International Journal of Science and Research (IJSR)
ISSN (Online): 2319-7064
Index Copernicus Value (2015): 78.96 | Impact Factor (2015): 6.391

Performance Analysis and Comparison of Reduced Order Systems using Chebyshev polynomial, Improved Pole Clustering and Fuzzy C-means Clustering Techniques

Maneesh Kumar Gupta

Assistant Professor, Electrical Department, LDC Institute of Technical Studies,Allahabad, U.P., India

Abstract: ***A mixed method for model order reduction of a linar, single input-single output systemis presented. The denominator of the original system is reduceedby using modified pole clustering techniques and fuzzy C-means clustering algorithm. The Chebyshev polynomial has been used for reducing the numerator of higher order transfer function.Then the result has been compared for boththe reduction techniques.***

Keywords: Model order reduction, Chebyshev polynomial, Modified Pole Clustering, Fuzzy C-means Clustering techniques, Integral square error

1. Introduction

Higher order models are very complicated to use in real-time system. The higher order model is complex and difficult to handle because of computational problem. The reduced order model makes simpler for controlling the system, reduces the complexity and gives the best result[1-12]. Here, we take input-output relationship of the system in the form of transfer function.

The Proposed method is a mixed method of Fuzzy C-means Clustering and Chebyshev polynomial and other mixed method is Modified Pole Clustering and Chebyshev polynomial. In literature [13], Chebyshev polynomial used with the pole clustering method in fourth order. It is a low order system, Now we are checking in higher order system's example. We are also used to reduce denominator by Fuzzy C-Means Clustering method for checking performance with pole clustering method.In the first mixed method, Poles are clustered by fuzzy C-Means Clustering algorithm andnumerator coefficientreduce by Chebyshev polynomial, and second, denominator reduces by Modified Pole Clustering and numerator reduce by Chebyshev polynomial.

2. Problem Formulation

Let the single-input single-output (SISO) higher order transfer function of the systemis:

$$G(s)=\frac{N(s)}{D(s)}=\frac{a_o+a_1s+a_2s^2+.........+a_ms^{n-1}}{b_0+b_1s+b_2s^2+.........+b_ns^n} \qquad (1)$$

Where a and b are scalar constants.

Let the corresponding reduce rth order model is

$$G_r(s)=\frac{\hat{N}(s)}{\hat{D}(s)}=\frac{d_o+d_1s+d_2s^2+.........+d_{r-1}s^{r-1}}{e_0+e_1s+e_2s^2+.........+e_rs^r} \qquad (2)$$

Where d and e are scalar constants.

3. Fuzzy C-Means Clustering

A large set of data are grouped into clusters of smaller sets of similar data is called Clustering of data [14]. Fuzzy c-means (FCM) is a method of clustering of data where any one piece of data allows to belongs more than one cluster.FCM method was developed by Dunn in 1973 and it was improved by Bezdek in 1981. It is based on minimization of objective function:

$$J=\sum_{i=1}^{n}\sum_{j=1}^{c}u_{ij}^{m}d_{ij}^{2} \qquad (3)$$

Where j is number of cluster, j=1,2,3,…c.
And i is the number of data set, i=1,2,3,… n and m is a weighting exponent, u_{ij} is a degree of membership.

$$d_{ij}^{2}=\left\|x_i-v_j\right\|_A^2 \qquad (4)$$

d_{ij} is the Euclidean distance between data points x_i to cluster center v_j and A is a symmetrical positive definite matrix, which means A=I matrix.

Updating the membership grades By iteration method, FCM iteratively moves the cluster centers to the right location in a data set.The initial centers are more important. For a durable approach, using an algorithm to determine all the cluster centers. FCM run several times,each starting with different value of membership grades of data.

If k pairs of complex conjugate poles in the cluster, using a same FCM algorithm for both real and imaginary parts individual the cluster is obtained as

$$[(\alpha_1\pm i\beta_1),(\alpha_2\pm i\beta_2),.....(\alpha_k\pm i\beta_k)]=\alpha_c\pm i\beta_c \qquad (8)$$

Where α_c and $ß_c$ are the cluster center of real and imaginary poles, respectively.

4. Chebyshev Polynomials

The chebyshev polynomial[5] and [13].Of the first kind are denoted by $T_n(y)$, where y of degree n which is defined as: $T_n(y)=\cos(n\cos^{-1}y)$

$y \in [-1,1]$.

