The Actual Speed Limit for Particles with Rest Mass Not Equal to Zero and the Highest Attainable Mass

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Abstract: *The actual velocity of objects/particles is not the speed of light, which is equal to c=299792458 meters per second, although the highest attainable mass is infinity.*

Keywords: Speed of light, Planck length, Planck length mass

1. Introduction

According to the Theory of Relativity, the speed limit for anything moving through space is equal to that of the speed of light when light moves in vacuum. Numerically, it is 299792458 meters per second (Wikipedia, Speed of light, 2017), and symbolically *c*. But, this speed limit possesses a problem as it, when used in calculations, results infinity. In other words, this speed is achievable only at infinity or never.

The other problem with this is that, although and because this speed limit is achievable at infinity, this allows object (or in general inertial mass) to acquire/change speeds of/by any amount.

This is a problem also because assuming the smallest possible length or distance, and which is according to quantum mechanics Planck length (approximately $1.616229e^{-35}$ meters (Wikipedia, Planck length, 2017)), the object cannot travel any distance at a given amount of time. So, the minimum distance any object can move is the Planck length. Also any distance is the integral multiple of the Planck distance. The thing to realize is that this limit in statement by theory of relativity is due to the limit in statement of quantum mechanics. Also, in the reverse way around, the limiting statement in the theory of relativity can serve for limits in quantum mechanics.

2. Solution

So, what is the actual speed limit for any object with some rest mass?

To answer this question let's get into a thought experiment. Let's consider a point A as in figure below. Now, let's consider another point B, which is 299792458 meters away from. Now, because of uncertainty principle, the position of anything cannot be measured exactly and the measurement must contain a least uncertainty of some value. This value is $\pm 1.616229e^{-35} meters$. So, the distance between points A and B, with uncertainty in consideration, is actually $299792458 \pm 1.616229e^{-35} meters$. But again, these points (points A and B) are the positions of the any object, and the object doesn't necessarily have inertial/rest mass but does obviously have mass-energy.
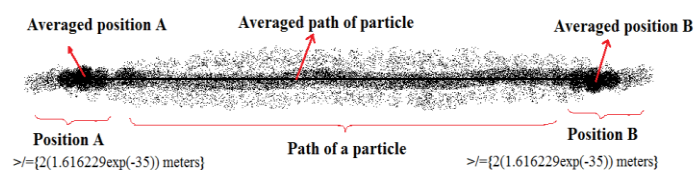

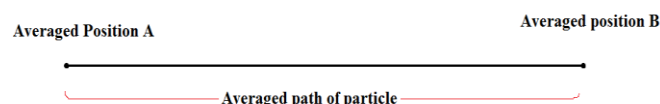Figure I: Actual **(quantum)** Position and path of particle in 1-Dimension.

Figure II: Averaged **(classical)** Position and path of particle in 1-Dimension.

And according to uncertainty principle, which in one form is:

$$\Delta x . \Delta p = \Delta x . \Delta(mv) = \Delta x . \Delta m \Delta v = \hbar$$

or we can take, $\quad \Delta x . m \Delta v = \hbar$

Where, Δx is uncertainty in position of particle/object.

Δv is uncertainty in velocity of particle/object.

m is the mass of the particle/object.

$\Delta p = \Delta(mv) = \Delta m \Delta v$ or taking $(m\Delta v)$ is uncertainty in momentum and $\hbar$ is called reduced Planck constant and which is equal to h/2π

Where again h= $6.62607004 \times 10^{-34}$ m^2 kg / s and π is approximately equal to 3.14159.

3. Mass and Uncertainty

As we can draw from the uncertainty principle above the uncertainty in position is inversely proportional to mass of the particle/object, as we can see from equation:

$$\Delta x . m \Delta v = \hbar \quad or, \Delta x = \frac{\hbar}{m \Delta v}$$

So, Δx proportional to $\frac{1}{mass\,(m)}$

Or, Δx inversely proportional to mass (m)

So, more the mass of a particle, the more it gets confined or can get confined and less the mass, less is the confinement. But, there is other parameter which is the is uncertainty in velocity (Δv) present with the mass inversely proportional to uncertainty in position (Δx). So, for smaller value of

uncertainty in position (or for more confinement), either mass of particle can increase or uncertainty in velocity can increase or both can increase. Also, uncertainty in mass (Δm) can increase, but we ignore it and take only the mass and uncertainty in velocity.

