

Cepstrum to Abate Noise in OFDM Symbols

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Abstract: *The ability of cepstral analysis to reveal periodicities in a signal resulted in its widespread use in signal processing applications. The paper focuses on the application of cepstrum on cancellation of noise in OFDM systems by using CRC. Cyclic prefix insertion induces periodicities in OFDM symbols. Cyclic Prefix mitigates completely the ISI and ICI in OFDM symbols. By cepstral analysis one could estimate the cyclic prefix length. Using this one could decide the length of redundancy bits for Cyclic Redundancy Check for error correction. To study the effect of frequency offset on subcarrier variance, the noise power is estimated using a 2D sliding window. The instantaneous error in estimating the noise power determines the window size, thereby elaborating the importance of cyclic prefix in preserving sub-carrier orthogonality. The cepstral analysis of OFDM symbol is generic in nature and can be used for other applications, such as analysis of the signals between the circuit components in larger circuits.*

Keywords: OFDM, Cepstrum, Frequency, Orthogonality, Carrier frequency Offset

1. Introduction

The growing demand of wireless applications and rigid spectrum allocation policy has put a lot of limitations on the available electromagnetic radio spectrum (3Hz-3000GHz). Modern wireless systems offering a wide variety of high data rate applications to various users at the same time, observation shows that usage of the spectrum is actually quite low. The underutilization problem was solved by the invention of Cognitive Radios. In such applications, acquiring awareness of the spectrum state such as identifying idle spectrum and classifying different users in the network is crucial. Cognitive Radio is an adaptive multi-dimensionally aware intelligent wireless communication system with the objectives of highly reliable communications whenever and wherever needed and efficient utilization of radio spectrum. Cognitive radios can sense its operational electromagnetic environment and can dynamically and autonomously adjust its radio operating parameters to modify system operation. The man-made signals are periodic; therefore the capability to distinguish signals with different periodicities is very useful in spectrum sensing.

The present as well as future wireless systems are benefited by the popular multicarrier modulation method of encoding digital data on multicarrier frequencies, Orthogonal Frequency Division Multiplexing (OFDM). The presence of Cyclic Prefix (CP) in OFDM signals induces periodicities. Therefore the cepstrum can be used to detect periodicities in OFDM caused by the presence of Cyclic Prefix (CP). For an OFDM system, the orthogonality among the sub-carriers is maintained only if the receiver uses a local oscillation signal that is synchronous with the carrier signal contained in the received signal. The mismatch in carrier frequency can cause Inter Carrier Interference (ICI). The effect of frequency offset resulting in Inter Carrier Interference (ICI) while receiving an OFDM modulated symbol has to be evaluated. In real time, noise affecting the OFDM spectrum unevenly, made up of white Gaussian noise along with correlated colored noise. Therefore taking into account the variation of

the noise statistics across the OFDM sub-carrier index, the noise power has to be estimated. The noise can be canceled from the periodic signals by using Cyclic Redundancy Check. CRC uses cyclic codes to correct errors. In CRC, if a code word is cyclically shifted or rotated, the result is another code word.

Our proposed method is based on including CRC to revoke noise in OFDM symbols, that has been proven periodic due to the presence of CP by means of cepstral analysis, along with it a study on the significance of addition of CP by exploring the impact of frequency offset resulting in ICI and estimation of noise power and noise variances at each subcarrier using a 2D sliding window.

In section 2 we describe the method overview and in section 3 we present the technique of cepstral analysis. In section 4 the impact of frequency offset on orthogonality of subcarriers is explained. In section 5 we describe the basic idea of OFDM and Cyclic Prefix the methods to compute the CP length. In section 6 we describe the method of error detection using Cyclic Redundancy Check. In section 7 we describe our experimental results. Finally in section 8 we present our conclusions and future work.

2. Method Overview

The input data signal is converted to OFDM symbol (combination of zeros and ones). The OFDM symbol is generated as some sort of random signal that is its CP length is unknown. The OFDM symbol is subjected to cepstral analysis, the mean for the symbol distributions are determined. CP length estimated from the cepstrum. Based on the CP length, the length of redundancy bits stream can be generated. These redundancy bits are used for error correction using cyclic codes. By appending the cyclic prefix the ICI and ISI are mitigated. Thus Carrier Frequency Offset can be avoided.