This polynomialis recursively expressed as $T_{n+1}(y) = 2yT_n(y) + T_{n-1}(y)$ (9)

Where $T_0(y)=1$ and $T_1(y)=y$

Expressing the Chebyshev polynomials as functions of y by equation 9 as:

$$T_2(y) = 2y^2 - 1,\quad T_3(y) = 4y^3 - 3y,\quad T_4(y) = 8y^4 - 8y^2 + 1 \quad \ldots \tag{10}$$

And also expression of reciprocal Chebyshevpolynomial by equation 9 and 10.

$$y^2 = 1/2[T_0(y) + T_2(y)]$$
$$y^3 = 1/4[3T_1(y) + T_3(y)]$$
$$y^4 = 1/8[3T_0(y) + 4T_2(y) + T_4(y)] \quad \ldots \tag{11}$$

Procedure for Reduction

Step 1: The denominator polynomial(s) of the reduced order model is derived by Fuzzy C-means clustering technique.

Step 2: Then the denominator $\hat{D}(s)$ is factories

$$\hat{D}(s) = \sum_{i=0}^{r} e_i s^i = \prod_{k=1}^{r} (s^k - p_k) \tag{12}$$

Where p_k is the reduce order poles.

Put s= jω then $\bar{D}$ (jω) Complex conjugate of D(jω), So that we can find

$D(\omega^2)=\bar{D}(j\omega)D(j\omega)$,

$$\hat{D}(\omega^2) = \prod_{k=1}^{r} (\omega^2 - p_k^2) \tag{13}$$

step 3: The function $\hat{D}$ (ω^2) is convertedinto chebyshev polynomial by equation 11.

$$\hat{D}(T(\omega^2)) = \sum_{i=0}^{r} \beta_i T_{2i}(\omega^2) \tag{14}$$

Step 4: Now The original n^{th}order system can be expressed as

$$G(s) = \frac{N(s)}{D(s)} = \zeta \frac{\prod_{i=1}^{n-1}(s - z_i)}{\prod_{i=1}^{n}(s - p_i)} \tag{15}$$

Where p_i and z_i are the poles and zeros of higher order system G(s), respectively and ζ is constant.

Step 5: Same as, we derived:

$G(\omega^2)=G(j\omega).\bar{G}(j\omega)$

$$G(\omega^2) = \zeta^2 \frac{\prod_{i=1}^{n-1}(\omega^2 - z_i^2)}{\prod_{i=1}^{n}(\omega^2 - p_i^2)} \tag{16}$$

Step 7: The function $G(\omega^2)$ is expanded into power series around $\omega^2=0$ as

$$G(\omega^2) = \sum_{i=0}^{\infty} C_i \omega^{2i} \tag{17}$$

Step 8: The function G (ω^2) is converted into Chebyshev polynomials as

$$G(T(\omega^2)) = \sum_{i=0}^{\infty} c_i T_{2i}(\omega^2) \tag{18}$$

Step 9:Then reduced order $\hat{G}$ (ω^2) can be expressed by the same method. After obtaining separate expansions in Chebyshev polynomial for numerator and denominator, the $\hat{G}(\omega^2)$ can be written as:

$$\hat{G}(T(\omega^2)) = \frac{\hat{N}(T(\omega^2))}{\hat{D}(T(\omega^2))} = \frac{\sum_{i=0}^{r-1}\alpha_i T_{2i}(\omega^2)}{\sum_{i=0}^{r}\beta_i T_{2i}(\omega^2)} \tag{19}$$

Step 10: reduce the order denominator$(T_{\hat{D}}(\omega^2))$ is already obtained by the equation 11. Then polynomial $\hat{N}$ $(T(\omega^2))$ is obtained by

$$\frac{\sum_{i=0}^{r-1}\alpha_i T_{2i}(\omega^2)}{\sum_{i=0}^{r}\beta_i T_{2i}(\omega^2)} = \sum_{i=0}^{\infty} c_i T_{2i}(\omega^2)$$

$$\sum_{i=0}^{r-1}\alpha_i T_{2i}(\omega^2) = \sum_{i=0}^{\infty} c_i T_{2i}(\omega^2) \sum_{i=0}^{r}\beta_i T_{2i}(\omega^2) \tag{20}$$

Step 11: Then comparing the coefficient of $T_{2i}(\omega^2)$ then we get

$$\alpha_0 = c_0\beta_0 + \frac{1}{2}\sum_{p=1}^{r} c_p \beta_p$$

and

$$\alpha_i = c_i\beta_0 + \frac{1}{2}c_0\beta_i + \frac{1}{2}\sum_{s=1}^{r}[c_{|s-i|} + c_{|s+i|}]\beta_s \tag{21}$$

