

# Graph Theory use in Transportation Problems and Railway Networks

Sanjay Kumar Bisen

Faculty of Mathematics, Government Post Graduate College, Datia (M.P.)  
(Affiliated to Jiwaji University Gwalior) India

**Abstract:** *One of the important issues in everyday life is optimization problem and reduction of the cost of distribution and transportation of goods. Researchers are always in line with this objective by providing search tools trying various approaches to minimize such cost. The purpose of this paper is to examine the problem and its solution to the modeling induced by graph theory. This provides the motivation for this paper in transportation problem and railway networks.*

**Keywords:** Graph Theory, Distance Balance Graph, Simple Graph, Multi Graph, Direct Graph, Null Graph

## 1. Introduction

During the last decades, graph theory has attracted the attention of many researchers. Graph theory has provided very nice atmosphere for research of provable technique in discrete mathematics for researches. Many application in the computing, industrial, natural and social science are studied by graph theory. It worth mentioning that all graph are usually classified when we encounter to special graph in modeling of phenomena in real life. The graph theory networks have always been important in transportation and telecommunication. They have become more important for all business today, especially because of the internet. The internet has connected virtually everything today. It has connected everybody, everything, everywhere into a network. Of course the internet has also changed how existing networks behave. Graph theoretic paper part 1 and part 2 discuss of certain transportation problem and railway networks.

### Use of Graph Theory in Transportation Networks

In solving problems in transportation networks Graph theory in mathematics is a fundamental tool. The term graph in mathematics has two different meaning. One is the graph of a function or the graph of a relation. The second usually related to 'graph theory' is a collection of 'vertices' or 'nodal' and 'links' or 'edges' for purpose of this paper we are concerned with the latter type graph theory has been closely tied to its applications and its use first can be credited to transport ant followed by its application to other fields. In transportation graph theory is most commonly used to study problems

**One way street problem: Robin's Theorem,** the first problem we consider has to do with movement of traffic. If traffic where to move more rapidly and with fewer delays in our cities, this would alleviate wasted energy and air pollution. It has sometimes been argued that making certain streets one-way would move traffic more efficiently. We consider the one-way and, if so, how to do it. Of course, it is always possible to make certain streets in a city one-way simply put up a one-way street sign. What is desired is to do in such a way that it is still possible to get from any place to any other place. Let us begin with that it is still possible to

get from any place to any other place. Let us begin with the simplified problem where every street is currently two-way and it is desired to make every street one-way in the future. We can formulate this problem graph theoretically by taking the street corners as the vertices of a graph. And drawing an edge between two street corners if only if these corners are currently joined by a two-way street. We wish to place a direction on each edge of this graph we speak of orienting each edge-so that in the resulting digraph. It is possible to go from any place to any other place.

### 1.1 Chinese Postman's Problem

In 1962, A Chinese mathematician called Kuan Mei-ko was interested in a postman delivering mail to a number of streets. Such that the total distance walked by the postman was as short possible. How could the postman ensure that the distance walked was minimum.

Following example: - A postman has to start at A, walk along all 13 streets and return to A. The numbers on each edge represent the Length, in meters, of each street. The problem is to find a train that uses all the edges of a graph with minimum Length

