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The Application of Mathematical Modeling in Several Financial Issues

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Abstract: ***Financial mathematics is an emerging discipline, which is the cross of finance and mathematics. This article described the basic concepts of financial mathematics and the history of its development briefly, and focused on the theoretical framework of financial mathematics and the application of mathematical modeling in finance. On the basis, this paper analyzed the problems that currently exist in this area, and the future prospects of financial mathematics.***

Keywords: Financial mathematics; financial issues; mathematical modeling

1. Introduction

With the development of the financial markets, the link between Mathematics and Finance is more and more closed. The development of modern finance promotes the development of certain branches of mathematics, at the same time, mathematical theories and methods also provide powerful tools for the development of Finance. Financial mathematics is an emerging discipline, its most significant feature is the effective use of mathematical methods to identify and demonstrate some rules of financial and economic operation [1, 2].

From a financial or economic hypothesis, Financial Mathematics uses abstract mathematical methods to build mathematical model of financial mechanisms. The main components of Financial Mathematics are the applications of mathematical concepts and methods (or any other method of natural sciences) in finance, especially in the theory of financial, and the purpose of which is to use mathematical language to express, reason and demonstrate the principle of Finance. Financial Mathematics is a branch of Finance, so financial mathematics is on the background and basis of financial theory firstly, but this does not meansthat the man who worked on financial mathematics must have formal academic training of finance. Though finance is independent from the economics because of its uniqueness, it was developed as a branch of Economics, so financial mathematics is on the background and basis of economic principles and techniques. The theoretical basis of financial mathematics also includes the modern mathematics theory and statistical learning theory, the important content is mathematical or statistical modeling, which means to extract the key factors from the complicated financial environment and distinguish correlation factor and irrelevant factor, and then on the basis of a series of assumptions to derive all relationships, at last get the conclusion and offer an explanation. Obviously, mathematical finance is an interdisciplinary subject of Finance, Mathematics, Statistics, Economics and other disciplines, it belongs to applied science.

In fact, mathematical finance is playing a more and more significant role in many aspects of modern economics. Several financial tools like mathematical modeling can help analyzing problems arising in finance markets and finding optimal solution. So it is necessary to pay attention to its application seriously. And since mathematical finance is quite a new subject, we should admit its defects and find ways to improve it.

Firstly, this article discussed the theoretical framework of financial mathematics, then described the application of mathematical modeling in a number of financial problems. On this basis, this paper analyzed the problems that exist in this area currently, and the future prospects of financial mathematics.

2. History and Theoretical Framework of Financial Mathematics

The history of financial mathematics dates back to 1900, when the French Mathematician L. Bachelier's doctoral thesis—Theory of Speculation, which proclaimed the birth of financial mathematics [3]. In this paper, he first uses Brownian motion to describe the changes of stock price. He believed that in the capital market the ones can buy and the other ones can sell, because the buyers are bullish and the sellers are bearish. Its price fluctuation is in line with Brownian motion and its statistical distribution is a normal distribution, which was five years earlier than Einstein's 1905 study of Brownian movement. However, Bachelet's work has not attracted the attention of the financial community for more than 50 years. In the early 1950s, Paul A. Samuelson rediscovered Baseler's work through statistician Savage, which marked the beginning of modern finance. Then modern finance went through two major revolutions, the first in 1952. That year, H. Markowitz (1952) published his doctoral thesis, and proposed the "mean-variance theory of portfolio selection" [4]. Its meaning is to lead the idea that people are looking for the "best" stocks to the understanding of the quantification and balance on risk and return. The second revolution in financial mathematics occurred in 1973. That year, F. Black and M. Scholes (1973) published the famous Black-Scholes formula, and showed the display expression of European option pricing[5].Now, with the development of the financial sector, mathematics is more and more widely used in financial. Meanwhile, the research of mathematical knowledge has become the main areas in the study of finance, math is constantly promote the development of financial practice.