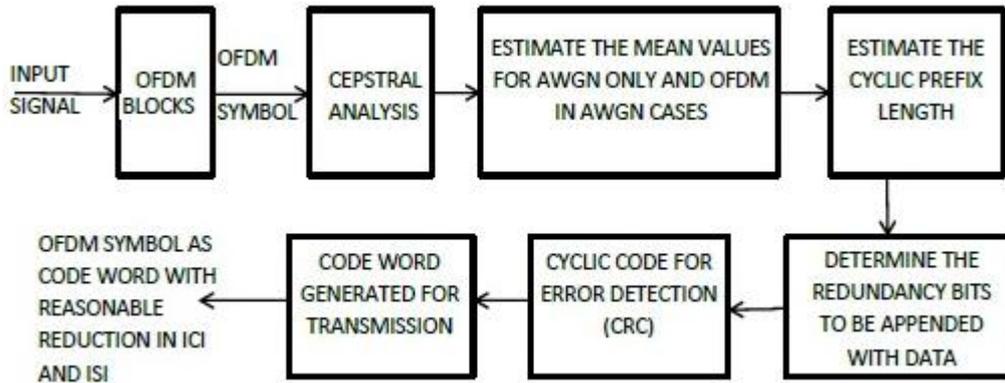


Figure 1: Method Overview

3. Cepstral Analysis

In signal processing two signals, signals can be combined in two ways:

- By Linear Addition (superposition principle)
- By Convolution

The signals combined by addition are transformed to frequency domain, then by applying a linear filter the signals can be separated. The frequency domain is easy to plot and conveys the information that one can find in a time domain plot. Advantage of the frequency domain is that we can immediately see the values of the frequency and peak amplitude. The convolved signals could not be easily separated, if so it leads to noise amplification. So to convert the signal obtained from convolution of two signals into the sum of two signals cepstrum can be used. Cepstrum is defined as the inverse discrete Fourier transform (IDFT) of the logarithm of the magnitude of the discrete Fourier transform of a signal.

$$c(n) = F^{-1}\{\log|F\{x(n)\}|\} \quad (1)$$

Log operation emphasize on the periodicity, since log denotes harmonics. In fact, the name ‘cepstrum’ comes from inverting the first syllable of the word ‘spectrum’. Similarly, the variable ‘n’ in c(n) is called ‘quefrequency’, which is the inversion of ‘frequency’. The properties of cepstrum are: (a) it has infinite duration, even if x(n) has finite duration and (b) it is real if x(n) is real. It is accurate and efficient. Advantages of cepstral analysis are simple algorithm, compactness and orthogonality. In the case of OFDM the distribution of OFDM for two cases has been derived

(i) AWGN only and

$$c(n) \sim \begin{cases} N\left(\frac{\log(\sigma_\omega^2) - \sqrt{N_r}}{2}, \frac{\pi^2}{24}\right) & n \neq 0 \\ N_c\left(0, \frac{\pi^2}{24}\right) & n = 0 \end{cases} \quad (2)$$

(ii) OFDM signal in AWGN

$$c(n) \sim N_c\left(\mu_c(n), \sigma_c^2(n)\right) \quad (3)$$

Equations (2) and (3) from [1] c(n) is the distribution of cepstral coefficient, shows that in addition to the DC peak, the mean of OFDM cepstrum has distinct peaks at integer multiple indices, number of samples in the data part of

OFDM symbols whereas the mean and variance of AWGN only cepstrum are constant-valued[1]. Cepstrum of OFDM abundant features and can be used to the parameters of OFDM. Thus we can demonstrate using cepstrum that the OFDM symbol is periodic in nature, thereby distinguishing it from AWGN signal.

4. Impact of Frequency Offset on Orthogonality

Carrier Frequency Offset (CFO) being one of the many non-idealities in baseband receiver design, often occurs when the local oscillator signal in the receiver does not synchronize with the carrier signal contained in the received signal. The presence of frequency offset results in loss of signal to noise ratio (SNR). The CP needs to be chosen optimally due to the loss of SNR and the bandwidth efficiency. Irrespective of current channel state the conventional OFDM uses a large and fixed CP length to tolerate worst case channel condition. In OFDM, the CP controller should attach the CP of adaptive length to mitigate both ISI and ICI introduced due to the channel. The noise variances at each subcarrier can be estimated using a two dimensional sliding window. These estimates can be used to detect and avoid narrowband interference. The common approach for estimation of noise power in OFDM systems is given by the difference between the noisy received sample in frequency domain and the best hypothesis of the noiseless received sample. It can be formulated as [3]

$$Z_{n,k} = Y_{n,k} - \hat{S}_{n,k} \hat{H}_{n,k} \quad (4)$$

A two dimensional sliding window is used to obtain the noise plus interference power. In this case, the estimate of the noise power at kth subcarrier of nth OFDM symbol can be written as [3]

$$\hat{\sigma}_{n,k}^2 = \frac{1}{L_t L_f} \sum_{l=n-\frac{L_t}{2}}^{n+\frac{L_t}{2}-1} \sum_{u=k-\frac{L_f}{2}}^{k+\frac{L_f}{2}-1} |Z_{l,u}|^2 \quad (5)$$

Estimation error at the kth subcarrier of nth OFDM symbol can be expressed as[3]

$$E(n,k) = \hat{\sigma}_{n,k}^2 - \sigma_{n,k}'^2 \quad (6)$$

This instantaneous error is a function of the window size.