$i=1,2,3,\ldots$

Using the equation 21, obtained the numerator parameter.

$$\hat{N}(T(\omega^2)) = \sum_{i=0}^{r-1}\alpha_i T_{2i} \tag{22}$$

Step 12: The numerator parameter converts in model parameter by equation 9 and 10 as:

$$\hat{N}(\omega^2) = \prod_{k=1}^{r-1}(\omega^2 - Z_k)$$

$$\hat{N}(s) = \sum_{k=0}^{r-1} e_k s^k \tag{23}$$

Step 13: now consider the gain adjustment factor

$$\psi = \frac{\hat{D}(s)}{\hat{N}(s)}\Big|_{s=0} = 1/\hat{G}(s)\Big|_{s=0} \tag{24}$$

Finally, we obtain the transfer function of the reduced order model.

$$\tilde{G}(s) = \psi.\hat{G}(s) \tag{25}$$

5. Improved Pole Clustering

Sinha and Pal [3] used the inverse distance measure(IDM) criterion for solving the pole clusters from the poles of the original system. The poles of the higher order system may be all real, all imaginary or a combination of complex poles. It solved individually as equation 8. Follow this step of modified pole clustering techniques:

Step 1: first, solve pole centers

$$p_c = \left[\left(\sum_{i=1}^{k} -1/|p_i| \right) \div k \right]^{-1} \qquad (26)$$

Step 2: Set c=c+1

Step 3: Find a modified cluster centre from

$$p_c = \left[\left(\frac{-1}{|p_1|} + \frac{-1}{p_{c-1}} \right) \div 2 \right]^{-1} \qquad (27)$$

Step 4: Is k=c? if No, and then go to step 4, otherwise go to step 5.

Step 5: Modified cluster centre of the k^{th} cluster as $p_k = p_c$

Where k is the number of pole, p is value of pole and p_c is pole cluster center. Pole is arrange in ascending order as

$|p_1|<|p_2|<|p_3|< \ldots\ldots | p_k|$.

6. Numerical Example

Consider aeighth order model [14].

$$G(s) = \frac{18s^7 + 514s^6 + 5982s^5 + 36380s^4 + 122664s^3 + 222088s^2 + 185760s + 40320}{s^8 + 36s^7 + 546s^6 + 4536s^5 + 22449s^4 + 67284s^3 + 118124s^2 + 109584s + 40320} \qquad (28)$$

CHEBYSHEV AND FCM CLUSTERING:

It is reduced by Fuzzy C-Means clustering and Chebyshev polynomial. Use the steps for obtaining reduced model. Follow us:

Step 1: Pole of the system G(s) is -1,-2,-3,-4,-5,-6,-7,-8. The Denominator is reduce by fuzzy C-means clustering. Then pole of reduce system is -2.3984, -6.6024.

$$\hat{D}(s) = s^2 + 9s + 15.835 \qquad (29)$$

Step 2:written in the form of complex conjugate.

$$\hat{D}(\omega^2) = \omega^4 + 49.344\,\omega^2 + 250.751 \qquad (30)$$

Expanded denominator polynomial into Chebyshev polynomial by equation 11 is

$$\hat{D}(T(\omega^2)) = 0.125\,T_4(\omega^2) + 25.172\,T_2(\omega^2) + 275.8T_0(\omega^2) \qquad (31)$$

Step 3: The complex conjugate of theoriginal net order system can be expressed as an equation (16)

$$G(\omega^2) = \frac{\omega^{14} + 5.57e7\omega^{12} + 7.76e14\omega^{10} + 1.081e17\omega^8 + 5.442e18\omega^6 + 1.1924e20\omega^4 + 9.79e20\omega^2 + 9.873e19}{\omega^{16} + 204\omega^{14} + 16422\omega^{12} + 669188\omega^{10} + 1.474e7\omega^8 + 1.737e8\omega^6 + 1.02e9\omega^4 + 2.483e9\omega^2 + 1.626e9} \qquad (32)$$

