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An Improved Algorithm to Obtain Initial Basic Feasible Solution for the Transportation Problem

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Abstract: ***This proposed method constitutes alternate method for initial solution in transportation problem. This method provides the initial solution. A transportation matrix is solved by a difference between maximum and next maximum element for each row and difference between maximum and minimum element for each column. In which the maximum value is marked and allocation is given to the least element.. Salient features of this method depict lesser calculation time, easy applicability. Depiction with examples provides easy understanding of this method.***

Keywords: Transportation problem, supply, Demand, Vogels method, initial solution

1. Introduction

Transportation problem is most referred and analyzed aspect where there is always efforts to match the demand-supply prerogative or to put them in balance. With each demand there needs to be sufficient supply to maintain balance and create optimal solution for transportation. The efficient way would be to provide a satisfactory logistical solution for the demand and supply with minimum cost incurred.

Considering the decision variable X_{ij} of model in transportation the i[th] supply at source to j[th] demand in destination.

The below listed methods are used to deduce feasible solutions with initial transportation problem:
1) North West Corner Method
2) Least Cost Method
3) Vogel's Approximation Method

The overtly popular method employed on solving this is from MODI and stepping stone method. The former being widely discussed and articulated. This new method can be applied instead of the above three methods to obtain an initial basic feasible solution.

The beginning of assignment problem dates back to 1941 by Hitchcockin, the subsequent evolution of the methods put forth Koopmans in 1949 and Dantzig in 1951. Though the Simplex method forms the basis of the problem solution, this would however be devoid of being implemented as it would not suit to the environment. With more research on its way in 1954, Charnes and Cooper introduced Stepping Stone Method. This proved to be more efficient, thereby driving the goal of being optimal. This one step further laid foundation towards our grit on further optimizing eventually giving birth to Heuristic method being formulated by Kirca and Stair through the VAM from Goyal;s version in the year 1958.

This article is formed in sections and flows as below: *Mathematical representation of transportation problem is depicted in Section 2, followed by algorithm in Section 3, numerical examples and conclusion follows in Section 4 and 5 respectively.*

2. Mathematical form for transportation problem

The LP problem as given below

Minimize Z $=\sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij} X_{ij}$

Subject to the constraints

$\sum_{j=1}^{n} X_{ij} \le S_i$ For all i

$\sum_{i=1}^{m} X_{ij} \ge d_j$ For all j

$X_{ij} \ge 0$

Transportation problem is considered to be balanced if $\sum_{i=1}^{m} S_i = \sum_{j=1}^{n} d_j$

3. Algorithm

Step 1
Construct the matrix of a transportation problem from given problem. In case if the problem is unbalanced we make it balanced.

Step 2
Find the difference between maximum and next maximum in each row which is called as row penalty and difference between maximum and minimum in each column as column penalty and write it in the side and bottom. From that select the maximum value. From the selected row / column we need to allocate the minimum of supply/demand in the minimum element of the row or column. Eliminate by deleting the columns or rows corresponding to where the supply or demand is satisfied.

Step 3
Thus obtained table is then discussed. The process is continued to the remaining table till m + n -1 cells are allocated with satisfaction to its supply and demand.

Step 4

If obtained condition in step 2 is contrary, that is if there is tie in maximum value select that value which has least element. If there is tie in the least element then allocate the least element which has minimum supply/demand.

Step 5

Repeating the steps 2 to step 4 until satisfaction of all the supply and demand is met.

Step 6

The total minimum cost is calculated by

Total cost = $\sum_{i=1}^{m}\sum_{j=1}^{n} C_{ij}$

4. Numerical Example

1. Obtain an initial basic feasible solution for the following problem.

		Distribution			
		D$_1$	D$_2$	D$_3$	Supply
Factories	F_1	6	4	1	50
	F_2	3	8	7	40
	F_3	4	4	2	60
	Req.	20	95	35	

Solution

Step1

Find the difference between the maximum and next maximum element in row-wise and difference between maximum and minimum in column-wise .For that row/column allocate the minimum supply/demand for the least element in that row/column.

