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On Shrinkage Estimation of the Stress – Strength Reliability of Exponentiaed Weibull Distribution

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Abstract: ***This paper deal with estimation the stress – strength reliability of Exponentiaed Weibull Distribution, using different estimation methods like, maximum likelihood, moment and shrinkage methods. Comparisons between the proposal estimators were made using simulation based on statistical indicter mean squared error (MSE).***

Keywords: Exponentiaed Weibull Distribution (EWD), Stress – Strength(S-S) Reliability, Maximum likelihood estimator (MLE), Moment estimator (MOM), Shrinkage estimator (sh) and mean squared error (MSE)

1. Introduction

Mudholkar and Srivastava (1993) proposed a modification to the standard weibull model through the introduction of an additional parameter v, such as G(x) is CDF of Weibull Distribution and F(x) is CDF of Exponentiaed Weibull Distribution
$F(x) = (G(x))^{v}$; $(0<v<\infty)$

That is mean, when v = 1, the model reduces to the standard two parameter Weibull model.

In (1993) also they introduced the Exponentiaed Weibull family distribution (EW) as an extension of Weibull family [10]. In (1995) above researchers introduced an applications of Exponentiaed Weibull (EW) distribution [11]. Mudholkar and Hutson in (1996) studied the reliability and survival about (EW) distribution [9]. Nassar and Eissa in (2003) studied in more detail the properties of (EW) distribution [12]. Pal and Woo in (2006) compared the Exponentiaed Weibull with two-parameter Weibull and gamma distributions with respect to failure rate, and also they showed that the EWD has an application in many fields such as health costs, civil engineering, economics and others [13].

The Stress – Strength (S-S) model for reliability is used in many application in engineering and physics. "The problem of estimation the (S –S) reliability (R =P(Y<X)) arises in the situation of mechanical reliability of component with strength X and stress Y"; [14]. The component fails if and only if at any time the stress exceeds the strength.

In this paper we estimate the stress – strength (S-S) reliability of the two parameters Exponentiaed Weibull distribution using different estimation methods and make a comparison between the proposed methods using simulation depends on the statistical indicator MSE .Our hypothesis in (S -S) model, the stress(Y) and the strength (X) are independent but not identically variables follows two – parameters Exponentiaed Weibull Distribution (EWD).

Therefore, the probability density function (PDF) of a r. v. X which is follows EWD is as below:

$$f(x;\alpha,\theta) = \alpha\theta x^{\theta-1}e^{-x^{\theta}}\left(1-e^{-x^{\theta}}\right)^{\alpha-1} \quad x>0,\ \alpha,\theta>0 \qquad (1)$$

Where, α is unknown shape parameter and θ is known shape parameter, and the cumulative distribution function CDF of X is as follows:-

$$F(x;\alpha,\theta) = (1-e^{-x^{\theta}})^{\alpha} \quad x>0,\ \alpha,\theta>0 \qquad (2)$$

Now, if the two random variables X and Y follows the EWD with parameters (α_1, θ) and (α_2,θ) as strength and stress respectively ,then the (S-S) reliability is defined as follows; [6], [7] and[8].

$$R = P(Y<X)$$
$$= \iint_{y<x} f(x)f(y)\,dy\,dx$$

Therefore,

$$R = \int_0^{\infty} \alpha_1\theta x^{\theta-1}e^{-x^{\theta}}(1-e^{-x^{\theta}})^{\alpha_1-1}dx \int_0^{x} \alpha_2\,\theta y^{\theta-1}e^{-y^{\theta}}(1-e-y\theta)\alpha 2-1\ dy$$

$$= \int_0^{\infty} \alpha_1\theta x^{\theta-1}e^{-x^{\theta}}(1-e^{-x^{\theta}})^{\alpha_2+\alpha_1-1}dx$$

$$= \frac{\alpha_1}{\alpha_1+\alpha_2} \qquad (3)$$

2. Estimation methods of R =P(Y<X)

2.1 Maximum Likelihood Estimator (MLE)

Let $x_1,x_2,\ldots\ldots,x_n$ be a random sample from EW(α_1,θ) and y_1 ,y_2 ,……, y_m be a random samples from EW(α_2,θ) then ,the likelihood function of the observed sample is given as