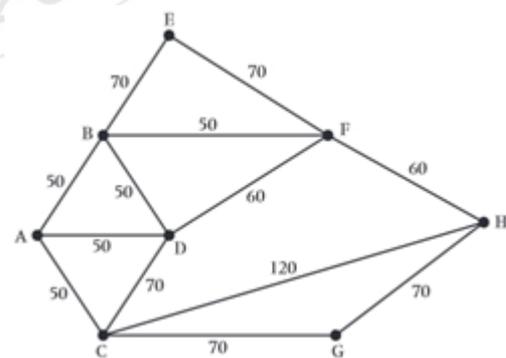
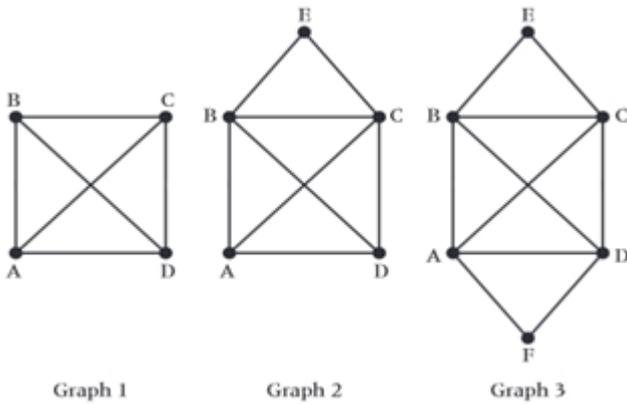


Figure 1

We will return to solving this actual problem later, but initially we will look at drawing various graphs. The Chinese postman is traversable graphs given below.



Graph 1

Graph 2

Graph 3

Vertex	Order
A	3
B	3
C	3
D	3

Graph 01

Vertex	Order
A	3
B	4
C	4
D	3
E	2

Graph 02

Vertex	Order
A	4
B	4
C	4
D	4
E	2
F	2

Graph 03

When the order of all the vertices is even the graph is Traversable. When there are two odd vertices we can draw the graph but the start and end vertices are different. When there are four odd vertices the graph can't be drawn without repeating an edge.

**Chinese postman algorithm :** An algorithm for finding an optimal Chinese postman route is.

**Step 1 :-** List all odd vertices.

**Step 2 :-** List all possible pairing of odd vertices.

**Step 3 :-** For each pairing find the edges that connect the vertices with the minimum weight.

**Step 4 :-** Find the pairing such that the sum of the weights is minimized.

**Step 5 :-** On the original graph add the edges that have been found in step 4.

**Step 6 :-** The length of an optimal Chinese postman route is the sum of all the edges added to the total found in step 4.

**Step 7 :-** A route corresponding to this minimum weight can then be easily found.

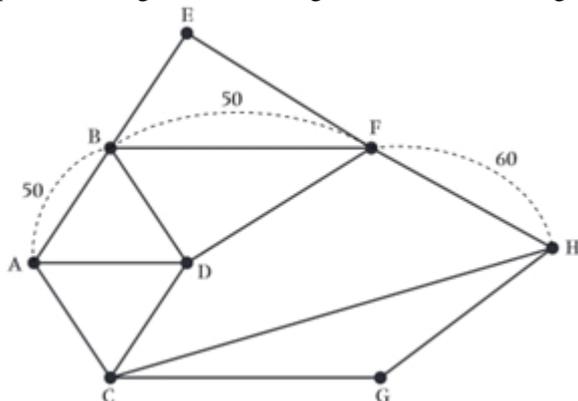
Now we apply the algorithm to the original problem in fig. 01.as;

**Step 01** the odd vertices are A and H.

**Step 02** there is only one way of pairing these odd vertices namely AH.

**Step 03** the shortest way of joining A to H is using the path AB, BF, FH a total length of 160.

**Step 04** these edges on to the original network in this fig



From these Graph we find;

- It is impossible to draw graph 1 without either taking the pen off the paper or re- tracing an edge.
- We can draw graph 2, but only by starting at either A or D-in each case the path will end at the other vertex of D or A.
- Graph 3 can be drawn regardless of the starting position and you will always return to the start vertex.

In order to establish the differences, we must consider the order of the vertices for each graph. The following

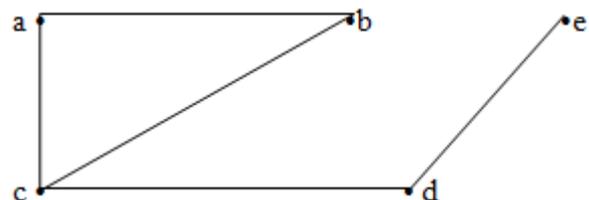
**Step 05** the length of the optimal Chinese postman route is the sum of all the edges in the original network. Which is 840 mtr Plus the answer found in step 4, which is 160 mtr., Hence the length of the optimal Chinese postman route is 1000 mtr.

