International Journal of Science and Research (IJSR)
ISSN (Online): 2319-7064
Index Copernicus Value (2015): 78.96 | Impact Factor (2015): 6.391

Interference Problem of Machines Embedded with Fuzziness

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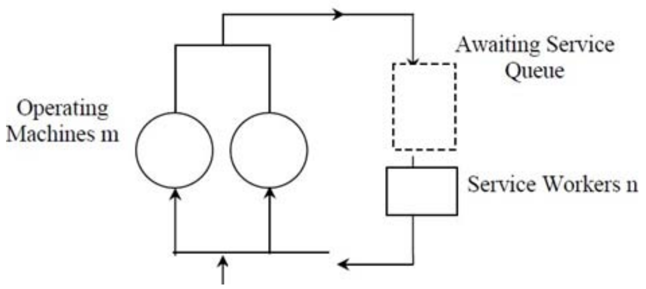
Abstract: ***Machine interference is a significant problem in many manufacturingsystem and client server computing. Machine interference problem involve manyparameters like break down rate, service rate, machine production rate, etc. Due touncontrollable factors parameters in the machine interference problem may befuzzy. This paper, proposes a methodology for constructing system performancemeasures, where breakdown rate and service rate are trapezoidal fuzzy numbers. Function principle is used as arithmetic operations of fuzzy trapezoidal numbers.Numerical example is solved successfully to illustrate the validity of the proposedapproach. Since the system characteristics being expressed as a fuzzy trapezoidalnumbers more information is provided for used by Management. By extending thefuzzy environment, the fuzzy queues can be represented more accurately by usingthe proposed approach, and the analysis of results for such queuing model will beuseful and significant for system designers and practitioners.***

Keywords: Machine Interference problem, Function principle, Graded MeanIntegration Representation

1. Introduction

Consider a system consisting of n technicians who support m machinessubject to stochastic failure where $n \le m$. Whenever a machine breaks down, it isrepaired by a technician, each repair keeps a technician busy for a period of timeduring which they cannot service other broken machines. The problem ofoperating such a system efficiently is commonly referred to as the Machine RepairProblem (MRP) or machine interference problem. This type of problemcan arise not only in maintenance operations but also in manufacturingapplications and in client server computing.



A MRP System of m machines and n workers

Machine interference is sometimes used to describe situations where machinesmay physically get in each other's way during operation rather than whileawaiting service. This includes for example the mechanical motion of roboticarms that are closely spaced on an assembly line. We do not consider thesesubjects or those that deals with "machine repair" in more general settings.

Analysis of MRP model typically begins by deriving the steady-stateprobability distribution $P_i \in \{0,1,2,...,n\}$, that describes the long run probabilityof i machines being in the failed (or) down state at any given point in time. Forexample, the machine operating lifetimes are exponentially distributed with meantime of $1/\lambda$ and that the service durations are likewise exponentially distributedwith mean time of $1/\mu$. Further assume that the service facility has ample bufferspace for machines to queue up while awaiting service machines are served inFirst come, First served order, and that a machine returns to operation "as good asnew" after being served. Under the assumption the system is easily modelled as afinite population M/M/m queue with n sources. The steady state distribution canbe used in turn to derive a variety of performance measures for the system such asaverage number of machines waiting for service, average number of machinesdown, average down time duration of a machines and average duration of waitingtime for repair etc. This descriptive analysis takes the model parameter (m, n, λ,μ) as given and then describes the system performance according to certainmetrics. Efficient methods have been developed for analyzing machineinterference problem with its parameters like breakdown rate and service rate areknown exactly. One commonly used type of solution methods is the queuingtheory approach in that the machine interference problem is modeled as a finite calling, population queuing system. The machines breakdowns are treated ascustomers and the repair persons are servers in the system. We can derive systemperformance measures of the machine interference problem and its variants whentheir parameters are known exactly. However, there are cases that theseparameters may not be presented precisely due to uncontrollable factors.

Specifically, in many practical applications, the statistical data may be obtained subjectively. The breakdown pattern and repair pattern are more suitably described by linguistic terms such as fast,moderate or slow rather than byprobability distributions based on statistical theory.To deal with imprecise information in making decision, Zadeh introduced the concept of fuzziness. Today, fuzzy set theory is well known formodelling imprecise data and the interest of many researchers is the discussion offuzzy queues. Thus, fuzzy queues are potentially much more useful and realisticthan the commonly used crisp queues. Li and Lee investigated the analyticalresults for two typical fuzzy queues M/F/1/∞and FM/FM/1/∞where F representsfuzzy time and FM represents fuzzified exponential distributions using a general approach based on Zadeh's extension principle. Negi and Lee proposed aprocedure using αcuts and two variable simulation using a cuts and two variablesimulations to analyse fuzzy queues. Using parametric programming Kao constructed the membership functions of the system characteristic for fuzzyqueues and successfully applied them to four simple fuzzy queue modelsM/F/1/∞, F/F/1/∞and FM/FM/1/∞. Recently Chen developed FM/FM/1/K and $FM/FM^{(k)}/1/\infty$ fuzzy systems using the same approach.