Financial mathematics main using random analysis, random control, mathematics planning, differential game, nonlinear analysis, mathematical statistics, functional analysis, martingale theory, backwards stochastic differential equations, fractal geometry and other modern mathematics toolsto research following several main aspects of problems: (1) how combination investment securities can get maximum returns or reduced investment risk; (2) capital assets pricing model and the optimal investment and consumption theory of incomplete financial markets securities (as futures, and option, derivative tool);(3) the term structure of interest rates and interest rate derivative pricing theory; (4) the risk management theory and risk control of incomplete markets

New types of nonlinear analysis tools, such as fractal geometry, chaos theory, Wavelet analysis, and pattern recognition, are also be introduced in the analysis of securities prices [6]. Neural network and artificial intelligence are used in stock selection and the prediction of stock type's methods, and there are also some people use simulated annealing and genetic algorithms in simulation study of innovation in the futures market. To sum up, several important theoretical frameworks in the field of financial mathematics are: modern portfolio theory, capital asset pricing model, arbitrage pricing theory, theory of hedging, option pricing theory, and term structure of interest rates and so on.

3. The Application of Mathematical Modeling in Financial Investment Risk and Revenue

Risk is considered to be the deviation from the expected value or the possibility of the average caused by the fluctuations of interest rates, exchange rates, commodity prices, stock price[7]. Therefore, the risk measurement has become a major component of financial engineering. The commonly used mathematical methods of measuring financial risk can be divided into deterministic mathematical methods and mathematical methods of uncertainty.

3.1Deterministic Mathematical Methods

This method abstracts the various factors and indicators of financial investment risk into deterministic mathematical variables and further abstracts the interrelationships between them into mathematical formulas, mathematics Functional or mathematical model, and then obtains numerical results by mathematical calculus. Based on these calculations, people can measure and assess the risk of financial investment, adjust and control financial transactions in order to achieve the purpose of preventing financial investment risks.

Bond yield, bond price, stock price and stock index are common indicators of investment risk analysis. Commonly used mathematical methods are as follows:

(1) Assuming that the yield on the new bond is S, the denomination rate is r, the denomination is M, the issue price is N, the repayment period is T, the yield on the new bond is:

$$S=\frac{\mathrm{r}+\frac{M-N}{T}}{N}\times 100\% \quad (1)$$

(2) Set the face value of the bond as C, R for the investment rate, N for the bond period, I for the interest rate, F for each interest ($F=I\times C$), the bond price is:

$$M=\frac{C}{(1+R)^{N}}+\sum_{k=1}^{\mathrm{n}}\frac{F}{(1+R)^{K}} \quad (2)$$

(3) Set the stock price as M, the expected dividend income as Q, the market interest rate as r, the stock par value as p, the dividend yield as c, then the stock price is:

$$M=\frac{Q}{r}=\frac{p\times c}{r} \quad (3)$$

(4) Set the base period stock price as p_{0i}, Q_{0i} for the base period of circulation, p_{1i} for the reporting period stock price, Q_{1i} for the reporting period circulation, $i=1,2,\cdots,n$ for the total period of historical data, the reporting period circulation Q_{0i} as the weight, the weighted stock index E is:

$$E=\frac{\sum_{i=1}^{n}p_{1i}Q_{1i}}{\sum_{i=1}^{n}p_{0i}Q_{0i}}\times 100\% \quad (4)$$

By calculating some of the common financial investment risk analysis indicators in these financial investments, you can assess and control financial transactions and prepare for further selection of the right financial portfolio.

3.2Mathematical Methods of Uncertainty

From the definition of financial investment risk, we can see that the impact of various uncertain factors is the cause of risk. Therefore, the usage of deterministic mathematical methods is not sufficient to accurately describe these factors and their interrelationships. In such circumstances, the uncertainty of mathematical methods such as probability theory, mathematical statistics, stochastic processes, etc. certainly play a role in the prevention of financial investment risk research.

The most basic application of uncertainty mathematics in controlling financial investment risk is to abstract the possible loss or yield of the financial investment process as a random variable, and then use mathematical expectations and variance or standard deviation to measure the average loss or yield Volatility. If financial investment involves two or more financial commodities, it is necessary to introduce random vectors and their covariance and correlation coefficients to measure it.