**Step 06** one possible route corresponding to this length is ADCGH CABDFBEHFBA, but many other possible routes of the sum minimum length can be found.

### 1.2 Konigsberg bridge problem

The town of Konigsberg had seven bridges and its people wanted to know if one could start at some point, cross bridge exactly once and return to the starting point. L. Euler whose name has been credited for solving this problem translated it into graph theory problem.

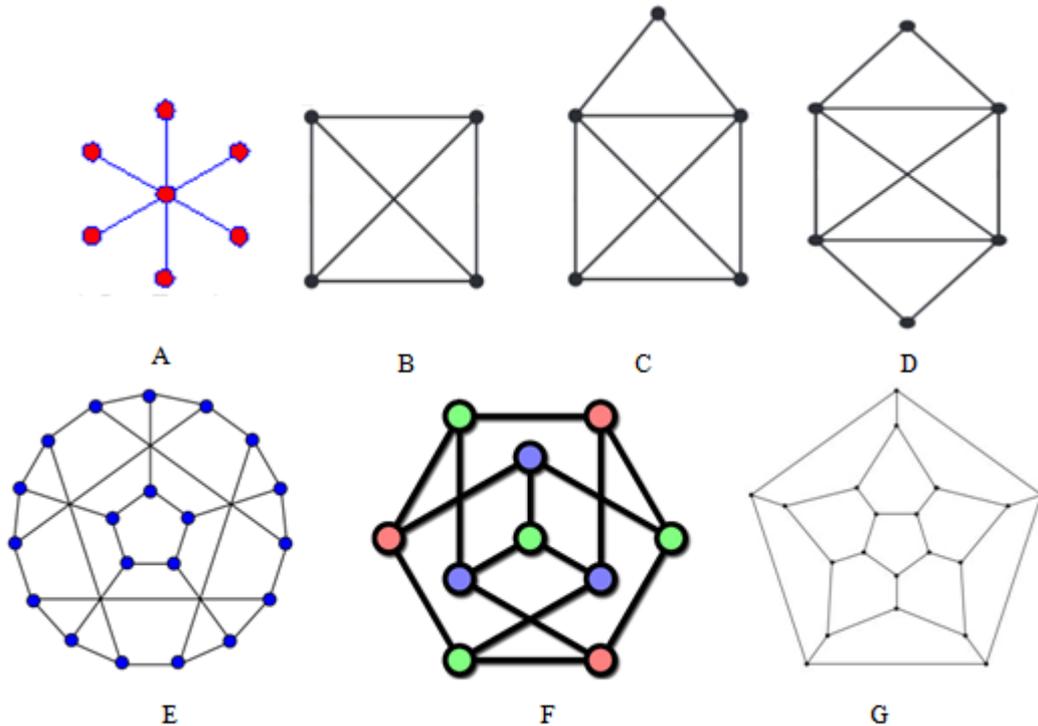
A graph 'G' in the above sense consists of two things : a set V whose elements are called 'vertices' of 'nodes' and a set E of unordered pairs of distinct vertices called 'edges'. It is commonly denoted as  $G(V, E)$ . A simple graph G below the set of vertices (points)  $V = \{a, b, c, d, e\}$  and the set of edges (lines)  $E = \{ab, ac, bc, cd, de\}$ .



If the edges have a sense of direction then it is usually referred to as a directed graph. Sometimes multiple connections between the same vertices are allowed and sometimes loops. Two vertices are said to be adjacent if there is an edge from one to the other. The degree of a vertex is the number of edges at that vertex. In the example above  $\text{deg}(a) = 2, \text{deg}(b) = 2, \text{deg}(c) = 3, \text{deg}(d) = 2, \text{deg}(e) = 1$ . If we add up the degrees from a graph we get twice the

number of edges because each edge gets counted twice, once for the vertex at either end.