		D$_1$	D$_2$	D$_3$	Supply	Row Penalty
Factories	F_1	6	4	[35] 1	15 ~~50~~	(2)
	F_2	3	8	7	40	(1)
	F_3	4	4	2	60	(0)
	Req.	20	95	~~35~~		
	Column Penalty	(3)	(4)	(6)*		

we take the row penalty as F_1 (6-4=2) and F_2 (8-7 =1) and F_3 (4-4=0) and column penalty as D_1 (6-3=3)and D_2 (8-4=4) and D_3 (7-1=6).

Here the difference is maximum in D_3.Hence allocate to the smallest element with minimum supply/demand.
Now delete D_3

Step 2

		D$_1$	D$_2$	Supply	Row Penalty
Factories	F_1	6	4	15	(2)
	F_2	[20] 3	8	~~40~~ 20	(5)*
	F_3	4	4	60	(2)
	Req.	~~20~~	95		
	Column Penalty	(3)	(4)		

we take the row penalty as F_1 (18-15=3) and F_2 (16-13 =3) and F_3 (17-12=5) and column penalty as D_1 (16-12=4)and D_2 (17-10=7) and D_4 (18-11=7)

Here the difference is maximum in F_2.Hence allocate to the smallest element with minimum supply/demand.
Now delete D_1

Step 3

		D$_2$	Supply
Factories	F_1	[15] 4	15
	F_2	[20] 8	20
	F_3	[60] 4	60
	Req.	95	
	Column Penalty	(4)	

Step 4

		D$_1$	D$_2$	D$_3$	Supply
Factories	F_1	6	[15] 4	[35] 1	50
	F_2	[20] 3	[20] 8	7	40
	F_3	4	[60] 4	2	60
	Req.	20	95	35	

Minimum cost = 15 × 4 + 35 × 1 + 20 × 3 +20 × 8 + 60 × 4

Minimum cost = 555

Initial Solution

Proposed Method	Vogels Method
555	555

2. **Obtain an initial basic feasible solution for the following problem.**

		Distribution				
		D$_1$	D$_2$	D$_3$	D$_4$	Supply
Factories	F_1	15	10	17	18	20
	F_2	16	13	12	13	60
	F_3	12	17	20	11	70
	Req.	30	30	40	50	

Solution

Step1

Find the difference between the maximum and next maximum element in row-wise and difference between maximum and minimum in column-wise .For that row/column allocate the minimum supply/demand for the least element in that row/column.

Factories		D1	D2	D3	D4	Supply	Row Penalty
	F_1	15	10	17	18	20	(1)
	F_2	16	13	40 / 12	13	~~60~~ 20	(3)
	F_3	12	17	20	11	70	(3)
	Req.	30	30	~~40~~	50		
	Column Penalty	(4)	(7)	(8)*	(7)		

we take the row penalty as F_1(18-15=3)and F_2(16-13 =3) and F_3 (17-12=5) and column penalty as D_1(16-12=4)and D_2 (17-10=7) and D_4 (18-11=7)

Here the difference is maximum in D_2.and D_4.Select the column D_2.which has least element 10. Hence allocate to the smallest element with minimum supply/demand.

Now delete F_1.

Step 2

Factories		D1	D2	D4	Supply	Row Penalty
	F_1	15	20 / 10	18	~~20~~	(3)
	F_2	16	13	13	20	(3)
	F_3	12	17	11	70	(5)
	Req.	30	~~30~~ 10	50		
	Column Penalty	(4)	(7)	(7)		

Step 3

		D1	D2	D4	Supply	Row Penalty
	F_2	16	13	13	20	(3)
	F_3	12	17	50 / 11	~~70~~ 20	(5)*
	Req.	30	10	~~50~~		
	Column Penalty	(4)	(4)	(2)		

we take the row penalty as F_2(16-13=3) and F_3 (17-12=5) and column penalty as D_1(16-12=4)and D_2 (17-13=4) and D_4 (13-11=2).

Here the difference is maximum in F_3. Hence allocate to the smallest element with minimum supply/demand.

