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Application of Sawi Transform for Solving Problems on Newton's Law of Cooling

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Abstract: ***The Newton's Law of Cooling arises in the field of Physics . In this paper , I use Sawi Transform for solving problems on Newton's Law of Cooling and one application is given in order to demonstrate the effectiveness of Sawi Transform for solving problems on Newton's Law of Cooling.***

Keywords: Sawi Transform, Inverse Sawi Transform, Newton's Law of Cooling, Temperature of environment, Temperature of body

AMS Subject Classification: 44A05, 44A10, 44A35.44A20

1. Introduction

Newton's Law of Cooling is a differential equation that predicts the cooling of a warm body placed in a cold environment. According to the law, the rate at which the temperature of the body decreases is proportional to the difference of temperature between the body and its environment. In symbols

$$\frac{dT}{dt} = -\,k\,(T - T_e) \qquad \ldots\ldots \quad \ldots\ldots \qquad (1)$$

with initial condition as $T(t_0) = T_0$……….. …. (2)

where T is the temperature of the object,
T_e is the constant temperature of the environment,
K is the constant of proportionality,
T_0 is the initial temperature of the object at time t_0.

In equation (1), the negative sign in the RHS is taken because the temperature of the body is decreasing with time and so the derivative $\frac{dT}{dt}$ must be negative

2. SAWI Transform

The Sawi Transform of the function f(t) is denoted by S{f(t)} and is defined by

$$S\{f(t)\} = \frac{1}{s^2}\int_0^\infty f(t)e^{-\frac{t}{s}}dt = g(s) \ \ldots\ldots\ldots (3)$$

Where S is the Sawi Transform operator.

The Sawi Transform of the function f(t) for t ≥ 0 exists if f(t) is piecewise continuous and of exponential order. These conditions are only sufficient conditions for the existence of Sawi Transform without large computational work.

3. Linear Property of SAWI Transform

S{α f(t) + β g(t)} = α S {f(t)} + β S {g(t)} where α and β are constants.

4. SAWI Transform of Some Standard Functions

Sl. No.	f(t)	S{f(t)}=g(s)
1	1	$\frac{1}{s}$
2	t	1
3	t^2	$2!\,s$
4	t^n, $n \in N$	$n!\,s^{n-1}$
5	e^{at}	$\frac{1}{s(1-as)}$
6	e^{-at}	$\frac{1}{s(1+as)}$
7	sinat	$\frac{a}{1+a^2s^2}$
8	cosat	$\frac{1}{s(1+a^2s^2)}$
9	sinhat	$\frac{a}{1-a^2s^2}$
10	coshat	$\frac{1}{s(1-a^2s^2)}$

5. Inverse SAWI Transform

If S {f(t)} = g(s), then f(t) is called the inverse Sawi Transform of g(s) and is denoted by $S^{-1}\{g(s)\}$.

6. SAWI Transform of Derivatives of the Function f(t) :

If S{f(t)} = g(s), then

i) $S\{f'(t)\} = \frac{1}{s}g(s) - \frac{1}{s^2}f(0)$

ii) $S\{f''(t)\} = \frac{1}{s^2}g(s) - \frac{1}{s^3}f(0) - \frac{1}{s^2}f'(0)$

iii) $S\{f'''(t)\} = \frac{1}{s^3}\ g(s) - \frac{1}{s^4}\ f(0) - \frac{1}{s^3}\ f'(0) - \frac{1}{s^2}\ f''(0)$

iv) $S\{f(t)\} = \frac{1}{s^n}\ g(s) - \frac{1}{s^{n+1}}\ f(0) - \frac{1}{s^n}\ f'(0) - \ldots \frac{1}{s^2}\ f^{n-1}(0)$

7. SAWI Transform for Newton's Law of Cooling

In this section, I present Sawi Transform for Newton's Law of Cooling given by (1) and (2). Applying Sawi Transform on both sides of (1) , we have

$S\{\frac{dT}{dt}\} = S\{-k(T - T_e)\}$ ……. (4)

⇨ $\frac{1}{s}\{T(t)\} - \frac{1}{s^2}\ T(o) = -kS\{T - T_e\}$

⇨ $\frac{1}{s}\ \{T(t)\} - \frac{1}{s^2}\ T(o) = -kS\{T(t)\} + kS\{T_e\}$

⇨ $\frac{1}{s}\{T(t)\} - \frac{1}{s^2}\ T_0 = -kS\{T(t)\} + kT_eS\{1\}$

⇨ $\frac{1}{s}\{T(t)\} - \frac{1}{s^2}\ T_0 = -kS\{T(t)\} + kT_e\frac{1}{s}$

⇨ $(\frac{1}{s} + k)\ S\{(t)\} = kT_e\frac{1}{s} + \frac{1}{s^2}\ T_0$

⇨ $\{T(t)\} = T_e\frac{K}{s\,(\frac{1}{s}+K)} + T_0\frac{1}{s^2(\frac{1}{s}+K)}$

⇨ $\{T(t)\} = T_e\frac{KS}{s\,(\frac{1}{s}+K)} + T_0\frac{1}{s^2(\frac{1}{s}+K)}$

