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Applications of Fourier Series in Double Sine and Cosine Series

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Abstract: ***In this paper we have introduce the new concept of double sine and cosine series in Fourier series of two variable functions and implementing these result by the examples of two variable Fourier series.***

Keywords: Fourier series, double sine series, double cosine series.

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1. Introduction

A Fourier series is an expansion of a periodic function f(x) in terms of an infinite sum of sines and cosines. Fourier series make use of the orthogonality relationships of the sine and cosine functions.

"Any functions f(x) can be expressed as a Fourier series

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n cosnx + \sum_{n=1}^{\infty} b_n sinnx$$

In the interval$(0,2\pi)$ or$(-\pi,\pi)$,$where a_0, a_n, b_n$ are constant.
(x) is periodic, single valued and finite
(x) has a finite number of finite discontinuities in any one period
(x) has a finite number of maxima and minima

$$a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)dx$$

Even function $a_n = \frac{2}{\pi}\int_0^{\pi} f(x)\ \cos(nx)dx$

Odd function $b_n = \frac{2}{\pi}\int_0^{\pi} f(x)\ \sin(nx)\,dx$

2. Main Result

2.1 Fourier series of two variable functions

Any functions f(x, y) can be expressed as a Fourier series

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n cosnx\ cosny + \sum_{n=1}^{\infty} b_n sinnx\ sinny$$

In the interval$(0,2\pi)$ or$(-\pi,)$,$here a_0, a_n, b_n$ are constant.

$$a_0 = \frac{1}{\pi}\int_0^{\pi}\int_0^{\pi} f(x,y)\,dx\,dy$$

Even function

$$a_n = \frac{2}{\pi}\int_0^{\pi}\int_0^{\pi} f(x,y)\ \cos(nx)\cos(ny)\,dx\,dy$$

Odd function $b_n = \frac{2}{\pi}\int_0^{\pi}\int_0^{\pi} f(x,y)\ \sin(nx)\sin(ny)\,dx$

2.2 Examples

Find the Fourier series of the function $f(x,y) = x^2 + y^2$ in $-\pi < x < \pi$

Solution: Let the Fourier series of the function $f(x,y) = x^2 + y^2$ be
$f(x,y) = x^2 + y^2 = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n cosnx\ cosny + bnsinnxsinny)$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi} f(x,y)\,dxdy$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi} (x^2+y^2)\ \ dxdy$$

$$\frac{1}{\pi}\left[\frac{x^3}{3}\right]_{-\pi}^{\pi} + \frac{1}{\pi}\left[\frac{y^3}{3}\right]_{-\pi}^{\pi}$$

$$\frac{1}{\pi}\left[\frac{\pi^3}{3}+\frac{\pi^3}{3}\right] = \frac{1}{\pi}\frac{4\pi^3}{3} = \frac{4\pi^2}{3}$$

$$a_0 = \frac{4\pi^2}{3}$$

$$a_n = \frac{2}{\pi}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi} f(x,y)\ \cos(nx)\cos(ny)\,dx\,dy$$

$$a_n = \frac{2}{\pi}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi} (x^2+y^2)\ \cos(nx)\cos(ny)\,dx\,dy$$

$$\frac{2}{\pi}\int_{-\pi}^{\pi} x^2 cosnx\ \ dx + \int_{-\pi}^{\pi} y^2 cosny\,dy$$

$$\int_{-a}^{a}\int_{-a}^{a} f(x,y)\,dxdy = \begin{cases} 2\int_0^a f(x)dx\ ,if\ f(x) is\ an\ even\ function\ of\ x \\ 0,\quad if\ f(x) is\ an\ odd\ function\ of\ x \end{cases}$$

$$= \frac{2}{\pi}\left[x^2\frac{\sin nx}{n} - 2x\left(-\frac{\cos nx}{n^2}\right) + 2\left(-\frac{sinnx}{n^3}\right)\right]_0^{\pi} +$$

$$\frac{2}{\pi}\left[y^2\frac{\sin ny}{n} - 2y\left(-\frac{\cos ny}{n^2}\right) + 2\left(-\frac{sinny}{n^3}\right)\right]_0^{\pi}$$

$\frac{4(-1)^n}{n^2}+\frac{4(-1)^n}{n^2}=\frac{8(-1)^n}{n^2}$ [$sinn\pi = 0$ and $cosn\pi = (-1)^n$]

$$\begin{cases} \frac{-8}{n^2}, when\ n\ is\ odd \\ \frac{8}{n^2}, when\ n\ is\ even \end{cases}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi} f(x,y) sinnx\ sinny\ dx\ dy$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi}\int_{-\pi}^{\pi}(x^2+y^2)\ sinnx\ sinny\ dx\ dy = 0$$

[$x^2$is an even function and $sinnx\ sinny$ is an odd function So that $x^2 sinnx$and $y^2 sinny$ is an odd function]