There are some names used for special types of graph. “null graphs” with no edges, “complete graphs” with every vertex joined to every other vertex, “cycles” which only join the outside of the vertices.



## 2. Use of Graph Theory in Railway Networks

One of the most important users of graphs with respect to applications in railway signaling systems is the derivation of paths.

A railway network in special graphs called double vertex graphs. A user can edit the networks topology graphically. Every element of the graph hold various attributes. An edge for example holds a track sections length, gradient, maximum speed for different train categories and much more. A user can create and manage objects for edges and vertices and also signals, switches, stations and router fig. (01) shows an example for a station.

### 2.1 Network Data

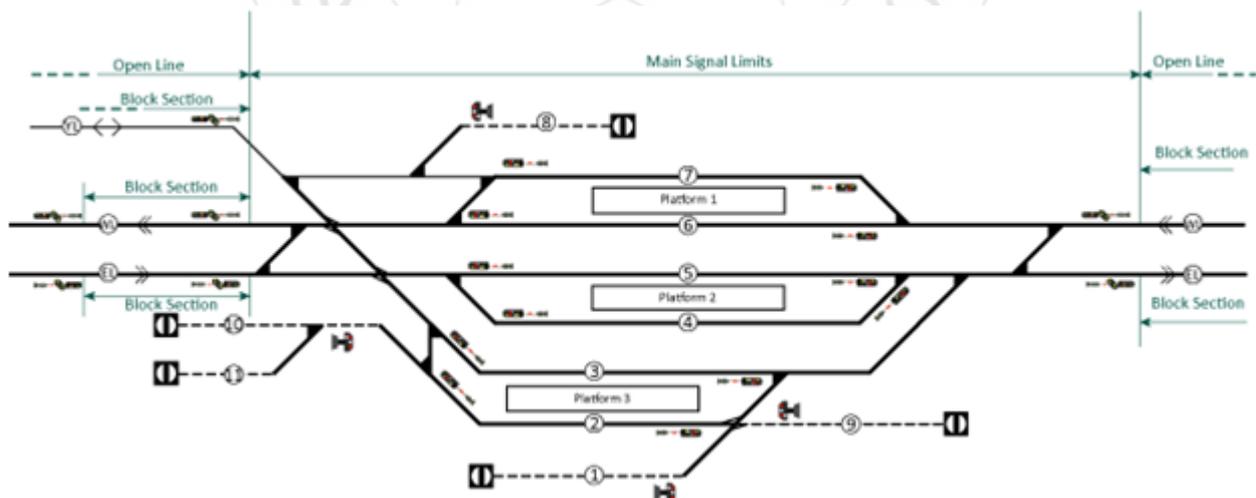


Figure 1

### 2.2 Time Table Data

The time table data base stores information for each train at each station, including arrival and departure times, minimal stop time and connections to other train. The differential equation for speed and distance are the basis for calculating a train’s movement. The signaling system of the railway

networks process constraints occupied tracks and restrictive signal aspects may impede a train’s progress. Every train continuously stores its speed acceleration, position, power consumption and other data. This data canbe evaluated after the simulation as show in fig. (02).