$$l = L(\alpha 1,\alpha 2,\theta;x,y)$$
$$= \prod_{i=1}^{n} f(x_i)\prod_{j=1}^{m} g(y_j)$$
$$= \prod_{i=1}^{n}\alpha_1\theta x_i^{\theta-1}e^{-x_i^{\theta}}(1-e^{-x_i^{\theta}})^{\alpha_1-1}\prod_{j=1}^{m}\alpha_2\theta y_j^{\theta-1}e^{-y_j^{\theta}}(1-e-yj\theta)\alpha 2-1$$

$$= \alpha_1^{n}\alpha_2^{m}\theta^{n+m}e^{-\sum_{i=1}^{n}x_i^{\theta}-\sum_{j=1}^{m}y_j^{\theta}}\prod_{i=1}^{n}x_i^{\theta-1}\prod_{j=1}^{m}y_j^{\theta-1}\prod_{i=1}^{n}(1-e-xi\theta)\alpha 1-1\ j=1m(1-e-yj\theta)\alpha 2-1 \qquad (4)$$

Take Ln to both sides we get:-

$$\mathrm{Ln}(l) = n\,\mathrm{Ln}\alpha 1 + m\,\mathrm{Ln}\alpha 2 + (n+m)\,\mathrm{Ln}\theta + \sum_{i=1}^{n} Lnx_i^{\theta-1} + \sum_{j=1}^{m} Lny_j^{\theta-1} - \sum_{i=1}^{n} x_i^{\theta} - \sum_{j=1}^{m} y_j^{\theta} +$$

$$\sum_{i=1}^{n} Ln\left(1-e^{-x_i^{\theta}}\right)^{\alpha_1-1} + \sum_{j=1}^{m} Ln\left(1-e^{-y_j^{\theta}}\right)^{\alpha_2-1} \quad (5)$$

The derivatives of Ln(l) with respect to α_1 and α_2 and equate to the zero are respectively given as follows:

$$\frac{dLn(l)}{d\alpha_1} = \frac{n}{\alpha_1} + \sum_{i=1}^{n} Ln\left(1-e^{-x_i^{\theta}}\right) = 0 \quad (6)$$

$$\frac{dLn(l)}{d\alpha_2} = \frac{m}{\alpha_2} + \sum_{j=1}^{m} Ln\left(1-e^{-y_j^{\theta}}\right) = 0 \quad (7)$$

The ML's estimator for the unknown shape parameters α_i (i=1, 2) is given by:

$$\hat{\alpha}_{1_{mle}} = \frac{-n}{\sum_{i=1}^{n} Ln\left(1-e^{-x_i^{\theta}}\right)} \quad (8)$$

$$\hat{\alpha}_{2_{mle}} = \frac{-m}{\sum_{j=1}^{m} Ln\left(1-e^{-y_j^{\theta}}\right)} \quad (9)$$

We note that, $\hat{\alpha}_{i_{mle}}$ is biased estimator, since$E(\hat{\alpha}_{i_{mle}}) = \frac{n\alpha}{n-1} \neq \alpha$.

Thus, $\hat{\alpha}_{i_{ub}} = \frac{n-1}{n}\hat{\alpha}_{i_{mle}}$ will be unbiased estimator ofα_i.

That is mean, $(\hat{\alpha}_{i_{ub}}) = \alpha$, $Var(\hat{\alpha}_{1_{ub}}) = \frac{(\alpha_1)^2}{(n-2)}$ and $Var(\hat{\alpha}_{2_{ub}}) = \frac{(\alpha_2)^2}{(m-2)}$.

i.e. ;

$$\hat{\alpha}_{1_{ub}} = \frac{n-1}{-\sum_{i=1}^{n} Ln\left(1-e^{-x_i^{\theta}}\right)} \quad (10)$$

and

$$\hat{\alpha}_{2_{ub}} = \frac{m-1}{-\sum_{j=1}^{m} Ln\left(1-e^{-y_j^{\theta}}\right)} \quad (11)$$

