{J^2}{Le^2}$$

For $\frac{J^2}{Le^2} \left[\left\{ 1 + JF + \frac{1}{Pr_1} \right\} J^2 + Q(J - a^2) \right] > a^2 \left[\frac{J}{Le} - \frac{J}{1 + NALeR_n} \right]$, ω^2 is negative, whenever for existence of oscillatory convection ω^2 must be positive. Then the above condition is sufficient for non-existence of oscillatory convection.

9. Graphical Observations

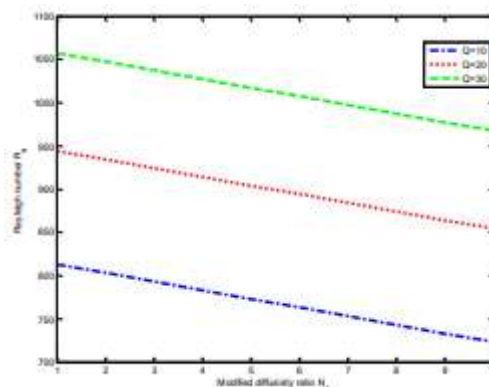
9.1 Change in (R_a) with respect to N_A



Different values	Fix values
$N_A = 10, 20, 30$	$Q = 5$
$L_e = 10, 20, 30$	$R_n = 1$

This shows that modified diffusivity ratio number has destabilizing impression on the stationary deportation.

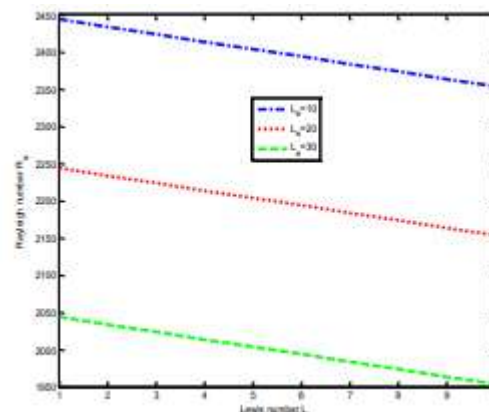
6.2 Change in (R_a) with respect to N_A



Different values	Fix values
$Q = 10, 20, 30$	$N_A = 5$
$L_e = 10, 20, 30$	$R_n = 1$

This shows that modified diffusivity ratio number has destabilizing impression on the stationary deportation

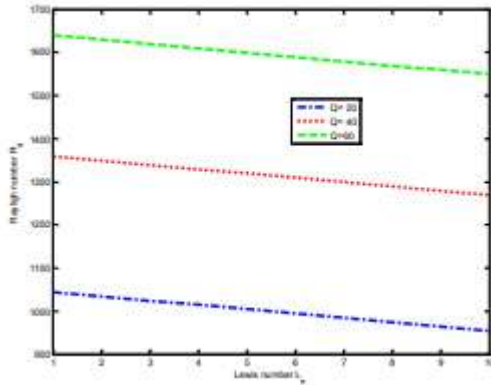
6.3 Change in (R_a) with respect to L_e



Different values	Fix values
$N_A = 20, 40, 60$	$Q = 100$
$L_e = 10, 20, 30$	$R_n = 10$

This shows that Lewis number has destabilizing impression on the stationary deportation.

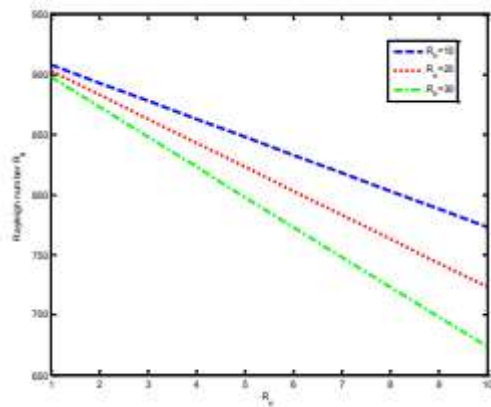
6.4 Change in (R_a) with respect to L_e



Different values	Fix values
$Q = 20, 40, 60$	$L_e = 10$
$N_A = 10, 20, 30$	$R_n = 10$

This shows that Lewis number has destabilizing impression on the stationary deportation.

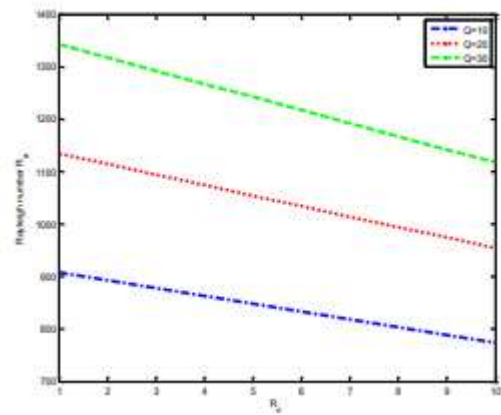
6.5 Change in (R_a) with respect to R_n



Different values	Fix values
$N_A = 5, 10, 15$	$Q = 10$
$R_n = 10, 20, 30$	$L_e = 10$

This shows that nano particle Rayleigh number has destabilizing impression on the stationary deportation.