⇨ $\{T(t)\} = T_e\frac{KS}{s\,(1+KS)} + T_0\frac{1}{s(1+KS)}$

⇨ $\{T(t)\} = T_e\left\{\frac{1}{S} - \frac{1}{s(1+KS)}\right\} + T_0\frac{1}{s(1+KS)}$

⇨ $\{T(t)\} = T_e\{S\{1\} - S\{e^{-kt}\}\} + T_0S\{e^{-kt}\}$

⇨ $\{T(t)\} = T_eS\{1\} - T_eS\{e^{-kt}\} + T_0S\{e^{-kt}\}$

⇨ $\{T(t)\} = S\{T_e - T_ee^{-kt} + T_0e^{-kt}\}$

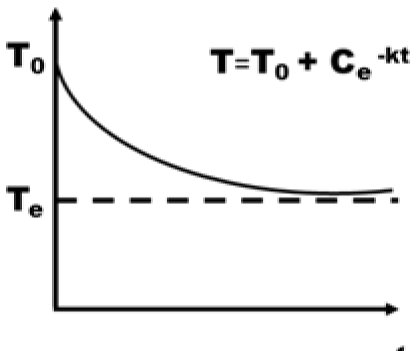
⇨ $(t) = T_e - T_ee^{-kt} + T_0e^{-kt}$

⇨ $(t) = T_e + (T_0 - T_e)e^{-kt}$

⇨ $T(t) = T_e + Ce^{-kt}$

where $C = (T_0 - T_e)$ …….. ………. ….. (5)

Where (t) *is the temperature of the object at any time* t *and* T_e *is the temperature of the environment.*



Cooling of a warm body

This function decreases exponentially, but approaches T_e as t→ ∞ instead of zero.

8. Applications

In this section, one application is given in order to demonstrate the effectiveness of Sawi Transfom for solving problems on Newton's Law of Cooling.

***Application*-1:** An apple pie with an initial temperature of 170^0C is removed from the oven and left to cool in a room with an air temperature of 20^0C . Given that the temperature of the pie initially decreases at a rate of 3.0^0C/ min. How long will it take for the pie to cool to a temperature of 30^0C ?

Solution: Assuming that the pie obeys Newton's Law of Cooling, we have the following information:

$\frac{dT}{dt} = -k(T-20)$, $T(0) = 170$, $T'0 = -3.0$

Where T is the temperature of the pie in degree Celsius,

T is the time in minutes and k is an unknown constant.

We can find the value of k by putting the information we know about t = 0 directly into the differential equation:

$-3 = -k(170 - 20)$

⇨ $k = \frac{1}{50} = 0.02$

So, the differential equation can be written as

$\frac{dT}{dt} = -\frac{1}{50} = (T\text{-}20)$

⇨ $S\{\frac{dT}{dt}\} = -\frac{1}{50}\{T - 20\}$

⇨ $S\{\frac{dT}{dt}\} = -\frac{1}{50}S\{T\} + \frac{2}{5}S\{1\}$

⇨ $\frac{1}{s}S\{T(t)\} - \frac{1}{s^2}(0) = -\frac{1}{50}S\{T(t)\}\frac{2}{5}\frac{1}{s}$

⇨ $\left(\frac{1}{s} + \frac{1}{50}\right)S\{T(t)\} = \frac{2}{5S} + 170\frac{1}{s^2}$

⇨ $\frac{(50+S)}{50S}\ S\{T(t)\} = \frac{(2S+850)}{5S^2}$

⇨ $S\{T(t)\} = \frac{(2S+850)}{5S^2}\ X\ \frac{50S}{(50+S)}$

⇨ $S\{T(t)\} = \frac{8500+20S}{S\,(50+S)}$

⇨ $S\{T(t)\} = \frac{1000+20S+7500}{S\,(50+S)}$

⇨ $S\{T(t)\} = \frac{20\,(50+S)+7500}{S\,(50+S)}$

⇨ $S\{T(t)\} = \frac{20\,(50+S)}{S\,(50+S)} + \frac{7500}{S\,(50+S)}$

⇨ $S\{T(t)\} = \frac{20}{S} + \frac{150}{S\,(1+\frac{S}{50})}$

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$$\Rightarrow S\{T(t)\} = 20S\{1\} + 150S\left\{e^{-\frac{1}{50}t}\right\}$$

$$\Rightarrow S\{T(t)\} = S\left\{20 + 150e^{-\frac{1}{50}t}\right\}$$

$$\Rightarrow T(t) = 20 + 150e^{-\frac{1}{50}t} \quad \ldots\ldots\ldots\ldots(6)$$

Putting T=30 in (6), we get

$$30 = 20 + 150e^{-\frac{1}{50}t}$$

$$\Rightarrow e^{-\frac{1}{50}t} = \frac{1}{15}$$

$$\Rightarrow e^{\frac{1}{50}t} = 15$$

$$\Rightarrow \frac{1}{50}t = \ln 15$$

$$\Rightarrow = 50\, ln15 = 50 * 2.7080502011 = 135.4 \quad min$$

Hence it will take 135.4 minutes for the pie to cool to a temperature of 30^0 C.

9. Conclusion

In this paper, I have successfully developed the Sawi Transform for solving problems on Newton's Law of Cooling. The given applications show the effectness of Sawi Transform for solving problems on Newton's Law of Cooling.

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