A Deterministic Inventory Model with Selling Price Dependent Demand Rate, Quadratic Holding Cost and Quadratic Time Varying Deteriorating Rate

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Abstract: ***In this paper we consider an inventory model for deteriorating products having demand which is function of selling price, here the deterioration rate is time varying with quadratic function of time. Holding cost is considered as a quadratic function of time. This model is developed to find the total cost of the inventory system. Suitable numerical example and sensitivity analysis also discussed. At the end of the paper, the optimum total cost and total order quantity has been calculated by the change of the value of the each parameter and graphs shows the behavior of the relation between parameter and total inventory cost.***

Keywords: Inventory, demand, Selling Price, Deteriorating items, Holding Cost Optimum cost

1. Introduction

In the Economic Order Quantity model, we assumed that demand is either constant or time dependent, but independent of stock status. However, in the market situation customers are attracted by display of units in the market. The presence of inventory has a motivational effect on the people around it and attracts the people to buy them.

Many mathematical models have been developed for controlling inventory. In several models, it is assumed that products have infinite shelf time but actually deterioration play a very important role in inventory.

In recent year, inventory problems for deteriorating items have been widely studied after Ghare and Schrader [1]. Philip [2] extended Ghare and Schrader [1] model with a three parameter Weibull distribution rate and no shortage. Deb and Chaudhari [4] derived inventory models with time dependent deterioration rate.

Another important factor is demand. It may be depend on selling price of items. A good number of authors have studied inventory models for deterioration, items considering different demand and deterioration rate.

Selling price is most important factor in selecting an item for use. Yadav.et.al [9] has developed a deterministic inventory model for deterioration items on which demand is taken as a function of selling price and variable rate of deterioration is taken as a linear function of time and a storage time dependent holding cost.
Many inventory models in past were developed under the assumption that the holding cost is constant for the entire inventory cycle but it is particularly false. In the storage of deterioration items such as food products the longer these food products are kept in storage the more sophisticated storage facilities and service needed and there for the higher holding cost should be assumed as a function of time. Various functions describing holding cost were considered by severed researchers like Roy [6], Sharma and Sharma [8], Weiss [3] and Goh [5].

In this paper, we have developed an inventory model for deteriorating items in which the deterioration rate and holding cost are quadratic and shortages are not allowed. The model is solved by minimizing the total cost. As a special case, this model reduces to well- known result. Demand rate is a function of selling price with permissible duly in payments.

2. Paper Structure

1. The demand rate is the function of selling price $D\{p(t)\} = a - b f(t)$ where 'a' is fixed demand; $a, b > 0$ and $a >> b$
2. $P(t)$ is the selling price of the item at time 't' and consider as $P(t) = pe^{rt}$ is the selling price per unit at time t and P is selling price of the item at time $t = 0$.
3. There is no repair or replacement of the deteriorated unit during the cycle time under consideration.
4. Shortages are not allowed and lead time is zero.
5. r is the inflation rate.
6. $I(t)$ is the level of inventory at any instant of time 't'.
7. C is the unit purchase cost & K is the ordering cost per order.
8. T is the cycle time.
9. The holding cost is quadratic with time dependent $H(t) = h + \delta t + \Delta t^2 \quad h > 0,\ \delta > 0,\ \Delta > 0$.
10. The deterioration rate is time varying $\theta(t) = \alpha + \beta t + \gamma t^2 \quad \beta > 0,\ t > 0,\ \alpha \geq 0,\ \gamma > 0$

3. Model Development

A variable function θ(t) of on hand inventory deteriorates per unit time. In the present model, the function θ(t) is assumed quadratic of the form

$$\theta(t) = \alpha + \beta t + \gamma t^2$$
$$\beta > 0,\ t > 0,\ \alpha \geq 0,\ \gamma > 0$$