Also, in other way, the uncertainty equation can be written as:

$$\Delta v = \hbar/m\Delta v$$

So, Δv proportional to $\dfrac{1}{mass\,(m)}$

Or, Δv inveresly proportional to mass (m)

Also, uncertainty in velocity depends inversely on uncertainty in position. So, points A and B aren't actually points but are very small distances or lengths, on the least scales, $\{\pm1.616229e^{-35}$ meters along the line containing points A and B and with magnitude $2(1.616229e^{-35})$ meters} where the particles can be considered to move (actually jump) to and fro from one to another end at particular speeds that has the value which lies within the uncertainty in velocity value (Δv) and which depends on the uncertainty in position (Δx) and mass (m) of that particle.

4. The minimum mass in minimum space and the maximum mass in minimum space

So, there are only certain masses that can be confined into the length equal to $2(1.616229e^{-35})$ meters. We can calculate the least mass that can accommodate in this length. Taking the speed limit equal to speed of light (for approximate value to the least mass that can confine into the Planck length, l_p), any particle cannot overcome this speed limit, the max velocity of particle is c. Now, let the uncertainty of velocity of a particle be $\Delta v = c = 299792458\ ms^{-1}$ and uncertainty in position along length be $\Delta x = l_p = 1.616229e^{-35}$ meters .

So, $\Delta x.\Delta v \geq \hbar/m$

Or, $m \geq \hbar/(\Delta x.\Delta m)$

Or,

$m \geq (1.054571596e^{-34})/(1.616229e^{-35}.299792458)$

Or, $m \geq 2.17646893e^{-8}$ Kgs.

So, only the masses or particle with masses greater or equal to $2.17646893e^{-8}$ Kgs (lets call it plank length mass) can confine within the Planck length (l_p), and as one confines the greater mass or particle with greater mass the velocity or average (most probable) velocity of the mass or particle is less. But again, if particle with less than plank length mass is gradually confined to the smaller and smaller length-position, the uncertainty in velocity increases and the average (most probable) velocity increases and thus does the mass of particle, eventually attaining a maximum mass (which is not infinity) which corresponds to the maximum velocity which is a bit less than speed of light c (which we will calculate later). But if the particle, which is now enormously heavy, is subjected to further confinement, the velocity cannot increase further because of the speed limit but, the mass can. So, even if the velocity doesn't increase,

the mass does on further confinement. Now, because there is an ultimate limit to how much any mass can be confined (and that is Plank length), the gradual increase only in the mass but not velocity due to further confinement seems to finally come to an end and does so, but only if equality sign in the uncertainty relation below is considered or inequality in the uncertainty relation is neglected.

.i.e. in the relation: $\Delta x.\Delta v = \dfrac{\hbar}{m}$

But the complete uncertainty relation comes with the inequality as:

$$\Delta x.\Delta v \geq \dfrac{\hbar}{m}$$

So, the term in the right hand side (RHS) of above inequality equation must be either equal to or less than the term in the left hand side (LHS) of the above inequality relation. Also, as the term in the RHS is defined by mass 'm', the RHS term gets smaller as mass 'm' gets bigger. And because RHS term be smaller than the LHS term is a necessary condition of the inequality equation, RHS term can get any smaller in value, allowing mass 'm' to get any bigger even if the term in LHS does not change at all. So, the LHS term doesn't necessarily have to change but remain greater than or equal to ($\geq$) the RHS term. Therefore, any amount of mass greater than or equal to $2.17646893e^{-8}$ Kgs can be confined within the length equal to Plank length ($\Delta x = l_p = 1.616229e^{-35}$ meters).

5. The actual speed limit

However, the limit to the confinement of any mass providing the limit to how much or to what value/limit the mass can grow with confinement limit and <u>velocity limit</u> is given by the equality equation:

$$\Delta x.\Delta v = \dfrac{\hbar}{m}$$

Thus limit to how much the mass grows or to what value mass can grow provides the limit to how much the mass/particle can speed up in accordance to relativity.

