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Evenly Divisible Composite Pseudo Intrinsic Vertex-Magic Graphs With Factorizable Property

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Abstract

In this paper, we introduce the new concept of fuzzy intrinsic vertex-magic in cycle and path graphs and based on this, we deal with the graphs which satisfy the conditions of evenly divisible composite pseudo intrinsic vertex-magic by the presence of a mock constant value. Also, we check whether the evenly divisible composite pseudo intrinsic graphs are fuzzy factorizable perfect intrinsic vertex-magic when looking at graphs order and size. We investigate the above properties on some graphs like C_4, 3-pan graph, $K_{2,2}$, and star graphs.

Keywords: Mock and Intrinsic constant, Evenly divisible composite pseudo graph, Factorizable.

AMS Subject Classification(2010): 05C78.

1 Introduction

Graph theory plays a prominent role in representing many issues in the physical world. Through the concepts and theories of graphs, there are many hurdles in representing the systems that were solved graphically. However, due to the uncertainty of the system parameters, graphs were unable to solve all the systems properly. Then the fuzzy set was initially defined by L.A.Zadeh[15] and kaufmann gave the first defintion for fuzzy graphs in 1973. To avoid the vagueness said before, the fuzzy graph model was designed very first by A.Rosenfeld[12]. Rosenfeld's basic graph concepts include paths, cycles, bridges, trees, etc which help for further findings and innovations in fuzzy graphs by other researchers. Bhuttani.K.R[3] and P.Bhattacharya[2] expanded

*R.Nishanthini,E-mail: nishamaths.1995@gmail.com the concepts of connectivity in fuzzy graphs which laid a strong foundation to attract more researchers in the fuzzy field.

The generalized crisp graph is a fuzzy graph that suggests those properties Similar to the crisp graph, deviation takes place in multiple points. Labeling[14] of acrisp graph is nothing but assigning of integer values to every vertices and edge of a graph. A graph is called a magic graph if edges are labeled by positive integers so that the sum over the edges incident with any vertex is the same, independent of the choice of a vertex. Through Kotzig and Rosa[9], the magic labeling concept originated and it was extended by the work of NagoorGani.A[10] as fuzzy magic labelings. Through these findings, we came to know that if the edges get smaller labels than the vertices then it is called super vertex-magic labeling. With the above results, other authors have found and elaborated on the fuzzy magic graph and its labelings[1],[5] & [12].

Kaliraja and sasikala previously addressed the fuzzy intrinsic edge magic graphs and its construction, factorizable on fuzzy intrinsic edge-magic graphs and prime & composite pseudo intrinsic edge-magic graphs in [6],[7],[8] & [9]. In this paper we have extended the concept to fuzzy intrinsic vertex magic graphs and also the concept of evenly divisible composite pseudo intrinsic vertex magic graphs and its factorizability on undirected fuzzy graphs.

2 Preliminaries

Definition 2.1 *A fuzzy graph G:*(σ, μ) *is a pair of functions* $\sigma : V \to [0,1]$ *and* $\mu : V \times V \to [0,1]$ *where for all u,v* $\in$ *V, we have* $\mu(u,v) \leq \sigma(u) \wedge \sigma(v)$.

Definition 2.2 *A path P in a fuzzy graph is a sequence of distinct vertices* $v_1, v_2, .., v_n$ *such that* $\mu(v_i, v_{i+1})$*>0 ;* $1 \leq i \leq n$ *;here* $n \geq 1$ *is called the length of the path P. The edge of the path is labelled consecutive pairs* (v_i, v_{i+1}).

Definition 2.3 *A path P is called a cycle if* $v_1 = v_n$ *and* $n \geq 3$ *and a cycle is called a fuzzy cycle if more than one weakest arc is in it. The degree of a vertex v is defined as* $d(v) = \sum_{u \neq v, u \in v} \mu(u,v)$.