By substituting equations (8) and (9) in equation (3) we get:-

$$\hat{R}_{mle} = \frac{\hat{\alpha}_{1_{mle}}}{\hat{\alpha}_{1_{mle}} + \hat{\alpha}_{2_{mle}}} \quad (12)$$

2.2 Moment Method (MOM)

The moment method was one of the exact method used to estimate the parameters. In this subsection we used the (MOM) to estimate the parameter α for EWD when the parameter θ is known and we need the populations moment for x and y of EWD, which is given below:-[13][2]

$$E(x^r) = \begin{cases} \alpha \sum_{i=0}^{\alpha-1} \binom{\alpha-1}{i} (-1)^i (i+1)^{-\frac{r}{\theta}-1} \Gamma\left(\frac{r}{\theta}+1\right) & , if\ \alpha \in N \\ \alpha \sum_{i=0}^{\alpha-1} \frac{\alpha-1^{P_i}}{i!} (-1)^i (i+1)^{-\frac{r}{\theta}-1} \Gamma\left(\frac{r}{\theta}+1\right) & , if\ \alpha \notin N \end{cases}$$

for r = 1,2,3 .... (13)

Where αP_i=α (α-1) (α-2)….. (α-i+1) and N is the set of natural number. Thus , the populations mean of x and y will be:

$$E(x) = \begin{cases} \alpha_1 \sum_{i=0}^{\alpha_1-1} \binom{\alpha_1-1}{i} (-1)^i (i+1)^{-\frac{1}{\theta}-1} \Gamma\left(\frac{1}{\theta}+1\right) & , if\ \alpha_1 \in N \\ \alpha_1 \sum_{i=0}^{\alpha_1-1} \frac{\alpha_1-1^{P_i}}{i!} (-1)^i (i+1)^{-\frac{1}{\theta}-1} \Gamma\left(\frac{1}{\theta}+1\right) & , if\ \alpha_1 \notin N \end{cases}$$

$$E(Y) = \begin{cases} \alpha_2 \sum_{j=0}^{\alpha_2-1} \binom{\alpha_2-1}{j} (-1)^j (j+1)^{-\frac{1}{\theta}-1} \Gamma\left(\frac{1}{\theta}+1\right) & if\ \alpha_2 \in N \\ \alpha_2 \sum_{j=0}^{\alpha_2-1} \frac{\alpha_2-1^{P_j}}{j!} (-1)^j (j+1)^{-\frac{1}{\theta}-1} \Gamma\left(\frac{1}{\theta}+1\right) & , if\ \alpha_2 \notin N \end{cases}$$

And when equating the sample mean with the corresponding population mean, we get

$$\bar{X} = \frac{\sum_{i=1}^{n} x_i}{n} = \alpha_1 \sum_{i=0}^{\alpha_1-1} \binom{\alpha_1-1}{i} (-1)^i \Gamma\left(\frac{1}{\theta}+1\right)(i+1)^{-\frac{1}{\theta}-1}$$

and ,

$$\bar{Y} = \frac{\sum_{j=1}^{m} y_j}{m} = \alpha_2 \sum_{j=0}^{\alpha_2-1} \binom{\alpha_2-1}{j} (-1)^j \Gamma\left(\frac{1}{\theta}+1\right)(j+1)^{-\frac{1}{\theta}-1}$$

By simplification, we obtain the estimation of the unknown shape parameters α_1, α_2 using moment method as follows

$$\hat{\alpha}_{1_{mom}} = \frac{\bar{X}}{\sum_{i=0}^{\alpha_1-1} \binom{\alpha_1-1}{i} (-1)^i \Gamma\left(\frac{1}{\theta}+1\right)(i+1)^{-\frac{1}{\theta}-1}} \quad (14)$$

and

$$\hat{\alpha}_{2_{mom}} = \frac{\bar{Y}}{\sum_{j=0}^{\alpha_2-1} \binom{\alpha_2-1}{j} (-1)^j \Gamma\left(\frac{1}{\theta}+1\right)(j+1)^{-\frac{1}{\theta}-1}} \quad (15)$$