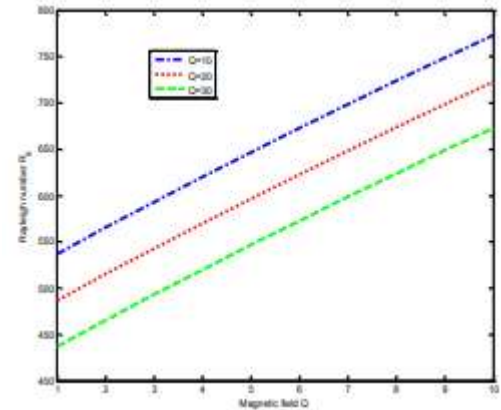
6.6 Change in (R_a) with respect to R_n



Different values	Fix values
$N_A = 5, 10, 15$	$R_n = 10$
$Q = 10, 20, 30$	$L_e = 10$

This shows that nano particle Rayleigh number has destabilizing impression on the stationary deportation.

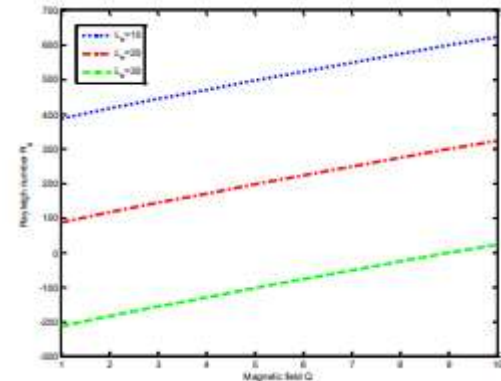
6.7 Change in (R_a) with respect to Q



Different values	Fix values
$N_A = 5, 10, 15$	$R_n = 10$
$Q = 10, 20, 30$	$L_e = 10$

This shows that Magnetic field has stabilizing impression on the stationary deportation.

6.8 Change in (R_a) with respect to Q



Different values	Fix values
$N_A = 20, 40, 60$	$Q = 100$
$L_e = 10, 20, 30$	$R_n = 10$

This shows that Magnetic field has stabilizing impression on the stationary deformation.

7. Conclusions

The magnetic field impression on the thermal impermanence of visco-elastic Rivlin-Ericksen nanofluid layer has been studied in this paper. The main conclusions from the observation are as follows:

(1) The magnetic field shows stability impression on the stationary deformation of fluid layer.

(2) The modified diffusivity ratio number, Lewis number, nano particle Rayleigh number having destabilized impression on the stationary deformation of fluid layer.

(3) The stability exists in the system if
$$\left[\frac{J^3}{Le} \left(\frac{1}{Pr_1} + F \right) + \left(1 + \frac{1}{Le} \right) (J^2 + QJ^2 - QJa^2) \right] > (a^2 Ra + R_n) \text{ and } \left(\frac{J}{Le} \right) \left(J + \frac{QJ}{Le} - \frac{Qa^2}{Le} \right) > \left\{ \left(1 + \frac{NALe R_n + a^2 Ra Le}{NALe R_n + a^2 Ra Le} \right) \right\}$$

(4) The system is unstable under the conditions
$$\left[\frac{J^3}{Le} \left(\frac{1}{Pr_1} + F \right) + \left(1 + \frac{1}{Le} \right) (J^2 + QJ^2 - QJa^2) \right] < (a^2 Ra + R_n) \text{ and } \left(\frac{J}{Le} \right) \left(J + \frac{QJ}{Le} - \frac{Qa^2}{Le} \right) > \left\{ \left(1 + \frac{NALe R_n + a^2 Ra Le}{NALe R_n + a^2 Ra Le} \right) \right\}$$

(5) The system undergoes destabilize if
$$\left[\frac{J^3}{Le} \left(\frac{1}{Pr_1} + F \right) + \left(1 + \frac{1}{Le} \right) (J^2 + QJ^2 - QJa^2) \right] < (a^2 Ra + R_n) \text{ and } \left(\frac{J}{Le} \right) \left(J + \frac{QJ}{Le} - \frac{Qa^2}{Le} \right) < \left\{ \left(1 + \frac{NALe R_n + a^2 Ra Le}{NALe R_n + a^2 Ra Le} \right) \right\}$$

(6) The sufficient condition for non-existence oscillatory deformation is

$$\frac{J^2}{Le^2} \left[\left\{ 1 + JF + \frac{1}{Pr_1} \right\} J^2 + Q(J - a^2) \right] > a^2 \left[\frac{J}{Le} - \frac{J + NALe R_n}{J + NALe R_n} \right]$$

8. Terminology

- A Dimensionless resultant wave number
- d^* Thickness of nanofluid layer
- D_B Brownian diffusion coefficient
- D_T Thermophoretic diffusion coefficient
- g Acceleration due to gravity
- n Growth rate of disturbances
- k_1 Medium permeability
- q_d Velocity vector
- R_a Rayleigh number
- R_n Nano particle Rayleigh number
- R_m Density Rayleigh number
- T Temperature

- T^* Reference temperature
- t Time
- (u, v, w) Velocity component
- (x, y, z) Space co-ordinate
- k_m Thermal conductivity
- Le Lewis number
- N_A Modified diffusivity ratio
- N_B Modified particle-density increment
- Pr_1 Prandtl number
- Pr_2 Magnetic Prandtl number
- P Hydrostatic pressure
- α Thermal expansion coefficient
- δ Viscosity
- δ' Kinematic viscoelasticity
- ρ Density of nanofluid
- σ Thermal capacity ratio
- $(\rho_c)_p$ Heat capacity of nano particles
- ρ_c Heat capacity of fluid
- ϕ Volume fraction of nano particle
- ρ_p Density of nano particles
- ρ_f Density of base fluid
- k Thermal diffusivity
- ω Dimensionless frequency
- Q Chandrasekhar number

Superscripts

- ` Non-dimensionless variables
- " perturbed quantities

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