Related mathematical method is:

Mind random variables X, and X have N historical data. Using these data and replace the probability with frequency, the probability distributions of X can be obtained: $p(x_1), p(x_2), \cdots p(x_n)$, $\sum_{i=1}^{n} p(\mathrm{x}_1) \equiv 1$,

（$n \le N$）

Mark mathematical expectation as $E(X)$,

$$E(X) = \sum_{i=1}^{n} x_1 p(x_1) \tag{5}$$

Mark standard deviation as $\alpha(X)$

$$\alpha(X) = \sqrt{E\left\{\left[X - E(X)\right]^2\right\}} \tag{6}$$

Mark random vector as (X,Y), and N sets of historical data $(x_i, y_i),\ i = 1,2,\cdots,N$

Mark covariance as $\mathrm{cov}(X,Y)$

$$\mathrm{cov}(X,Y) = E\left\{\left[X - E(X)\right]\left[Y - E(Y)\right]\right\} \tag{7}$$

Mark correlation coefficient as ρ_{xy}

$$\rho_{xy} = \frac{\mathrm{cov}(X,Y)}{\alpha(X)\alpha(Y)} \tag{8}$$

The measure of risk in a financial portfolio is the variance of the expected return. In the portfolio of n securities, mark the expected return on the portfolio as, mark the venture capital as σ^2. We can conclude it from the above uncertain method:

$$R = \sum_{i=1}^{n} w_i R_i \tag{9}$$

$$\sigma^2 = \sum_{i=1}^{n} w_i^2 \sigma_i^2 + \sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \rho_{ij} \sigma_i \sigma_j \tag{10}$$

where ω_i is the weight of the Type i securities in the portfolio, R_i is the expected return for Type i securities; σ_i and σ_j are the standard deviations of Type i securities and Type j securities; ρ_{ij} is the correlation coefficient between the i-type securities and the j-type securities, range $[-1,1]$.

From the above analysis and calculation, we can see that increasing the number of portfolio and the choice of negative securities portfolio can reduce the investment risk.

4. The Application of Mathematical Method in Financial Forecasting and Decision-making

There are many uncertain factors exist in financial activities, the forecastof future financial variables such as deposits, posted rates and inflation rateconcernsthat whether policy makers can make the right decision or not. the common mathematical methods in Financial forecasts are least squares, single and double moving average method, single and double and triple exponential smoothing method, modified index curve method and growth curve prediction method, linear regression method, the three-point method, Markov forecasting method, Kalman filtering, two-staged prediction method and so on.

The common Mathematical methods in financial decision making are extreme selection decision method, linear programming decision method, expected value method, marginal analysis method, the indifference curve method, maximum yield combination method, least-cost combination method and so on.

5. Conclusions

With rapid development of mathematical finance, there are still some problems existing in this filed, such as mathematical models of financial markets were premised on the assumption of rational expectations, this assumption thinks that market participants (institutions or individuals) can use and process all the information available, in addition to random factors, people are completely predictable. In practice, however, financial markets expectations are irrational, short-term expectations tend to the impact of speculation. In addition, the mathematical model of

financial markets is built in the new classical economics framework, and it is based on the market equilibrium model and linear mathematical methods, but financial market equilibrium is actually short, unequilibrium is absolute. Furthermore, the conclusion got by mathematical models and quantitative analysis method is semi-empirical and semi-rational, and so on. The existence of these problems causeda deviation of mathematical model calculation and actual price, it can fit well with historical data, but don't have a very high success rate in the future forecast.

It can be foreseen that the application of mathematics in the field of finance should be based on the existing theoretical framework, and enrich theoretical research methods and results, so the mathematical model can better explain the reality. In addition, it should also be based on the whole financial market and consider the fundamentals, technical, information and so on various aspects, with the analysis of combination of qualitative and quantitative, can we build the mathematical model to reflect the nature of the financial market system as much as possible, and then it can play an optimized role.

Financial mathematics has injected great impetus into the development of financial economy, and promoted financial theory, financial practice management and financial innovation. With the development of financial globalization, financial mathematics will become a wonderful work in the field of international finance, which is highly valued by the international financial and applied mathematics circles. We believe that financial mathematics will be more in-depth development and wider application in the 21st century. So financial mathematics is worth deeper study not only for institutions but also for individual researchers.

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