Definition 2.4 *A bijection* $\star$ *is a fuction from the set of all nodes and edges of to* $[0,1]$ *which assign each nodes* $\sigma^\star(a), \sigma^\star(b)$ *and edge* $\mu^\star(a,b)$ *a membership value such that* $\mu^\star(a,b) \leq \sigma^\star(a) \wedge \sigma^\star(b)$ *for all* $a, b \in V$ *is called a fuzzy labeling.*

A graph is said to be fuzzy labeling graph if it has fuzzy labelings and it is denoted as $G^\star$.

Definition 2.5 *A fuzzy labeling graph G is said to be fuzzy intrinsic labeling if G:* $\sigma \to [0,1]$ *and G:* $\mu \to [0,1]$ *is bijective such that the membership values of edges and vertices are* $\{z, 2z, 3z, ..., Nz\}$ *where N is the total number of vertices and edges and let z=0.1 for* $N \leq 6$ *and z = 0.01 for* $N \geq 6$.

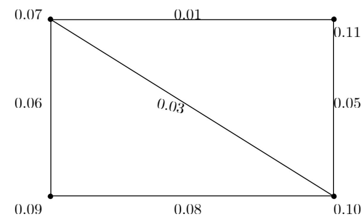

Figure 1: Fuzzy labeling

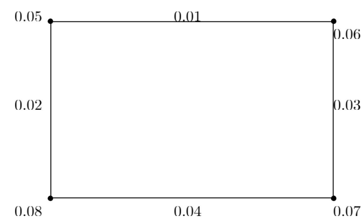

Figure 2: Fuzzy intrinsic graph

Definition 2.6 *A fuzzy labeling graph G is said to be fuzzy perfect intrinsic labeling if $g : \sigma \to [0,1]$ and $g : \mu \to [0,1]$ is bijective such that the membership values are $\{z, 2z, 3z, \cdots, \in z\}$ and vertices are $\{(\varepsilon+1)z, (\varepsilon+2)z, \cdots, (\varepsilon+v)z\}$ where $\varepsilon + v = N$ is the total number of vertices and edges and let z=0.1 for $N \leq 6$ and z = 0.01 for N>6.*

Definition 2.7 *A fuzzy perfect intrinsic labeling is said to be an vertex-magic labeling if it has an intrinsic constant $\lambda_c = \mu(u,v) + \sigma(v) + \mu(v,w)$ for all $u, v, w \in V$.*

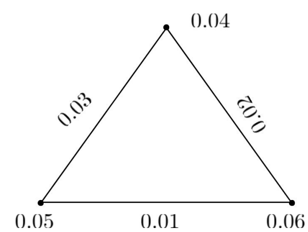

Figure 3: Fuzzy intrinsic vertex-magic graph

Definition 2.8 *A vertex-magic constant in a fuzzy perfect intrinsic vertex-magic graph is said to be mock constant λ_m if it is equal to $\mu(u,v) + \sigma(v) + \mu(v,w)$ for some $u, v, w \in V$ with $\lambda_c \neq \lambda_m$.*

Definition 2.9 *A fuzzy graph is said to be a pseudo-intrinsic vertex-magic graph if it contains a mock constant λ_m which is denoted by G_p.*

Necessary condition: For intrinsic vertex-magic, the necessary condition is that the intrinsic vertex-magic graph satisfies only intrinsic vertex-magic labeling.

Sufficient condition: A sufficient condition for intrinsic vertex-magic is that if it has the same intrinsic constant for all vertices.

Definition 2.10 *Let G be a fuzzy pseudo intrinsic vertex-magic (FPIVM) graph. If the mock constant* $'\lambda'_m$ *is not prime, then the graph is called the composite pseudo intrinsic vertex-magic.*

Definition 2.11 *Let G be a FPIVM graph. The size and order of G is denoted by* α *and* β *respectively, where* $\alpha = \sum_{u \neq v} \mu(u,v)$ *and* $\beta = \sum_{v \in V} \sigma(v)$ *and G is called a factorizable graph if it has both composite order and composite size.*