5. OFDM & Cyclic Prefix

Orthogonal frequency division multiplexing (OFDM), digital modulation method of encoding digital data on multicarrier frequencies, the high bit rate serial bit stream is converted into parallel lower rate bit streams, then each bit stream is segmented into a group of bits called symbols. Cyclic prefix (CP) induces periodicities in OFDM signals. A continuous time signal is said to be periodic, if it completes a pattern within a time frame and repeats that pattern over identical subsequent periods. The sum of any number of signals, all of which are periodic with period T, is also periodic with the same time period. CP insertion implies that the last part of the OFDM symbol is copied and inserted at the beginning of the OFDM symbol. It is used to completely eliminate both Inter-Symbol Interference (ISI) and Inter-Carrier Interference (ICI) as long as the CP length is greater than the channel delay spread. But it consumes a considerable amount of the scarce spectrum and the power. CP is discarded at the receiver, since it is pure overhead and reduces throughput.

5.1 Calculation of Cyclic Prefix Length [2]

By keeping the subcarrier spacing fixed such that the useful symbol length (Tu) should be the inverse of the subcarrier spacing Δf across different channel bandwidth. Then, the CP length is (Tg = G*Tu), where G is (Tg/Tu) ratio. The G has been chosen according to channel parameters. The total OFDM symbol length consists of the useful symbol length and the CP length (TOFDM = Tg + Tu).

5.2 Calculation of CP Length using Cepstrum[1]

When an OFDM symbol is present the variances of the different cepstral coefficients follow a periodic pattern. By finding the highest correlation for different lags τ the period of triangular peaks is N_s samples given as[1]

$$N_s = \arg \max_{\tau \in A} \sum_{n=N_d}^{N_d+M_v} V_c(n)V_c(n+\tau) \quad (7)$$

The index of the highest real cepstral coefficient value excluding the DC peak can give the length of the payload data size N_d. The estimate of N_d is given by [1]

$$N_d = \arg \max_n R\{c(n)\}, n = 1, \dots, N_r \quad (8)$$

Therefore, the estimate of number of samples in the CP is given by[1]

$$N_c = N_s - N_d \quad (9)$$

6. Cyclic Redundancy Check

Data transmission can contain errors. The errors can be detected by sending more information with data that satisfies a special relationship or by adding redundancy frequencies. Cyclic Redundancy Check is a parity based error detection scheme. Cyclic Redundancy Check using Polynomials is preferred namely CRC-8 (generator polynomial $x^8 + x^2 + x + 1$). CRC is a powerful error detection

scheme. Rather than addition, binary division is used. It can be easily implemented with small amount of hardware. Let us assume k message bits and n bits of redundancy. The redundancy bits are of the Length of Cyclic Prefix. Then associate bits with coefficients of a polynomial. Let M(x) be the message polynomial, P(x) be the generator polynomial and P(x) is fixed for a given CRC scheme, also P(x) is known both by sender and receiver. Create a polynomial block F(x) based on M(x) and P(x) such that F(x) is divisible by P(x).

5.1 Sending

- Multiply M(x) by x^n
- Divide $x^n M(x)$ by P(x)
- Ignore the quotient and keep the remainder C(x)
- Form and send $F(x) = x^n M(x) + C(x)$

5.2 Receiving

- Receive F'(x)
- Divide F'(x) by P(x)
- Accept if remainder is 0, reject otherwise

7. Experimental Results

MATLAB R2015b is used as the implementation tool. The difference in mean for both the cases is evaluated from the cepstrum.