Substitution the equations (14) and (15) in the equation (3), we get the estimation of (S-S) reliability using moment method as below:

$$\hat{R}_{mom} = \frac{\hat{\alpha}_{1_{m0m}}}{\hat{\alpha}_{1_{mom}} + \hat{\alpha}_{2_{mom}}} \quad (16)$$

2.3 Shrinkage Estimation Method (Sh):

Shrinkage estimation method is the Bayesian approach depending on prior information regarding the value of specific parameter α from past experiences or previous studies as initial value α_0 and classical (unbiased) estimator $\hat{\alpha}_{ub}$ through merge them as a linear combination using shrinkage weight factor $\varphi(\hat{\alpha}_i)$, $0 \leq \varphi(\hat{\alpha}_i) \leq 1$ as below:

$$\hat{\alpha}_{sh} = \varphi(\hat{\alpha}_i)\hat{\alpha}_{ub} + \left(1-\varphi(\hat{\alpha}_i)\right)\alpha_0 \text{ , i=1,2} \quad (17)$$

Where, $\varphi(\hat{\alpha}_i)$ refers to the believe of $\hat{\alpha}_{ub}$, and $(1-\varphi(\hat{\alpha}_i))$ represents to believe of α_0, which may be constant or a function of $\hat{\alpha}_{ub}$, function of sample size or may be found by minimizing the mean square error for $\hat{\alpha}_{sh}$;

As Thompson said, the important reasons to use initial value refer to:

1) Suppose that the initial value α_0 is very close to the true value , and then it is necessary to used it.
2) Something bad may be happened if not usedα_0, especially when the initial value is very close to the true value of parameter. See [1], [3], [4], [15] and [16].

There is no doubt, if our assumption to take the moment method as initial value instated of α_0 in this work.

2.3.1 Shrinkage weight function (sh1)

In this sub section we consider the shrinkage weight factor as a function of n and m respectively in equation (17) $\varphi(\hat{\alpha}_1) = K_1 = e^{-n}$, and $\varphi(\hat{\alpha}_2) = K_2 = e^{-m}$.

$$\hat{\alpha}_{i_{sh1}} = K_i\, \hat{\alpha}_{i_{ub}} + (1-K_i)\hat{\alpha}_{i_{mom}} \text{ for i= 1,2} \quad (18)$$

The corresponding (S-S) reliability using above shrinkage method $\hat{R}_{sh1}$ we get

$$\hat{R}_{sh1} = \frac{\hat{\alpha}_{1_{sh1}}}{\hat{\alpha}_{1_{sh1}} + \hat{\alpha}_{2_{sh1}}} \quad (19)$$

2.3.2 Constant shrinkage factor (sh2)

In this subsection the constant shrinkage weight factor will be assumed as $\varphi(\hat{\alpha}_1) = K_3 = 0.3$, and $\varphi(\hat{\alpha}_2) = K_4 = 0.3$, and this implies to the following shrinkage estimators

$$\hat{\alpha}_{1_{sh\,2}} = K_3\hat{\alpha}_{1_{ub}} + (1-K_3)\hat{\alpha}_{1_{mom}} \quad (20)$$

$$\hat{\alpha}_{2_{sh\,2}} = K_4 \hat{\alpha}_{2_{ub}} + (1 - K_4)\hat{\alpha}_{2_{mom}} \quad (21)$$

When substitution the equations (20) and (21) in the equation (3), lead to the estimation of (S-S) reliability using shrinkage estimator $\hat{R}_{sh2}$ as below:

$$\hat{R}_{sh2} = \frac{\hat{\alpha}_{1_{sh2}}}{\hat{\alpha}_{1_{sh2}} + \hat{\alpha}_{2_{sh2}}} \quad (22)$$

2.3.3 Modified thompson type shrinkage weight factor (th):

In this subsection we modified the shrinkage weight factor considered by Thompson in 1968 as follows:

$$\emptyset(\hat{\alpha}_i) = \frac{\left(\hat{\alpha}_{i_{ub}} - \hat{\alpha}_{i_{mom}}\right)^2}{\left(\hat{\alpha}_{i_{ub}} - \hat{\alpha}_{i_{mom}}\right)^2 + var\left(\hat{\alpha}_{i_{ub}}\right)}(0.01) \quad \text{for i=1,2} \quad (23)$$

Where, Var($\hat{\alpha}_{i_{ub}}$) defined in section (2-1).