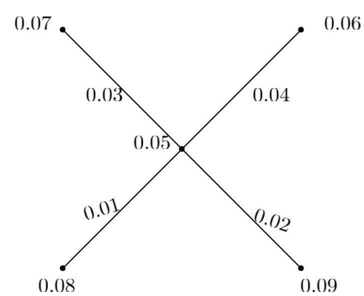

Figure 4: Factorizable graph

Definition 2.12 *Let G be a fuzzy pseudo intrinsic vertex-magic graph. If the mock constant* $'\lambda'_m$ *is evenly divisible then G is said to be evenly divisible.*

3 Evenly Divisible Composite Pseudo Intrinsic Vertex-Magic Graphs With Factorizable Property

Theorem 3.1 *A cycle* C_n *is fuzzy intrinsic vertex-magic if n is odd.*

Proof. Consider the cycles of length 3 and 5 (Since a cycle needs at least three vertices and it must be intrinsic labeling, we take n=3 &5).

From the figure 5, we came to know that C_3 and C_5 are vertex-magic since at every vertex of C_3 and C_5, we get a unique intrinsic constant (λ_c) as 0.09 and 0.14 respectively.

Hence, cycle C_n is fuzzy intrinsic vertex-magic if n is odd.

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Theorem 3.2 *A path P_n is fuzzy intrinsic vertex-magic if n is odd.*

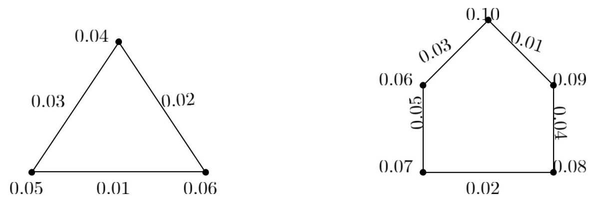

Figure 5: Fuzzy intrinsic vertex-magic for C_3 and C_5

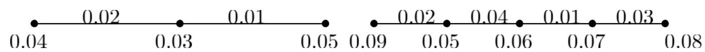

Figure 6: Fuzzy intrinsic vertex-magic for P_3 and P_5

Proof. Consider the paths of length 3 and 5 (Since a path needs at least two vertices and it must be intrinsic labeling, we take n=3 &5).

From the figure 6, we came to know that P_3 and P_5 are vertex-magic since at every vertex of P_3 and P_5, we get a unique intrinsic constant (λ_c) as 0.06 and 0.11 respectively.

Hence, P_n is fuzzy intrinsic vertex-magic if n is odd.

Theorem 3.3 *A cycle C_n is an evenly divisible composite pseudo intrinsic vertex-magic graph only if n=4 and also it is factorizable.*

Proof. Consider the fuzzy intrinsic vertex-magic labeling in C_4 as follows:

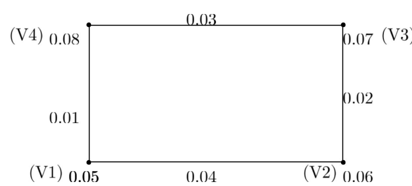

Figure 7: Fuzzy intrinsic vertex-magic labeling for C_4

From the figure 7,

$$\mu(v_1,v_4)+\sigma(v_1)+\mu(v_1,v_2) = 0.01+0.05+0.04 = 0.10 = \lambda_m$$
$$\mu(v_1,v_2)+\sigma(v_2)+\mu(v_2,v_3) = 0.04+0.06+0.02 = 0.12 = \lambda_c$$
$$\mu(v_2,v_3)+\sigma(v_3)+\mu(v_3,v_4) = 0.02+0.07+0.03 = 0.12 = \lambda_c$$
$$\mu(v_3,v_4)+\sigma(v_4)+\mu(v_1,v_4) = 0.03+0.08+0.01 = 0.12 = \lambda_c$$

Clearly, this shows that, at the first vertex(v_1) of C_4, the mock constant (λ_m) = 0.10 exist and at the last three vertices(v_2, v_3, v_4) intrinsic constant(λ_c) = 0.12 exists. Here $\lambda_m = 0.10$, which is evenly divisible but not prime.