$H(t)$ holding cost of the item at time t
$H(t) = h + \delta t + \Delta t^2 \quad h > 0,\ \delta > 0,\ \Delta > 0$ Generally the inventory level decreases mainly due to demand and

partly due to deterioration of units. the differential equation governing the system in the interval (0, T) is given by

$$\frac{d\,I(t)}{dt} = -\theta(t)I(t) - D\{(p(t)\},\quad 0 \le t < T \quad (1)$$

$$= -(\alpha + \beta t + \gamma t^2)I(t) - (a - bpe^{rt}) \quad (2)$$

Solution of the differential equation after adjusting constant of integration and initial condition, $t = 0,\ I(t) = I(0)$

$$I(t) = \exp\left\{-\left(\alpha t + \beta\frac{t^2}{2} + \gamma\frac{t^3}{3}\right)\right\}\left[-a\left(t + \alpha\frac{t^2}{2} + \beta\frac{t^3}{6} + \gamma\frac{t^4}{12}\right) + bp\left\{t + \left(\frac{\alpha + r}{2}\right)t^2 + \left(\frac{\beta + 2rd + r^2}{6}\right)t^3 + \left(\frac{\gamma}{12} + \frac{r\beta}{8} + \frac{r^2\beta}{8} + \frac{r^3}{24}\right)t^4 + \left(\frac{\gamma r}{15} + \frac{r^2\beta}{20} + \frac{r^3\alpha}{30}\right)t^5 + \left(\frac{r^2\gamma}{36} + \frac{\beta r^3}{72}\right)t^6 + \frac{\gamma r^3}{126}t^7\right\} + I(0)\right] \quad (3)$$

inventory without decay $I_w(t)$ at time ' t ' is given by

$$\frac{d}{dt}I_w(t) = -(a - bpe^{rt})$$

$$\Rightarrow \quad I_w(t) = -at + \frac{bpe^{rt}}{r} + I_0 - \frac{bp}{r} \quad (4)$$

[using initial condition at $t = 0, I(t) = I(0)$]

The stock loss Z (t) due to decay in [0, T] is given by

$$Z(t) = I_w(t) - I(t)$$

$$= -at - \frac{bp(1 - e^{rt})}{r} + I(0) - I(t) \quad (5)$$

Equation (3) gives

$$I(0) = I(t)\exp\left(\alpha t + \beta\frac{t^2}{2} + \gamma\frac{t^3}{3}\right) + a\left(t + \alpha\frac{t^2}{2} + \beta\frac{t^3}{6} + \gamma\frac{t^4}{12}\right) - bp\left\{t + \left(\frac{\alpha + r}{2}\right)t^2 + \left(\frac{\beta + 2rd + r^2}{6}\right)t^3 + \left(\frac{\gamma}{12} + \frac{r\beta}{8} + \frac{r^2\beta}{8} + \frac{r^3}{24}\right)t^4 + \left(\frac{\gamma r}{15} + \frac{r^2\beta}{20} + \frac{r^3\alpha}{30}\right)t^5 + \left(\frac{r^2\gamma}{36} + \frac{\beta r^3}{72}\right)t^6 + \frac{\gamma r^3}{126}t^7\right\} \quad (6)$$

Substituting value of I (0) from (6) in equation (5), we get

$$Z(t) = -at - \frac{bp[\![(1 - e]\!]^{rt})}{r} + I(t)\left[\exp\left(\alpha t + \beta\frac{t^2}{2} + \gamma\frac{t^3}{3}\right) - 1\right] + a\left(t + \alpha\frac{t^2}{2} + \beta\frac{t^3}{6} + \gamma\frac{t^4}{12}\right) - bp\left\{t + \left(\frac{\alpha + r}{2}\right)t^2 + \left(\frac{\beta + 2rd + r^2}{6}\right)t^3 + \left(\frac{\gamma}{12} + \frac{r\beta}{8} + \frac{r^2\beta}{8} + \frac{r^3}{24}\right)t^4 + \left(\frac{\gamma r}{15} + \frac{r^2\beta}{20} + \frac{r^3\alpha}{30}\right)t^5 + \left(\frac{r^2\gamma}{36} + \frac{\beta r^3}{72}\right)t^6 + \frac{\gamma r^3}{126}t^7\right\} \quad (7)$$

At $t = T$, we get

$$Z(t) = -aT - \frac{bp[\![(1 - e]\!]^{rT})}{r} + a\left(T + \alpha\frac{T^2}{2} + \beta\frac{T^3}{6} + \gamma\frac{T^4}{12}\right) - bp\left\{T + \left(\frac{\alpha + r}{2}\right)T^2 + \left(\frac{\beta + 2rd + r^2}{6}\right)T^3 + \left(\frac{\gamma}{12} + \frac{r\beta}{8} + \frac{r^2\beta}{8} + \frac{r^3}{24}\right)T^4 + \left(\frac{\gamma r}{15} + \frac{r^2\beta}{20} + \frac{r^3\alpha}{30}\right)T^5 + \left(\frac{r^2\gamma}{36} + \frac{\beta r^3}{72}\right)T^6 + \frac{\gamma r^3}{126}T^7\right\} \quad (8)$$