Now returning to point A and B which are at distance $c\pm l_p = 299792458\pm1.616229e^{-35}$ meters away from each other. So, particle is at A actually means it can be anywhere within $\pm1.616229e^{-35}$ meters including point A. i.e. particle is either at A or to the (right) $+1.616229e^{-35}$ meters from A or to the (left) - $1.616229e^{-35}$ meters from A. Therefore uncertainty in position of particle at A is twice $1.616229e^{-35}$ meters .i.e. $3.232458e^{-35}$ meters. However we take the average position of the particle at A. Similarly, we take average position of particle at point B $\pm1.616229e^{-35}$ meters to be at exact B.
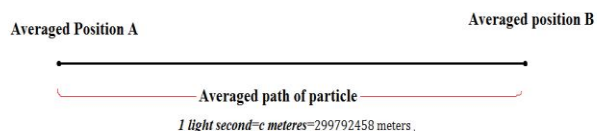
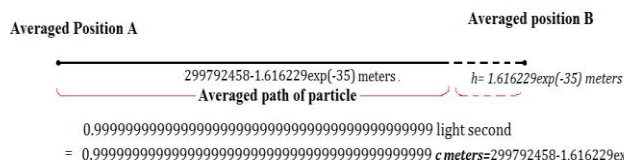**Figure III:** Maximum Distance of path travelled by photon in 1 second i.e. 100% .

Figure IV: Maximum Distance of path travelled by particle/object with rest mass in 1 second i.e. [SUM (n=-41 to n=1) 9exp(n)]%.

In this manner we just calculate the distance travelled and time taken for distance travelled and thus velocity in averages, without calculating the uncertainties in values.

So, considering the particle has no rest mass .i.e. for example if it is a photon, it will travel from point A to point B in a second (without calculating and including the uncertainties in that time .i.e. on an average).

Now, because the particle with some rest mass has limit to speed it can attain and because that speed limit is not $c = 299792458 ms^{-1}$ but less than this value, the particle for example an electron will travel from point A to point B in more than one seconds. Also, if it requires more than a second to get to B from A, and because there is a limit to how much any particle can travel $(= 1.616229 e^{-35} \text{ meters} = l_p)$, in time one second, the particle with rest mass can travel to distance $1.616229 e^{-35}$ meters less than 299792458 meters .i.e. $1.616229 e^{-35}$ meters away from B and nearer to A. So the maximum distance travelled by the rest mass particle in a second is $(299792458 - 1.616229 e^{-35})$ meters, which is:

$$\left[\frac{299792458 - 1.616229\,e^{-35}}{299792458}\right] 100\ \% \ of\ speed\ of\ light\ distance\ (= 299792458\ meters)$$

Therefore the maximum speed of the rest mass particle is:

$$\left[\frac{299792458 - 1.616229\,e^{-35}}{299792458}\right] 100\ \% \ of\ speed\ of\ light\ (= 299792458\ meters\ per\ second)$$

which is obtained by calculation to be 99.99999999999999999999999999999999999999 % the speed of light.

6. Maximum relativistic mass

Also, if the rest mass of the particle is m_0, then by relativistic mass *m* relation of the theory of Special relativity, the mass at 99.99999999999999999999999999999999999999 % the speed of light is:

$$m = \frac{m_0}{\sqrt{1 - \frac{(99.99999999999999999999999999999999999999c)^2}{c^2}}}$$

Or, $m \approx 10^{21} m_0$
This is the maximum attainable mass of any particle/object with rest mass equal to m_0 at the speed limit 99.99999999999999999999999999999999999999 % the speed of light (*c*).

7. Mass-limit extension

But again, as discussed earlier, by the inequality relation, $\Delta x . \Delta v \geq \frac{\hbar}{m}$, the mass can change by any amount until it keeps the inequality relation maintained; left-hand side (LHS) should not necessarily be equal to but greater than the right-hand side (RHS). So, in the most confined state (state with $\Delta x = l_p = 1.616229 e^{-35}$ meters and $\Delta v = c = 299792458 \text{ ms}^{-1}$), the mass can be greater or **increase to be greater** than value $2.17646893 e^{-8}$ Kgs .i.e. $m \geq 2.17646893 e^{-8}$ Kgs.

8. Result and Conclusion:

Hence the discussions made above shows that there is a practically attainable limit to the speed of any rest-mass body and this limit is not equal but is less than the speed at which light travels in vacuum. Also, the highest relativistic mass that any finite rest mass can achieve is not infinite but finite. However, the actual mass-limit for any rest-mass body is not given by or isn't the relativistic mass-limit. This actual mass-limit is beyond relativistic mass-limit to infinity .i.e. relativistic mass-limit is practically attainable, as resulted in above discussions, and doesn't increase further after the limit has been attained. But, the mass of the body does increase further than the relativistic mass-limit and can increase forever, but without any relativistic contribution. Also, as the particle is confined (decrease in uncertainty of position), the mass-energy of that particle grows due to increase in uncertainty of velocity and thus increase in average velocity. For, particles without the rest-mass, this average velocity limit is speed of light in vacuum and for particles with rest-mass not equal to zero, this limit is less than the speed of light in vacuum.

References

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