Hence pseudo intrinsic fuzzy C_n is an evenly divisible composite vertex-magic graph only if n=4. Also, $\alpha=\sum_{u\neq v}\mu(u,v)=0.10$ and $\beta=\sum_{v\in V}\sigma(v)=0.26$, which implies that C_4 is factorizable.

For n=3,5 & 6 in a cycle C_n, the above results for fuzzy composite intrinsic vertex-magic with factorizable property doesn't satisfy and so there is no need to check for evenly divisible.

Theorem 3.4 *A n-pan graph is an evenly divisible composite pseudo intrinsic vertex-magic only if n=3 and also it is factorizable.*

Proof. Consider a n-pan graph and take n=3. Consider the fuzzy intrinsic vertex-magic labeling in 3-pan graph as follows:

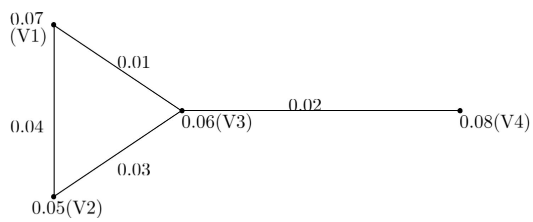

Figure 8: fuzzy intrinsic vertex-magic labeling in 3-pan graph

From the figure 8,
$\mu(v_1,v_2)+\sigma(v_1)+\mu(v_1,v_3)=0.04+0.07+0.01=0.12=\lambda_c$
$\mu(v_1,v_2)+\sigma(v_2)+\mu(v_2,v_3)=0.04+0.05+0.03=0.12=\lambda_c$
$\mu(v_1,v_3)+\mu(v_2,v_3)+\sigma(v_3)+\mu(v_3,v_4)=0.01+0.03+0.06+0.02=0.12=\lambda_c$
$\mu(v_3,v_4)+\sigma(v_4)=0.02+0.08=0.10=\lambda_m$

Here, at first three vertices(v_1,v_2,v_3) of 3-pan graph, we get an intrinsic constant $(\lambda_c)=0.12$ and at the last vertex(v_4), mock constant$(\lambda_m)=0.10$ exist. Here $\lambda_m=0.10$, which is evenly divisible but not prime.

Hence pseudo intrinsic fuzzy n-pan graph is an evenly divisible composite vertex-magic graph.

Also, $\alpha=\sum_{u\neq v}\mu(u,v)=0.10$ and $\beta=\sum_{v\in V}\sigma(v)=0.26$, which implies that 3-pan graph is factorizable.

For n=4,5 & 6 in a n-pan graph, the above results for fuzzy composite intrinsic vertex magic with factorizable property doesn't satisfy and so there is no need to check for evenly divisible.

Theorem 3.5 *A complete bipartite $K_{2,2}$ is an evenly divisible composite pseudo intrinsic vertex-magic graph and also it is factorizable.*

Proof. Consider a $K_{2,2}$ graph.

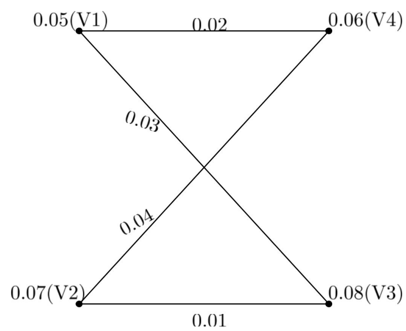

Figure 9: Fuzzy intrinsic vertex-magic labeling in $K_{2,2}$

Consider the fuzzy intrinsic vertex-magic labeling in $K_{2,2}$ as follows:

From the figure 9,

$$\mu(v_1,v_3)+\sigma(v_1)+\mu(v_1,v_4) = 0.03 + 0.05 + 0.02 = 0.10 = \lambda_m$$
$$\mu(v_2,v_3)+\sigma(v_2)+\mu(v_2,v_4) = 0.01 + 0.07 + 0.04 = 0.12 = \lambda_c$$
$$\mu(v_2,v_3)+\sigma(v_3)+\mu(v_1,v_3) = 0.01 + 0.08 + 0.03 = 0.12 = \lambda_c$$
$$\mu(v_2,v_4)+\sigma(v_4)+\mu(v_1,v_4) = 0.04 + 0.06 + 0.05 = 0.12 = \lambda_c$$

Clearly, this shows that, at the first vertex(v_1) of $K_{2,2}$, the mock constant(λ_m) = 0.10 exist and at the last three vertices(v_2, v_3, v_4) intrinsic constant(λ_c) = 0.12 exists. Here $\lambda_m = 0.10$, which is evenly divisible but not prime.