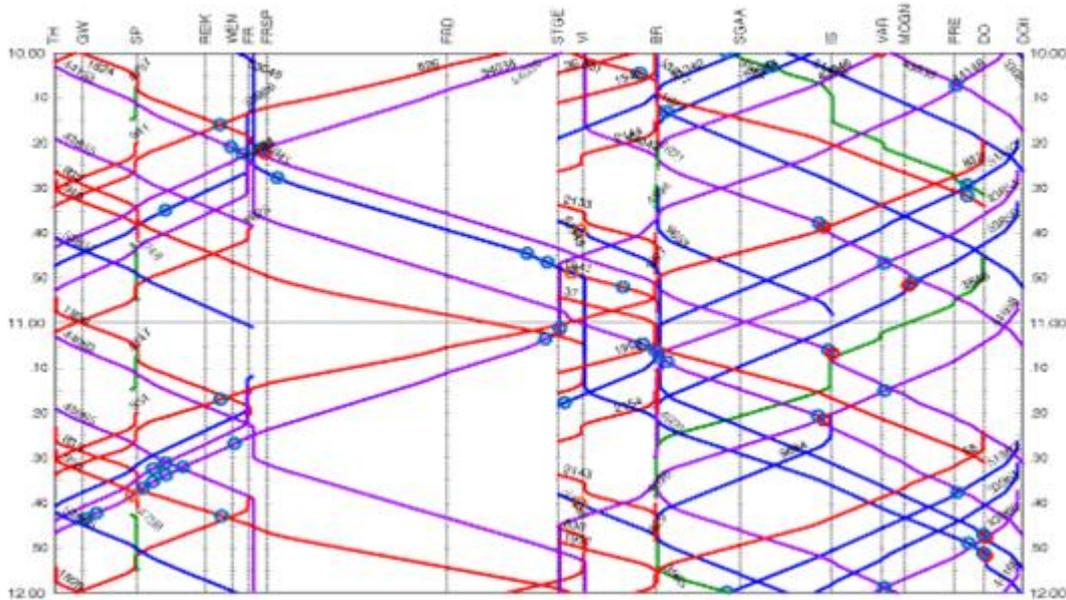


Figure 2

**Output data**

There is evaluation in the form of diagrams of train movement fig. (03), track occupation fig. (04) and line profiles. Every station produces output about all the trains. That used it, including arrival, stopping and departure times. The user can view output data in either a diagram or and excel table and ASC II table.

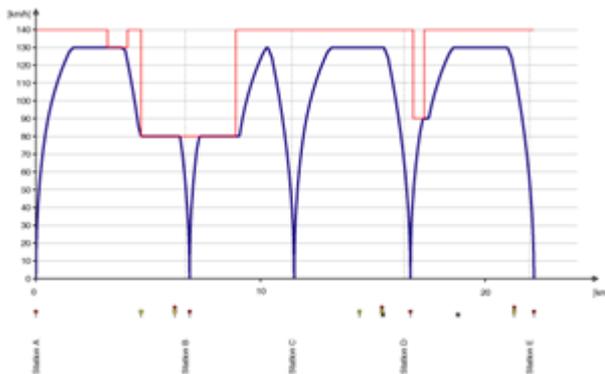


Figure 3

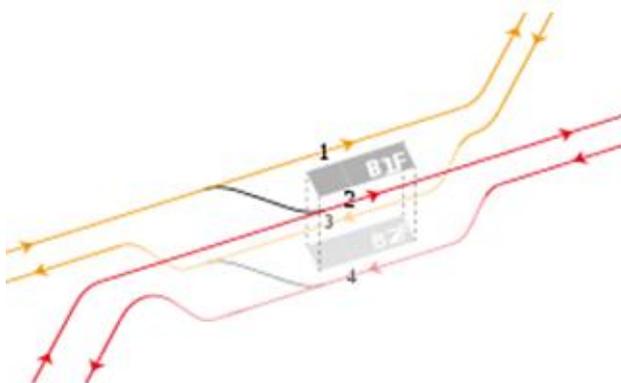


Figure 4

**3. Conclusions**

The main aim of this paper is to present the importance of graph theoretical idea in transportation networks. A Transportation Problems and Railway Networks are especially to project the idea of graph theory. Researcher may get some information related to graph theory and Transportation Problems and Railway Networks field and can get some ideas related to their field of research. This paper is designed to benefit the students of computer science to gain depth knowledge of transportation problems and railway networks.

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