Clearly when the machine breakdown or service rate are fuzzy, the systemperformance measures of the machine interference problem will be fuzzy as well.To conserve the fuzziness of input information completely, the performancemeasure should be fuzzy. In this paper, we introduce fuzzy machine interferenceproblem in which the breakdown rate and service rate are all trapezoidal fuzzynumbers.

In order to simplify the calculation of trapezoidal fuzzy numbers, Chen's Function Principle is introduced to calculate the fuzzy system performancemeasures of our proposed model. Function principle is proposed as the fuzzyarithmetic operations of fuzzy numbers in 1985. Also the principle is proven thatit doesnot change the type of membership function under fuzzy arithmeticoperations of fuzzy numbers. In the fuzzy sense, it is reasonable to discuss thegrade of each point of support set of fuzzy numbers for representing fuzzynumbers. Therefore Chen and Hsieh's Graded Mean Integration Representation Method adopted grade as the important degree of each point of support setof generalized fuzzy number. We use it to defuzzify the trapezoidal fuzzy systemperformance measure. First we shall see some of the basic aspects of Fuzzy sets and other related concepts.

2. Fuzzy Set

In a universe of discourse X, a fuzzy subset $\tilde{A}$ on X is defined by themembership function $\mu_{\tilde{A}}(x)$ which maps each element *x* into X to a real numberin the interval [0, 1]. $\mu_{\tilde{A}}(x)$ denotes the grade or degree of membership and it isusually denoted as $\mu_{\tilde{A}}: X \to [0,1]$.

2.1 Fuzzy Number

The fuzzy number $\tilde{A}$ is said to be a trapezoidal fuzzy number if it is fully determined by (a_1, a_2, a_3, a_4) of crisp numbers such that $a_1 < a_2 < a_3 < a_4$ with membership function, representing a trapezoid of the form

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1}, a_1 \le x \le a_2 \\ 1, a_2 \le x \le a_3 \\ \dfrac{x-a_4}{a_3-a_4}, a_3 \le x \le a_4 \end{cases}$$

where a_1, a_2, a_3 and $a_4$are the lower limit, lower mode, upper mode and upperlimit respectively of the fuzzy number. When $a_2 = a_3$, the trapezoidal fuzzynumber becomes a triangular fuzzy number.

3. Machine Interference Fuzzy Model

Consider a conventional machine interference model that consists of mmachines and n repairmen. At any instant of time, a particular machine is in eithergood or bad condition. When a machine breaks down it must be repaired by anyone of the available n repairmen. Normally a repair man is in charge of more thanone machine at a time. When a machine breaks at the time when all repair men arebusy it has to wait and is interfered by the machine being repaired. Suppose themean time to repair a machine is 1/μand the mean time between failures for asingle machine is1/λ. The machine interference problem is a queueing model withfinite calling population in which machines are customers. Suppose thebreakdown rate λand service rate μare represented as fuzzy sets $\tilde{\lambda}, \tilde{\mu}$ respectively. Let $\eta_{\tilde{\lambda}}$ and $\eta_{\tilde{\mu}}$ denote their membership function, we then have

$$\tilde{\lambda} = \{(\lambda, \eta_{\tilde{\lambda}}(\lambda) / \lambda \in X\}$$

$$\tilde{\mu} = \{(\mu, \eta_{\tilde{\mu}}(\mu) / \mu \in Y\}$$

where X and Y are the crisp universal sets of the breakdown rate and the servicerate respectively. The performance measures of the system obtained by usingFunction Principle arithmetic operation of $\tilde{\lambda}, \tilde{\mu}$.

4. Measures Related to Performance

In this study, we consider the following fuzzy performance measures thatare commonly used in traditional queuing theory.
(i) Operator utilization
(ii) Machine availability
(iii) Average number of machines waiting for service
(iv) Average number of machines down
(v) Average downtime duration of a machine
(vi) Average duration of waiting time for repair.

Once a machine is repaired it returns to good condition and is againsusceptible to breakdown. The length of time that a machine remains in goodcondition follows an exponential distribution with breakdown rate $\tilde{\lambda}$ and repairrate $\tilde{\mu}$. Both $\tilde{\lambda}$ and $\tilde{\mu}$ are trapezoidal fuzzy numbers. Each machine has grossproduction rate $\tilde{G}$ that would be achieved if each machine were always available.

Operator fuzzy utilization $\tilde{\rho} = 1 - \tilde{P}_0$.