Hence pseudo intrinsic fuzzy $K_{2,2}$ is an evenly divisible composite vertex-magic graph. Also,α=$\sum_{u\neq v}\mu(u,v) = 0.10$ and $\beta = \sum_{v\in V}\sigma(v) = 0.26$, which implies that $K_{2,2}$ is factorizable.

Theorem 3.6 *A star graph $K_{1,n}$ is an evenly divisible composite pseudo intrinsic vertex-magic graph if n=3,4 and also it is factorizable.*

Proof. Consider a $K_{m,n}$ graph.

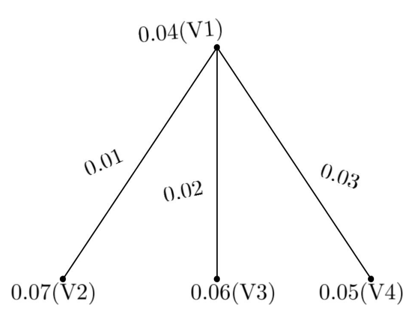

Figure 10: Fuzzy intrinsic vertex-magic labeling in $K_{1,3}$

Now fix n=3 and consider the fuzzy intrinsic vertex-magic labeling in $K_{1,3}$ as follows: From the figure 10,

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$\mu(v_1, v_2) + \sigma(v_1) + \mu(v_1, v_3) + \mu(v_1, v_4) = 0.01 + 0.04 + 0.02 + 0.03 = 0.10 = \lambda_m$
$\sigma(v_2) + \mu(v_1, v_2) = 0.07 + 0.01 = 0.08 = \lambda_c$
$\sigma(v_3) + \mu(v_1, v_3) = 0.06 + 0.02 = 0.08 = \lambda_c$
$\sigma(v_4) + \mu(v_1, v_4) = 0.05 + 0.03 = 0.08 = \lambda_c$

Clearly, this shows that, at the first vertex(v_1) of $K_{1,3}$, the mock constant$(\lambda_m) = 0.10$ exist and at the last three vertices(v_2, v_3, v_4) intrinsic constant$(\lambda_c) = 0.08$ exists. Here $\lambda_m = 0.10$, which is evenly divisible but not prime.

Hence pseudo intrinsic fuzzy $K_{1,3}$ is an evenly divisible composite vertex-magic graph. Also, α=$\sum_{u \neq v} \mu(u, v) = 0.06$ and $\beta = \sum_{v \in V} \sigma(v) = 0.22$, which implies that $K_{1\,3}$ is factorizable.

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Now, fix n=4 and consider the fuzzy intrinsic vertex-magic labeling in $K_{1,4}$ as follows:

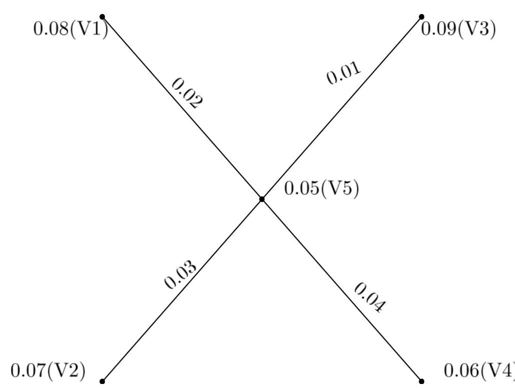

Figure 11: Fuzzy intrinsic vertex-magic labeling in $K_{1,4}$

From the figure 11,

$\sigma(v_1) + \mu(v_1, v_5) = 0.08 + 0.02 = 0.10 = \lambda_c$
$\sigma(v_2) + \mu(v_2, v_5) = 0.07 + 0.03 = 0.10 = \lambda_c$
$\sigma(v_3) + \mu(v_3, v_5) = 0.09 + 0.01 = 0.10 = \lambda_c$
$\sigma(v_4) + \mu(v_4, v_5) = 0.06 + 0.04 = 0.10 = \lambda_c$
$\mu(v_1, v_5) + \mu(v_2, v_5) + \sigma(v_5) + \mu(v_3, v_5) + \mu(v_4, V_5) = 0.02 + 0.03 + 0.05 + 0.01 + 0.04$ $= 0.15 = \lambda_m$

Clearly, this shows that, at the first four vertices(v_1, v_2, v_3, v_4) intrinsic constant(λ_c) $= 0.10$ exists and at the last vertex(v_5) mock constant$(\lambda_m) = 0.15$ exist. Here $\lambda_m =$ 0.15, which is evenly divisible but not prime.

Hence pseudo intrinsic fuzzy $K_{1,4}$ is an evenly divisible composite vertex-magic graph. Also, α=$\sum_{u \neq v} \mu(u, v) = 0.10$ and $\beta = \sum_{v \in V} \sigma(v) = 0.30$, which implies that $K_{1,4}$ is factorizable.

Now, fix n=5 and consider the fuzzy intrinsic vertex-magic labeling in $K_{1,5}$ as follows:

From the figure 12,

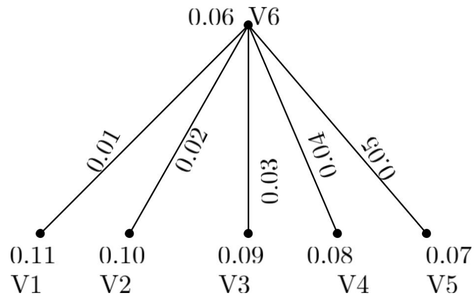

Figure 12: Fuzzy intrinsic vertex-magic labeling in $K_{1,5}$

$\sigma(v_1) + \mu(v_1, v_6) = 0.11 + 0.01 = 0.12 = \lambda_c$
$\sigma(v_2) + \mu(v_2, v_6) = 0.10 + 0.02 = 0.12 = \lambda_c$
$\sigma(v_3) + \mu(v_3, v_6) = 0.09 + 0.03 = 0.12 = \lambda_c$
$\sigma(v_4) + \mu(v_4, v_6) = 0.08 + 0.04 = 0.12 = \lambda_c$
$\sigma(v_5) + \mu(v_5, v_6) = 0.07 + 0.05 = 0.12 = \lambda_c$
$\sigma(v_6) + \mu(v_1, v_6) + \mu(v_2, v_6) + \mu(v_3, v_6) + \mu(v_4, v_6) + \mu(v_5, v_6) = 0.06 + 0.01 + 0.02 + 0.03 + 0.04 + 0.05 = 0.21 = \lambda_m$
Clearly this shows that, at the first five vertices(v_1, v_2, v_3, v_4, v_5) intrinsic constant(λ_c) = 0.12 exists and at the last vertex(v_6) mock constant(λ_m) = 0.21 exist. Here λ_m = 0.21, which is evenly divisible but not prime.

Hence pseudo intrinsic fuzzy $K_{1,5}$ is an evenly divisible composite vertex-magic graph. Also, $\alpha = \sum_{u \neq v} \mu(u, v) = 0.15$ and $\beta = \sum_{v \in V} \sigma(v) = 0.51$, which implies that $K_{1,5}$ is factorizable. Hence, $K_{1,n}$ is an evenly divisible composite pseudo intrinsic vertex magic if n=3,4,5 and also it is factorizable.

4 Conclusion

We explored the fuzzy intrinsic vertex-magic concept on cycles and path graphs. Also, on fuzzy vertex magic graphs like C_4, $K_{2,2}$ and star graphs we have investigated the composite pseudo intrinsic characteristical and evenly divisible property. In addition, these graphs are known to be fuzzy factorizable perfect intrinsic vertex-magic.

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