Note that I(t)= 0

Order quantity is given by $Q_T = Z(T) + \int_0^T (a - bpe^{rt})dt$

$$= a\left(T + \alpha\frac{T^2}{2} + \beta\frac{T^3}{6} + \gamma\frac{T^4}{12}\right) - bp\left\{T + \left(\frac{\alpha + r}{2}\right)T^2 + \left(\frac{\beta + 2rd + r^2}{6}\right)T^3 + \left(\frac{\gamma}{12} + \frac{r\beta}{8} + \frac{r^2\beta}{8} + \frac{r^3}{24}\right)T^4 + \left(\frac{\gamma r}{15} + \frac{r^2\beta}{20} + \frac{r^3\alpha}{30}\right)T^5 + \left(\frac{r^2\gamma}{36} + \frac{\beta r^3}{72}\right)T^6 + \frac{\gamma r^3}{126}T^7\right\} \quad (9)$$

Also $I(0) = Q_T$ implies

$$I(t) = \exp\left[-\left(\alpha t + \beta\frac{t^2}{2} + \gamma\frac{t^3}{3}\right)\right]\left[a\left((T - t) + \frac{\alpha(T^2 - t^2)}{2} + \beta\frac{(T^3 - t^3)}{6} + \frac{\gamma}{12}(T^4 - t^4)\right) - bp\left\{(T - t) + \left(\frac{\alpha + r}{2}\right)(T^2 - t^2) + \left(\frac{\beta + 2rd + r^2}{6}\right)(T^3 - t^3) + \left(\frac{\gamma}{12} + \frac{r\beta}{8} + \frac{r^2\beta}{8} + \frac{r^3}{24}\right)(T^4 - t^4) + \left(\frac{\gamma r}{15} + \frac{r^2\beta}{20} + \frac{r^3\alpha}{30}\right)(T^5 - t^5) + \left(\frac{r^2\gamma}{36} + \frac{\beta r^3}{72}\right)(T^6 - t^6) + \frac{\gamma r^3(T^7 - t^7)}{126}\right\}\right] \quad (10)$$

As stated earlier the holding cost is assumed to be a quadratic function of time i.e.

$$H(t) = \text{h} + \delta t + \Delta t^2$$

In this case

$$C(T,P) = \frac{K}{T} + \frac{CQ_T}{T} + \frac{1}{T}\int_0^T h(t).I(t)dt \qquad (11)$$

$$= \frac{K}{T} + C\left[a\left(1 + \frac{\alpha T}{2} + \frac{\beta T^2}{6} + \gamma\frac{T^3}{12}\right) - bp\left\{1 + \left(\frac{\alpha + r}{2}\right)T + \left(\frac{\beta + 2rd + r^2}{6}\right)T^2 + \left(\frac{2\gamma + 3r\beta + 3r^2\alpha + r^3}{24}\right)T^3\right\}\right] + h(a - bp)\frac{T}{2} + \left(\frac{a - bp}{6}\right)(h\alpha + \delta)T^2 - \frac{bprh}{3}T^2 + \left(\frac{2h\beta + a\delta}{24}\right)(a - bp)T^3 - \frac{bpr}{8}h\alpha T^3 - \frac{bpr^2T^3}{8} - \frac{bprT^3}{8} + \left(\frac{\Delta}{12} - \frac{h\alpha^2}{8}\right)(a - bp)T^3$$

For minimum total average cost, the necessary criterion is

$$\frac{d}{dT}\{C(T,P)\} = 0$$

$$\Rightarrow -K + T^2C\left[a\left(\frac{\alpha}{2} + \frac{\beta T}{3} + \gamma\frac{T^2}{4}\right) - bp\left\{\left(\frac{\alpha + r}{2}\right) + \left(\frac{\beta + 2rd + r^2}{6}\right)T + \left(\frac{2\gamma + 3r\beta + 3r^2\alpha + r^3}{24}\right)T^2\right\}\right] + h(a - bp)\frac{T^2}{2} + \left(\frac{a - bp}{3}\right)(h\alpha + \delta)T^2 - \frac{bprh}{3}T^2 + \left(\frac{2h\beta + a\delta}{24}\right)(a - bp)T^3 - \frac{bpr}{8}h\alpha T^3 - \frac{bpr^2T^3}{8} - \frac{bprT^3}{8} + \left(\frac{\Delta}{12} - \frac{h\alpha^2}{8}\right)(a - bp)T^4 \qquad (12)$$

for fixed p which can be solved for C(T, P) numerically by using theory of equations.