Thus, the modified Thompson type shrinkage estimator will be

$$\hat{\alpha}_{i_{th}} = \emptyset(\hat{\alpha}_i)\hat{\alpha}_{i_{ub}} + \left(1 - \emptyset(\hat{\alpha}_i)\right)\alpha_{i_{mom}} \quad , \quad \text{for } i = 1,2 \quad (24)$$

By substituting equation (24) in the equation (3) , we get the modified Thompson type shrinkage estimation of the (S-S) reliability as below

$$\hat{R}_{th} = \frac{\hat{\alpha}_{1_{th}}}{\hat{\alpha}_{1_{th}} + \hat{\alpha}_{2_{th}}} \quad (25)$$

3. Simulation Study

In this section we numerical results were studied to compare the performance of the different estimators of reliability which is obtained in section 2, using different sample size =(10, 30, 50 and 100), based on 1000 replication via MSE criteria. For this purpose Mote Carlo simulation was used the following steps:-[5]

Step1: we generate the random sample which is follows the continuance uniform distribution defined on the interval (0,1) as $u_1,u_2,\ldots,u_n$.

Step2: we generate the random sample which is follows the continuance uniform distribution defined on the interval (0, 1) as $w_1, w_2,\ldots, w_m$.

Step3: transform the above uniform random samples to random samples follows EWD using the cumulative distribution function (c.d.f.) as follow:

$$F(x) = (1 - e^{-x_i^{\theta}})^{\alpha_1}$$

$$Ui = (1 - e^{-x_i^{\theta}})^{\alpha_1}$$

$$x_i = [-\ln(1 - U_i^{\frac{1}{\alpha_1}})]^{\frac{1}{\theta}}$$

And, by the same method, we get

$$y_j = [-\ln(1 - W_j^{\frac{1}{\alpha_2}})]^{\frac{1}{\theta}}$$

Step4: we compute the maximum likelihood estimator of R using equation (12).

Step5: we compute the moment method of R using equation (16).

Step6: we compute the three shrinkage estimators of R using equations (19), (22) and (25).

Step7: based on (L=1000) Replication, we calculate the MSE as follows:

$$MSE = \frac{1}{L}\sum_{i=1}^{L}(\hat{R}_i - R)^2$$

Where $\hat{R}$ refer the proposed estimators of real value of Reliability R.

All the results are given in table (1), (2), (3) and (4) below

Table 1: Shown estimation value of R, when α_1=5, α_2=2, θ =3

n	m	R				
10	10	0.71429	0.70972	0.70177	0.71429	0.71426
	30	0.71429	0.71371	0.70909	0.71429	0.71436
	50	0.71429	0.71595	0.71172	0.71429	0.71435
	100	0.71429	0.71728	0.71379	0.71429	0.71434
30	10	0.71429	0.70251	0.69933	0.71429	0.71423
	30	0.71429	0.71076	0.70817	0.71429	0.71429
	50	0.71429	0.71326	0.71215	0.71429	0.71427
	100	0.71429	0.71305	0.71266	0.71429	0.71432
50	10	0.71429	0.69488	0.69573	0.71429	0.71422
	30	0.71429	0.71181	0.70978	0.71429	0.71428
	50	0.71429	0.71211	0.70994	0.71429	0.71323
	100	0.71429	0.71493	0.71237	0.71429	0.71427
100	10	0.71429	0.69923	0.69923	0.71429	0.71420
	30	0.71429	0.71238	0.71178	0.71429	0.71424
	50	0.71429	0.71105	0.71044	0.71429	0.71428
	100	0.71429	0.71343	0.71266	0.71429	0.71431

Table 2: Shown MSE values when α_1=5, α_2=2, θ =3 and R=0.71429

n	m					Best
10	10	0.00893	0.00497	79E-14	46E-8	
	30	0.00530	0.00155	75E-14	51E-8	
	50	0.00469	0.00084	77E-14	56E-8	
	100	0.00446	0.00042	77E-14	50E-8	
30	10	0.00618	0.00512	51E-14	50E-8	
	30	0.00295	0.00161	45E-14	54E-8	