Machine availability is given by $\tilde{\eta} = \tilde{\rho} \otimes \tilde{\mu} \% m \otimes \tilde{\lambda}$, where $\otimes$ denote co-ordinate wise multiplication. In particular, if $\tilde{A} = (a_1, a_2, a_3, a_4), \tilde{B} = (b_1, b_2, b_3, b_4)$ then $\tilde{A} \otimes \tilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4)$

Production rate of each finished items is given by $\tilde{H} = \tilde{\rho} \otimes \tilde{\mu} \otimes h \% \tilde{\lambda}$

Fuzzy Average number of machines waiting for service is given by $\tilde{L} = m \otimes (1 ! \tilde{\eta}) ! \tilde{\rho}$

Fuzzy Average number of machines down is given by $\tilde{N} = m \otimes (1 ! \tilde{\eta})$

Fuzzy Average downtime duration of a machine is given by $\tilde{T} = 1 ! \tilde{\eta} \% \tilde{\eta} \otimes \tilde{\lambda}$

Fuzzy Average duration of waiting time for repair is given by $\tilde{W} = (1 ! \tilde{\eta}) \% \tilde{\eta} \otimes \tilde{\lambda} ! 1 \% \tilde{\mu}$

Fuzzy Average number of failures per unit is given by $\tilde{\lambda} \otimes \tilde{\eta} \otimes m$

Since all the system performance measures are described by trapezoidalfuzzy numbers the value conserves completely all of the fuzziness of thebreakdown rate, service rate. However manager or practitioners would prefer onecrisp value rather than fuzzy number. In order to overcome this problem wedefuzzify the fuzzy performance measures using Graded Mean IntegrationRepresentation Method based on the integral value of Graded Mean h-level ofgenerated fuzzy number.

Let $\tilde{B}$ be a trapezoidal fuzzy number and be denoted by $\tilde{B}$ = (b1, b2, b3,b4) then we get the Graded Mean Integration Representation of $\tilde{B}$ as

$$P(\tilde{B}) = \frac{\int_0^1 h\left(\frac{b_1 + b_4 + (b_2 - b_1 - b_4 + b_3)h}{2}\right) dh}{\int_0^1 h\,dh} = \frac{b_1 + 2b_2 + 2b_3 + b_4}{6}$$

5. Numerical Example

To demonstrate the validity of the proposed approach, a numerical exampleinspired by Gross and Harris is solved.The W.E. Finish Machine Shop Company have five turret lathes. Thesemachine are break down periodically and the company has one repairmen toservice the lathes when they breakdown. When the lathe is fixed, the time until thenext breakdown is exponentially distributed with a fuzzy rate that can berepresented by a trapezoidal fuzzy number $\tilde{\lambda}$ = (3, 4, 5, 6). The repair time foreach repairman is exponentially distributed with a fuzzy rate that can berepresented a trapezoidal fuzzy number $\tilde{\mu}$ = (7, 8, 9, 10). The shop managerwishes to know the average number of lathes operational at any given time, theexpected "down time" of a lathe that requires repair and expected idle time ofeach repairman is given by $\tilde{\lambda} * \tilde{\mu} = (0.3, 0.44, 0.625, 0.857)$, where $*$ represent co-ordinate wise division. In particular, if $\tilde{A} = (a_1, a_2, a_3, a_4), \tilde{B} = (b_1, b_2, b_3, b_4)$ then $\tilde{A} * \tilde{B} = (a_1 / b_1, a_2 / b_2, a_3 / b_3, a_4 / b_4)$.

Fuzzy time of operator utilization $\tilde{\rho}$ = (0.8608, 0.9464, 0.982, 0.994)

Fuzzy Machine availability $\tilde{\eta}$ = (0.201, 0.3028, 0.4419, 0.6629)

Fuzzy number of machines waiting for service $\tilde{L}_q$ = (0.6911, 1.8085, 2.5396,3.1342)

Fuzzy average number of machines down $\tilde{N}$ = (1.6855, 2.7905, 3.486, 3.995)

Using Graded mean Integration Method we find that, ρ = 0.951, η= 0.392, Lq= 2.0869,
N = 3.0389

6. Conclusion

When the breakdown rate and service rate are fuzzy numbers, theperformance measures of the machine interference system is also fuzzy numbers.By using Function Principle as a fuzzy arithmetical operator of fuzzy trapezoidalnumbers, the system performance measures can be derived.

Clearly fuzzy average number of machines down is [1.6855, 3.995]indicating that the expected waiting time of lathes for repair will never below1.6855 or exceed 3.995 approximately. Consider the fuzzy average number oflathes waiting for repair $\tilde{L}_q$ is [0.6911, 3.1342]. Similarly, fuzzy machineavailability is [0.201, 0.6629]. The above information obtained from theproposed approach completely maintain the fuzziness of input date, thus they candescribe the machine interference problem more accurately. It will be useful indesigning machine interference system. In this paper thus, all fuzzy performancemeasures are expressed by a fuzzy number that completely conserves the fuzzinessof input information when some parameter in the machine interference model arefuzzy.

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