Now delete D_4

Step 4

Factories		D1	D2	Supply	Row Penalty
	F_2	10 / 16	10 / 13	20	(3)
	F_3	20 / 12	17	20	(5)*
	Req.	30	10		
	Column Penalty	(4)	(4)		

Step5

Factories		D1	D2	D3	D4	Supply
	F_1	15	20 / 10	17	18	20
	F_2	10 / 16	10 / 13	40 / 12	13	60
	F_3	20 / 12	17	20	50 / 11	70
	Req.	30	30	40	50	

Minimum cost = 20 × 10 +10 × 16 + 10 × 13 +40 × 12 + 20 × 12 +50 × 11

Minimum cost = 1760

Initial Solution

Proposed Method	Vogels Method
1760	1760

3. Obtain an initial basic feasible solution for the following problem.

		Distribution				
Factories		D1	D2	D3	D4	Supply
	F_1	4	19	22	11	100
	F_2	1	9	14	14	30
	F_3	6	6	16	14	70
	Req.	40	20	60	80	

Solution

Step-1

Find the difference between the maximum and next maximum element in row-wise and difference between maximum and minimum in column-wise .For that row/column allocate the minimum supply/demand for the least element in that row/column.

		D_1	D_2	D_3	D_4	Supply	Row Penalty
Factories	F_1	4	19	22	11	100	(3)
	F_2	1	9	14	14	30	(0)
	F_3	6	20 6	16	14	~~70~~ 50	(2)
	Req.	40	~~20~~	60	80		
	Column Penalty	(5)	(13)*	(8)	(3)		

we take the row penalty as F_1(22-19=3) and F_2(14-14=0) and F_3 (16-14=2) and column penalty as D_1(6-1=5)and D_2 (19-6=13) and D_3 (22-14=8) and D_4 (14-11=3)

Here the difference is maximum in D_2.Hence allocate to the smallest element with minimum supply/demand.
Now delete D_2

Step 2

		D_1	D_3	D_4	Supply	Row Penalty
Factories	F_1	40 4	22	11	~~100~~ 60	(10)*
	F_2	1	14	14	30	(0)
	F_3	6	16	14	50	(2)
	Req.	~~40~~	60	80		
	Colum Penalty	(5)	(8)	(3)		

we take the row penalty as F_1(22-11=10)and F_2(14-14 =0) and F_3 (16-14=2) and column penalty as D_1(6-1=5)and D_3 (22-14=8) and D_4 (14-11=3)

Here the difference is maximum in F_1.Hence allocate to the smallest element with minimum supply/demand.
Now delete D_1

Step-3

		D_3	D_4	Supply	Row Penalty
Factories	F_1	22	60 11	~~60~~	(10)*
	F_2	14	14	30	(0)
	F_3	16	14	50	(2)
	Req.	60	~~80~~ 20		
	Column Penalty	(8)	(3)		

we take the row penalty as F_1(22-11=10)and F_2(14-14=0) and F_3 (16-14=2) and column penalty as D_3(22-14=8)and D_4 (14-11=3)
Here the difference is maximum in F_1. Hence allocate to the smallest element with minimum supply/demand.
Now delete F_1

Step-4

		D_3	D_4	Supply	Row Penalty
Factories	F_2	30 14	14	30	(0)
	F_3	30 16	20 14	50	(2)*
	Req.	60	20		
	Column Penalty	(2)	(0)		

Step-5

		Distribution				
		D_1	D_2	D_3	D_4	Supply
Factories	F_1	40 4	19	22	60 11	100
	F_2	1	9	30 14	14	30
	F_3	6	20 6	30 16	20 14	70
	Req	40	20	60	80	

Minimum cost = 40 ×41 +60 ×11 +30 × 14 +20 ×6 +30 × 16 +20 × 14
Minimum cost = 2120

Initial Solution

Proposed Method	Vogels Method
2120	2170

5. Conclusion

The above discussed method on initial solution gives another paradigm on solutions with more efficient way. This method further qualifies to be more efficient and initial in solving the problems being a game to further research and development through this. This proposed method gives result exactly or even lesser to VAM method. All necessary qualities of being time efficient, easy applicability etc., forms the core of being implemented successfully.

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