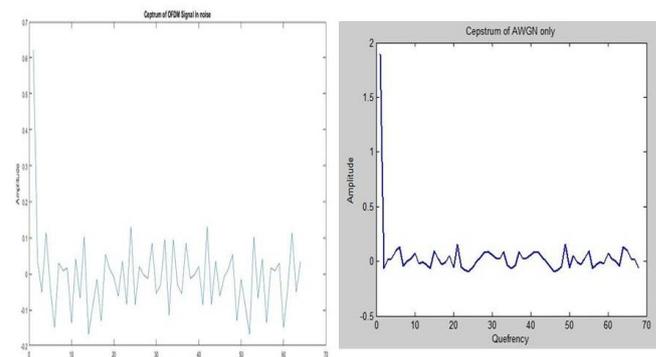


Figure 1: Cepstrum of OFDM signal in AWGN and AWGN only

Mean of AWGN signal 0.001787
 Mean of OFDM Signal with AWGN signal 0.023144

Figure 2: Showing variation in mean of signals

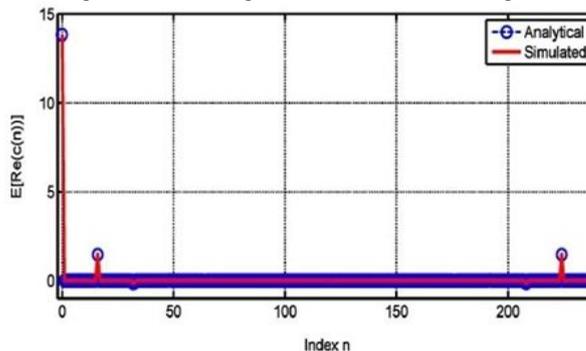


Figure 3: Analytical and simulated mean values of R(c(n))

The effect of frequency offset on sub-carrier variance is

studied and plotted. Figure 4 shows simulated results are better than theoretical results, because using average error for all subcarriers the simulated results are computed.

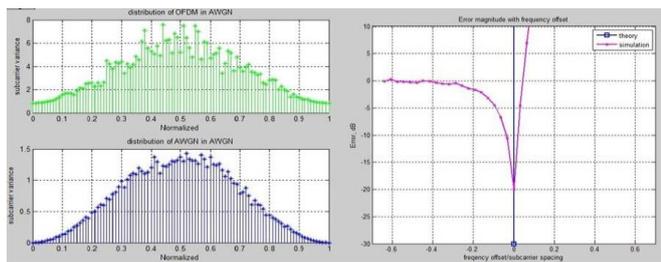


Figure 4: Demonstrated noise variance of sub-carriers and the effect of frequency offset with error.

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Corrupted_frame_code =
1C
Power of the Signal from Time domain 0.500000
Total Power of the Signal from DFT 0.500000
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Figure 5: Received code word and estimated power.

The window size of the noise subcarrier variance determined by the instantaneous error in estimating the noise plus interference power of OFDM symbol. Hence, the distribution of subcarrier variance for both the cases for which cepstral co-efficient has been derived is plotted in Figure 4.

The code word that has to be transmitted was generated as the part of including the Noise Cancellation module in to the work. At the receiver section, it would get divided by the complement of the polynomial block and accepted if the remainder is 0, otherwise rejected.

8. Conclusion

Cepstrum gives the information about the periodicities in signals. Using cepstral analysis more accurate estimate of the OFDM parameters can be obtained. Thus, the CP length can be estimated. By using the CP length, the lengths of the redundancy bits for CRC are determined. By the inclusion of CP the ISI and ICI can be mitigated. Thus the effects of frequency offset on sub-carrier variance are reduced. The system thus revoked the noise by using CP length obtained from cepstral analysis. The cepstral analysis of OFDM symbol is generic in nature and can be used for other applications, such as analysis the signals between the circuit components in larger circuits.

9. Acknowledgment

I would like to express profound gratitude to our Head of the Department, Asst. Prof. Ms. Sangeeta T.R., for her encouragement and for providing all facilities for my work. I express my highest regard and sincere thanks to my guide, Asst. Prof. Ms. Ann Nita Netto, who provided the necessary guidance and serious advice for my work.

References

- [1] J. Jantti, S. Chaudhari, and V. Koivunen "Detection and Classification of OFDM Waveforms using Cepstral Analysis", IEEE Transaction on Signal Processing vol.63, No.16, August 2015.
- [2] Amar Al-Jzari and Kostanic Iviva, "Cyclic Prefix Length Determination for Orthogonal Frequency Division Multiplexing System over Different Wireless Channel Models Based on the Maximum Excess Delay Spread", American Journal of Engineering and Applied Sciences, April 2015.
- [3] Tevfik Yucek and Huseyin Arslan, "Noise Plus Interference Power Estimation in Adaptive OFDM Systems".
- [4] R. Prasad, OFDM for Wireless Communications Systems. Norwood, MA, USA: Artech House, 2004.
- [5] J. Lunden, V. Koivunen, A. Huttunen, and H. Poor, "Collaborative cyclostationary spectrum sensing for cognitive radio systems," IEEE Trans. Signal Process., vol. 57, no. 11, pp. 4182–4195, Nov. 2009.
- [6] J. Jantti, S. Chaudhari, and V. Koivunen, "Cepstrum based detection and classification of OFDM waveforms," in Proc. IEEE Int. Conf. Acoust., Speech, Signal Process. (ICASSP), May 2014, pp. 8063–8067.
- [7] E. Axell, G. Leus, E. Larsson, and H. Poor, "Spectrum sensing for cognitive radio: State-of-the-art and recent advances," IEEE Signal Process. Mag., vol. 29, no. 3, pp. 101–116, 2012.

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