Expanded in Chebyshev polynomial as

$$G(T(\omega^2)) = \frac{6.35e20T_0(\omega^2) + 5.52e20T_2(\omega^2) + 1.595e19T_4(\omega^2) + 1.768e17T_6(\omega^2) + 8.596e14T_8(\omega^2) + 1.515e12T_{10}(\omega^2) + 27197T_{12}(\omega^2) + 1.22e-4T_{14}(\omega^2)}{3.307e9T_0(\omega^2) + 1.838e9T_2(\omega^2) + 1.629e8T_4(\omega^2) + 6.41e6T_6(\omega^2) + 128756T_8(\omega^2) + 1405.5T_{10}(\omega^2) + 8.37T_{12}(\omega^2) + 0.0254T_{14}(\omega^2) + 3.052e-5T_{16}(\omega^2)} \qquad (33)$$

The function $G(\omega^2)$ is expanded into power series

$$G(s) = 19.1977e10T_0(\omega^2) + 6.0185e10T_2(\omega^2) - 3.8087e10T_4(\omega^2) + 1.788e10T_6(\omega^2) - 8.187e9T_8(\omega^2). \qquad (34)$$

Then comparing the coefficient of $T_{2i}(\omega^2)$ by equation (20) and (21) then find numerator polynomial

$$\hat{N}(T(\omega^2)) = 21.43e12T_2(\omega^2) + 52.947e12T_0(\omega^2) \qquad (35)$$

Translated into model parameter by equation 23 as

$$\hat{N}(\omega^2) = 42.863\,e12\,\omega^2 + 31.515\,e12$$

$$\hat{N}(s) = 6.547\,e6\,s + 5.6138\,e6 \qquad (36)$$

Now we are finding gain adjustment factor by equation (24) obtain as:

$$\Psi = 15.8352/5.6138e6 = 2.821e-6 \qquad (37)$$

We got transfer function of the reduced order system

$$\tilde{G}(s) = \psi\hat{G}(s) = \frac{18.4676s + 15.835}{s^2 + 9s + 15.835} \qquad (38)$$

CHEBYSHEV AND IMPROVED POLE CLUSTERING:

We consider same example and numerator reduce by Chebyshevpolynomial, anddenominator reduces by pole clustering method. Then pole of reduce system is

$P_1 = 1.0637$

$P_2 = 5.1327$

$$\hat{D}(s) = s^2 + 6.1964\,s + 5.4597 \qquad (39)$$

The denominator converts in Chebyshev for obtaining the numerator by same above method:

$$\hat{D}(T(\omega^2)) = 0.125T_4(\omega^2) + 14.238T_2(\omega^2) + 43.92T_0(\omega^2) \qquad (40)$$

By equation (15), expanded in Chebyshev polynomial as

$$G(T(\omega^2)) = 0.552T_0(\omega^2) - 0.0861T_2(\omega^2) + 0.0198T_4(\omega^2) - 0.0048T_6(\omega^2) + \ldots\ldots \qquad (41)$$

Then comparing the coefficient of $T_{2i}(\omega^2)$ with equation(34) by same above methods

$$\hat{N}(T(\omega^2)) = 5.3767e12T_2(\omega^2) + 8.4316e12T_0(\omega^2) \qquad (42)$$

$$\hat{N}(s) = 3.279\,e6s + 1.748\,e6 \qquad (43)$$

Translated into model parameter by equation 20 and 21 as

$$\tilde{G}(s) = \frac{10.241\,s + 5.4597}{s^2 + 6.1964\,s + 5.4597} \qquad (44)$$

We got transfer function of the reduced order system

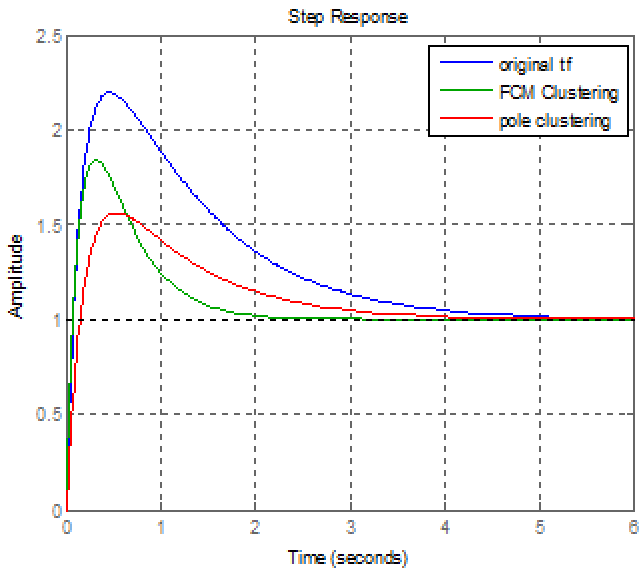

Figure 1: Comparison of Step responses of system

7. Comparison of Methods

The performance comparison of both the proposedalgorithm is given in Table 1.The step responses of both reduce order models are compared with the original model are shown in Figure 1.In theTable 1,comparingthe integral square error of transient part of step response isin between original and reduced order models. The ISE is

$$ISE = \int_0^{\infty} [y_o(t) - y_r(t)]^2 dt \qquad (45)$$

Where y_o and y_r are unit step response of original reduce order system respectively.