$$\frac{d^2[C(T,P)]}{dT^2} = C\left[a + bp\left\{(a + r)T + T^2(\beta + 2r\alpha + r) + \left(\frac{2\gamma + 3r\beta + 3r^2\alpha + r^3}{2}\right)T^3\right\} + h(a - bp)T + (a - bp)(h\alpha + \delta)T^2 - 2bprhT^2 + \frac{3}{8}(2h\beta + \alpha\delta)(a - bp)T^2 + \frac{3}{2}bprh\alpha T^3 + \frac{3}{2}bpr^2T^3 + \frac{3}{2}bpr\delta T^3 + 12\left(\frac{\Delta - 3h\alpha^2}{24}\right)(a - bp)T^3\right] \qquad (13)$$

Furthermore, Equation (13) shows that C(T,P) is convex with respect to T, so For a given Positive integer T, the optimal value of P can be obtained from equation (12).

$$\frac{d^2[C(T,P)]}{dT^2} > 0$$

Special case:

$a = R, \quad \alpha = 0, \quad \beta = 0, \quad \gamma = 0, \quad b = 0, \quad \delta = 0, \quad \Delta = 0$

Substituting all the above values in equation (11) we get

$$C(T) = \frac{K}{T} + CR + \frac{hRT}{2}$$

which is the standard result for non decaying inventory.

4. Numerical Example

To illustrate the results obtained for the suggested model, a numerical example with the following parameter value is considered.

Let $\alpha = 0.002$, $\beta = 1$, $\gamma = 2$, $a = 1.5$, $b = 1.9$, $P = 2$, $r = 0.52$, $T = 0.75$, $C = 2.5$, $\delta = 00.7$, $\Delta = 2$, h=0.52. Substitute these values in equation (11) the optimal solution (total inventory cost) is 323.9744 with the help of Matlab software.

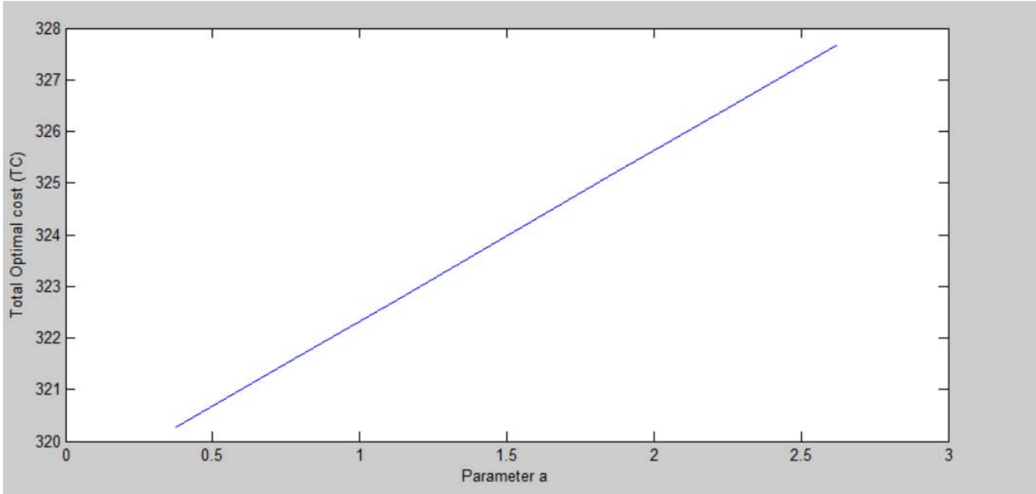
5. Sensitivity Analysis

Now we will test the sensitivity of the optimal solution with respect to demand parameter a & b and α, β, γ in deterministic model.

Table 1: Relation between demand parameter a and optimal cost (TC)

Parameter a	% Change	Value of the parameter a	Optimal cost or total cost for deterministic model (TC)
1.5	+75%	2.625	327.6817
1.5	+50%	2.25	326.4459
1.5	+25	1.875	325.2102
1.5	-25%	1.125	322.7387
1.5	-50%	0.75	321.5029
1.5	-75%	0.375	320.2672

From the table-1, conclusion is that when the value of the parameter a decreases, the total cost of the deterministic model will also decreases.