Substituting the above values in (1) ,we obtain the required Fourier series of the function $f(x,y)=(x^2+y^2)$ as given by

$$x^2+y^2 = \frac{\pi^2}{3}+\sum_{n=1}^{\infty}\left(\frac{4(-1)^n}{n^2}cosnx+\frac{4(-1)^n}{n^2}cosny\right) + \sum_{n=1}^{\infty} 0.sinnx\ sinny$$

$$x^2+y^2 = \frac{\pi^2}{3}+4\left[-cosx+\frac{1}{4}cos2x-\frac{1}{9}cos3x+\cdots..\right] + 4\left[-cosy+\frac{1}{4}cos2y-\frac{1}{9}cos3y+\cdots..\right]$$

Putting $x=0, y=0$,

$$0 = \frac{\pi^2}{3}+4\left[-1+\frac{1}{4}-\frac{1}{9}+\frac{1}{16}-\right]+4\left[-1+\frac{1}{4}-\frac{1}{9}+\frac{1}{16}-\right]$$

$$= \frac{\pi^2}{3}+8\left[-2+\frac{1}{2}-\frac{2}{9}+\frac{1}{8}-\cdots.\right]$$

$$\frac{\pi^2}{24} = 2-\frac{1}{2}+\frac{2}{9}-\frac{1}{8}+\cdots$$

2.3 Examples

Obtain the Fourier series representing the function $f(x,y) = x + y$in the interval $0 < x < 2\pi$ and $0 < y < 2\pi$.

Solution Letthe Fourier series representing the functions

$$f(x,y) = x+y$$

$$f(x) = \frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n cosnx + b_n sinnx)(1)$$

$$a_0 = \frac{1}{\pi}\int_0^{2\pi}\int_0^{2\pi} f(x,y)\ dx = \frac{1}{\pi}\int_0^{2\pi}\int_0^{2\pi}(x+y)\ \ dx\ dy$$

$$\frac{1}{\pi}\left[\frac{x^2}{2}+\frac{y^2}{2}\right]_0^{2\pi} = 2\pi+2\pi = 4\pi$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi}\int_0^{2\pi} f(x,y)cosnx\ cosny\ dx\ dy$$

$$= \frac{1}{\pi}\int_0^{2\pi}\int_0^{2\pi}(x+y)cosnx\ cosny\ dx\ dy$$

$$\frac{1}{\pi}\int_0^{2\pi} x\ cosnx\ dx + \frac{1}{\pi}\int_0^{2\pi} y\ cosnydy$$

$$\frac{1}{\pi}\left[x.\frac{sinnx}{n}-1.\left(\frac{-cosnx}{n^2}\right)\right]_0^{2\pi}+\frac{1}{\pi}\left[\left[y.\frac{sinny}{n}-1.\left(\frac{-cosny}{n^2}\right)\right]_0^{2\pi}\right.$$

$$\frac{1}{\pi}\left[\frac{\cos 2n\pi}{n^2}-\frac{1}{n^2}\right]+\frac{1}{\pi}\left[\frac{\cos 2n\pi}{n^2}-\frac{1}{n^2}\right]$$

$$\frac{1}{n^2\pi}(1-1)+\frac{1}{n^2\pi}(1-1) = 0$$

$$a_n = 0$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi}\int_0^{2\pi} f(x,y)\ sinnx\ \ sinny\ dx\ dy$$

$$\frac{1}{\pi}\int_0^{2\pi} x\ sinnx\ dx + \frac{1}{\pi}\int_0^{2\pi} y\ sinny\ dy$$

$$\frac{1}{\pi}\left[x\left(-\frac{\cos nx}{n}\right)-1.\left(-\frac{sinnx}{n^2}\right)\right]_0^{2\pi}+\frac{1}{\pi}\left[y\left(-\frac{\cos ny}{n}\right)-1.\left(-\frac{sinny}{n^2}\right)\right]_0^{2\pi}$$

$$\frac{1}{\pi}\left[-\frac{2\pi\cos 2n\pi}{n}\right]+\frac{1}{\pi}\left[-\frac{2\pi\cos 2n\pi}{n}\right] = -\frac{4}{n}$$

Putting the value's of $a's and\ b's$ in (1), we get

$$x = \pi - 2\left[sinx+\frac{1}{2}\sin 2x+\frac{1}{3}sin3x+\cdots\right]$$

$$y = \pi - 2\left[siny+\frac{1}{2}\sin 2y+\frac{1}{3}sin3y+\cdots\right]$$

3. Result

To solve the examples of two variable Fourier series we see that the result are obtained as similar in two variable Fourier series.

4. Conclusions

The necessity of dealing with Fourier series by double sine and cosine series in different applications for Engineering together with the difficulty of computing them in double valued function. In this paper to be self-contained, where the basic theoretical concepts on double sine and cosine series are solved by Fourier series. These theoretical concepts have been used to compute Fourier series of double valued functions.

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