8. Conclusion

An algorithm which combined the advantage of two systems with Chebyshev polynomial, one is fuzzy C-means clustering and other is improved pole clustering. This methodchecked by an example, which reduced the numerator by Chebyshev polynomialand denominator reduced by Fuzzy c-means clustering. Now the second method apply on same example, which reduced the numerator by Chebyshev polynomial and denominator reduced by the improved pole clustering method. The Result ofStep response and ISE error show in figure 1 and table 1respectively. ISE of pole clustering method is minimum than fuzzy c-means clustering method. Reduced order model obtaineed is stable, mathematically simple, as well as in quality.

Method of order reduction	*Reduced model*	*ISE*
FCM clustering	$\tilde{G}(s) = \dfrac{18.4676s + 15.835}{s^2 + 9s + 15.835}$	0.540
Improved Pole clustering	$\tilde{G}(s) = \dfrac{1.0241s + 5.4597}{s^2 + 6.1964s + 5.4597}$	0.467

References

[1] Dharmendra Singh, Om Prakash Gujela. "Performance Analysis of Time Moments, Markov'sParameters and Eigen Spectrum Using Matching Moments,"*Int. J. of Innovative Research in Computer and Com. Eng. Vol. 3, Issue 3, 2015.*

[2] Y. Shamash, "Stable reduced order model using padè type approximations," *IEEE Trans. Automatic control, vol. 19 pp. 615-616, 1974.*

[3] K. Sinha, J. Pal, "simulation based reduced order modeling using a clustering techniques," *computer and electrical engg. 16(3), 159-169, 1990.*

[4] R. Parthasarathy and K. N. Jayasimhaj. "System reduction using stability-equation method and modified cauer continued fraction" proceedings of the IEEE, vol. 70, no. 10, 1982.

[5] Y. Bistritz andG. Langholz. "Model Reduction by Chebyshev Polynomial Techniques" IEEE Trans. On automatic control, vol ac-24, no. 5, *1979.*

[6] N. Sai Dinesh, M. Siva Kumar, SrinivasaRao, "order reduction of discrete time system using modified pole clustering technique," Int. J. of Eng. Research and Apps., vol. 3pp.565-569, 2013.

[7] V. Singh, D. Chandra and H. Kar, "Improved Routh Pade approximants: A Computer aided approach", IEEE Trans. Autom. Control, 49(2), pp.292-296, 2004

[8] .R. Prasad and J. Pal, "Stable reduction of linear system by continued fractions," Jr. Inst. Eng. India, vol. 72, 113-116, 1991.

[9] Shamash Y, "Stable reduced order models using Pade type approximants", IEEE Trans. Autom. Control, Vol.AC-19, No.5, pp.615-616, October 1974.

[10] R. Prasad and J. Pal, "Stable reduction of linear systems by continued fractions," Jr. Inst. Eng. India, IE(I)J.EL, vol. 72, pp. 113-116, 1991.

[11] L. S. Shieh and Y. J. Wei, "A mixed method for multivariable system reduction," IEEE Autom. Contr., vol. 20, pp. 429-432, 1975.

[12] G. Parmar, R. Prasad and S. Mukherjee, "order reduction of linear dynamic systems using stability equation methods and GA," world academy of science, eng. And tech., 2007.

[13] V. P. Singh, Prateek Chaubey and D. Chandra, "model order reduction of continuous time systems using pole clustering and chebyshev polynomial", IEEE 978-1-4673-0455,9/ 2012

[14] Anirudha Narain, Dinesh Chandra, and R. K. Singh "Model order reduction using fuzzy C-means clustering" trans. of institute of measurement and control, 2014.

[15] B. Vishwakarma, "Order reduction using modified pole clustering and padè approximation," world academy of science, eng. and tech 56, 2011

[16] Awadhesh Kumar and D. Chandra, "improved padè pole clustering approximant" International Conference on Computer Science and Electronics Engineering.

[17] J. C. Mason and D. C. Handscomb, Chebyshev Polynomials, Chapman & Hall/CRC, 2003.