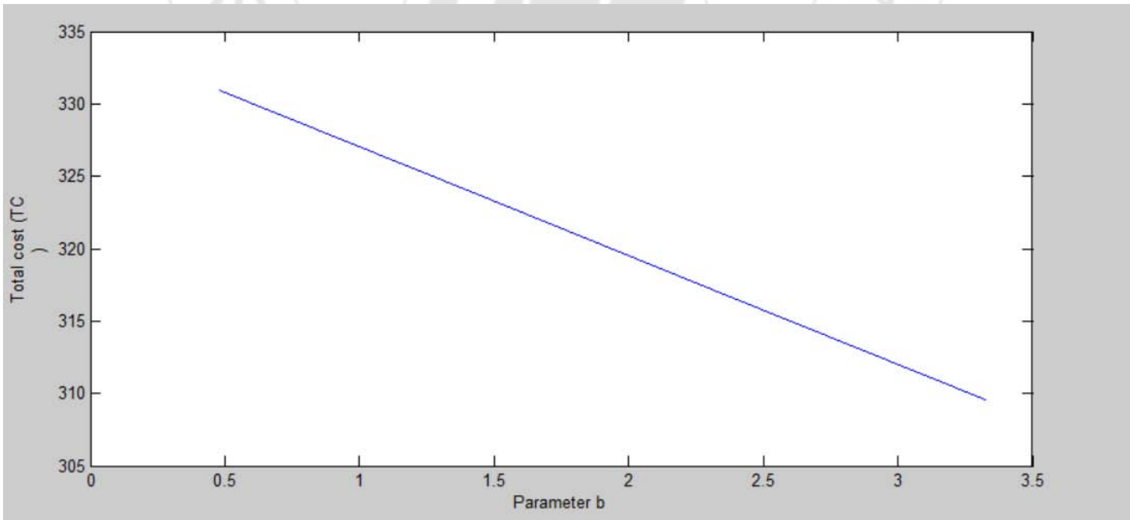
Graph 1

Since a is fixed demand parameter and if demand is increasing then total cost also increases so linear graph is showing the relation between the demand and the total cost.

Table 2: Relation between *b* and *TC*

Parameter *b*	% Change	Value of the parameter *b*	Optimal cost or total cost for deterministic model (*TC*)
1.9	+75%	3.325	309.5408
1.9	+50%	2.85	313.1163
1.9	+25%	2.375	316.6917
1.9	-25%	1.425	323.8427
1.9	-50%	0.95	327.4181
1.9	-75%	0.475	330.9936

From the table-2, conclusion is that when the value of the parameter *b* decreases, the total cost of the deterministic model increases.



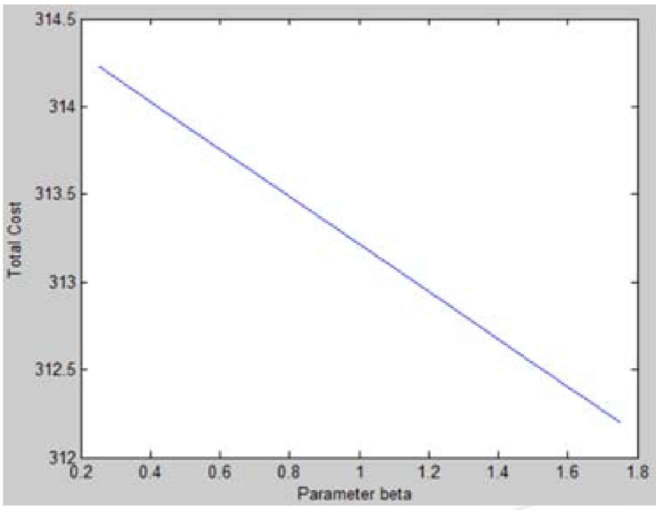
Graph 2

Graph 2: Shows that if parameter *b* increases then the total cost decreases of the deterministic inventory model.

Table 3: Relation between β and *TC*

Parameter β	% Change	Value of the parameter β	Optimal cost or total cost for deterministic model (*TC*)
1	+75%	1.75	312.2018
1	+50%	1.5	312.5406
1	+25	1.25	312.8782
1	-25%	0.75	313.5547
1	-50%	0.5	313.8929
1	-75%	0.25	314.2311

From the table-3, conclusion is that when the value of the parameter β decreases, the total cost of the deterministic model increases.