	50	0.00222	0.00083	44E-14	56E-8	
	100	0.00177	0.00044	44E-14	57E-8	
50	10	0.00584	0.00481	43E-14	54E-8	
	30	0.00201	0.00137	38E-14	52E-8	
	50	0.00160	0.00091	38E-13	55E-8	
	100	0.00125	0.00043	39E-14	58E-8	
100	10	0.00486	0.00455	40E-14	53E-8	
	30	0.00195	0.00145	35E-14	60E-8	
	50	0.00132	0.00091	35E-14	60E-8	
	100	0.00081	0.00042	34E-14	58E-8	

Table 3: Shown estimation values of R when α_1=2.1, α_2=4.3 ,θ =1.5

n	M	R				
10	10	0.328125	0.33326	0.31876	0.32812	0.32809
	30	0.328125	0.33568	0.32523	0.32812	0.32817
	50	0.328125	0.34033	0.32777	0.32812	0.32823
	100	0.328125	0.34102	0.32719	0.32812	0.32820
30	10	0.328125	0.32775	0.32371	0.32812	0.32808
	30	0.328125	0.33190	0.32698	0.32812	0.32807
	50	0.328125	0.33094	0.32578	0.32812	0.32815
	100	0.328125	0.33097	0.32645	0.32812	0.32813
50	10	0.328125	0.32475	0.32212	0.32812	0.32808
	30	0.328125	0.33084	0.32771	0.32812	0.32810
	50	0.328125	0.32804	0.32589	0.32812	0.32808
	100	0.328125	0.33038	0.32743	0.32812	0.32812
100	10	0.328125	0.32398	0.32300	0.32812	0.32803
	30	0.328125	0.32953	0.32850	0.32812	0.32810
	50	0.328125	0.32788	0.32802	0.32812	0.32812
	100	0.328125	0.32595	0.32608	0.32812	0.32815

Table 4: Shown MSE values when α_1=2.1, α_2=4.3 ,θ =1.5 and R=0.328125

n	m					Best
10	10	0.00884	0.00422	82E-14	55E-8	
	30	0.00661	0.00146	57E-14	60E-8	
	50	0.00606	0.00095	60E-14	64E-8	
	100	0.00647	0.00049	57E-14	64E-8	
30	10	0.00613	0.00488	86E-14	60E-8	
	30	0.00326	0.00158	55E-14	66E-8	
	50	0.00267	0.00102	54E-14	64E-8	
	100	0.00208	0.00046	54E-14	66E-8	
50	10	0.00602	0.00513	88E-14	62E-8	
	30	0.00253	0.00156	54E-14	65E-8	
	50	0.00202	0.00093	53E-14	69E-8	
	100	0.00159	0.00051	53E-14	67E-8	
100	10	0.00527	0.00465	82E=14	62E-8	
	30	0.00196	0.00151	55E-14	63E-8	
	50	0.00148	0.00095	53E-14	61E-8	
	100	0.00089	0.00047	54E-14	67E-8	

4. Numerical Results

For all n=(10,30,50,100) and for all m=(10,30,50,100), in this work, the minimum mean square error (MSE) for the (S-S) reliability estimator of the Exponentiaed Weibull distribution is holds using the shrinkage estimator based on constant shrinkage weight function ($\hat{R}_{sh2}$). This implies that that shrinkage estimator of (S-S) reliability ($\hat{R}_{sh2}$) is the best and follows by using Thompson type shrinkage estimator $\hat{R}_{th}$. For any n, some of the proposed estimator (mle, sh_1, and sh_2) is decreasing with m and the other methods are vibration. Finely, when n=m=100, the third order best estimator is $\hat{R}_{sh1}$after$\hat{R}_{sh2}$, $\hat{R}_{th}$.

5. Conclusion

From the numerical results, one can find the proposal shrinkage estimation method using constant shrinkage weight function ($\hat{R}_{sh2}$) which is depend on unbiased estimator and prior estimate (moment method) as a linear combination, performance good behavior and it is the best estimator than the others in the sense of MSE.

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