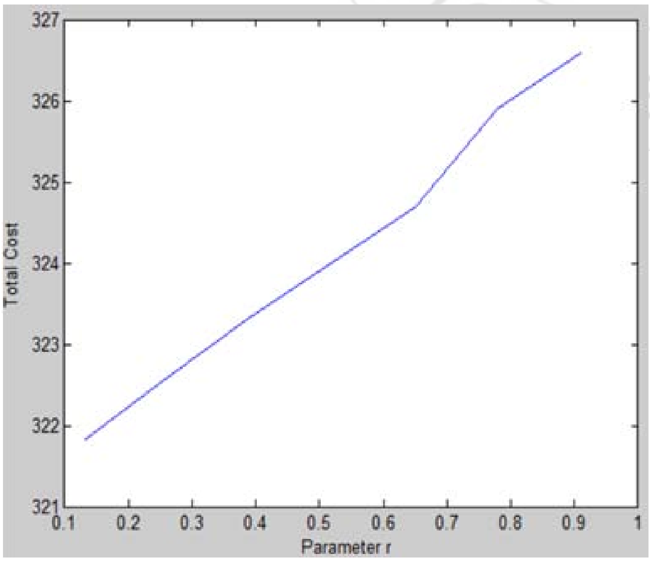
Graph 3

When Parameter β increases then total cost decreases and graph is showing the relation between β and the total cost.

Table 4: Relation between parameter *r* (i.e. inflation rate) and *TC*

Parameter ***r***	% Change	Value of the parameter *r*	Optimal cost or total cost for deterministic model (*TC*)
0.52	+75%	0.91	326.5904
0.52	+50%	0.78	325.9103
0.52	+25%	0.65	324.6954
0.52	-25%	0.39	323.3213
0.52	-50%	0.26	322.5904
0.52	-75%	0.13	321.823

From the table-4, conclusion is that when the value of the parameter *r* decreases, the total cost of the deterministic model decreases.



Graph 4

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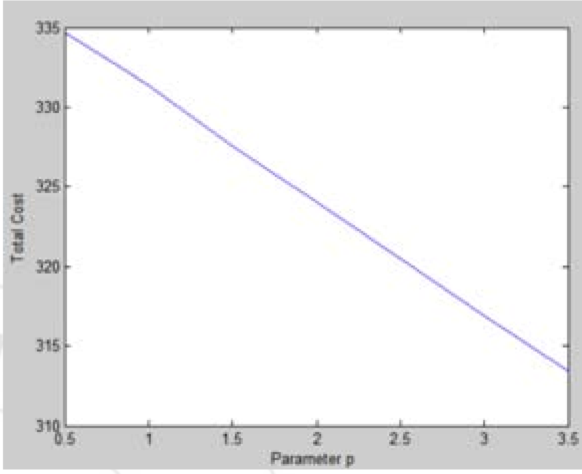
When Parameter *r* increases then total cost increases and graph is showing the relation between parameter *r* and the total cost.

Table 5: Relation between parameter *p* and *TC*

Parameter *p*	% Change	Value of the parameter *p*	Optimal cost or total cost for deterministic model (*TC*)
2	+75%	3.5	313.3696
2	+50%	3	316.9239
2	+25%	2.5	320.4783
2	-25%	1.5	327.5896
2	-50%	1	331.3696
2	-75%	0.5	334.6956



Graph 5

When Parameter *p* increases then total cost decreases and graph is showing the relation between parameter *p* and the total cost.

6. Conclusion

The prime objective of this study is the formulation of a deterministic inventory model for the deteriorating items under price dependent demand rate and time dependent holding cost and when the supplier offers a trade credit for a specified period. Deterioration is taken as quadratic function of time. The model is solved for cost minimization. Inflation is considered as constant and shortages are not allowed. Special case is also discussed in which this model reduces to standard result for non-decaying inventory. Our research implies that the effect of inflation and time value of money on the present value of total cost is more significant and highlights that total cost decreases as the inflation rate increases.

Thus, this model incorporates some realistic features that are likely being associated with some kinds of inventory. Finally, we provide the numerical example and sensitivity analysis for the illustration and inference of the theoretical results. Behaviors of different parameters have been illustrated through the numerical example and sensitivity analysis.

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