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Axially Symmetric Bianchi Type-I Bulk Viscous Cosmological Model with Lyra Geometry

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Abstract: ***We have discussed in this paper spatially homogeneous and anisotropic axially symmetric Bianchi type-I cosmological model with Lyra geometry in the presence of bulk viscous fluid. Here we have considered the coefficient of bulk viscosity ζ as a quadratic function of Hubble parameter H (i,e $\zeta = \zeta_0 + \zeta_1 H + \zeta_2 H^2$),Where $\zeta_0, \zeta_1, \zeta_2$ are constant. The physical and kinematical properties of the models are discussed.***

Keywords: Bianchi space-time, Hubble's parameter, Bulk viscous fluid, Lyra geometry

1. Introduction

It has been discussed in this literature that during the evolution of the universe bulk viscosity could arise in many circumstances and could lead to an effective mechanism of galaxy formation. The possibility of bulk viscosity leading to inflationary-like solutions in general relativistic FRW models is discussed by padmanabhan and chitre. Johri and Sudharsan have pointed out that the bulk viscosity leads to inflationary solutions in Brans-Dicke theory. Bulk viscocity is supposed to play very important role in the early evolution of the universe. There are many circumstances during the evolution of the universe in which bulk viscosity could arise. The bulk viscosity coefficient determines the magnitude of the viscous stress relative o the expansion. Bulk viscosity is associated with the GUT phase transition and string creation. Thus, we should consider the presence of a material distribution other than a perfect fluid to have realistic cosmological models for a review on cosmological models with bulk viscosity. On the other hand, cosmological models of a fluid with viscosity play a significant role in the study of evolution of universe. It is well known that at an early stage of the universe when neutrino decoupling occurs, the matter behaves like a viscous fluid. The coefficient of viscosity is known to decrease as the universe expands. Viscous fluid cosmological models in early universe have been widely

The Bianchi cosmologies play an important role in theoretical cosmology and have been much studied since the 1960s. A Bianchi cosmology represents a spatially homogeneous universe, since by definition the spacetime admits a three parameter group of isometries whose orbits are space like hyper-surfaces. These models can be used to analyze aspects of the physical Universe which pertain to or which may be affected by anisotropy in the rate of expansion, for example , the cosmic microwave background radiation, nucleo synthesis in the early universe, and the question of the isotropization of the universe itself Spatially homogeneous cosmologies also play an important role in attempts to understand the structure and properties of the space of all cosmological solutions of Einstein field equations.

Spatially homogeneous and anisotropic cosmological models play a significant role in the description of large scale behaviour of universe and such models have been widely studied in framework of general relativity in search of a realistic picture of the universe in its early stages. Recently Pradhan et al. and Saha et al. have studied homogeneous and anisotropic B-I space time in different context. In this paper we have investigated a new B-I cosmological model with bulk viscous fluid in Lyra geometry. The simplest of anisotropic models describe the anisotropic effects are Bianchi type-I spatially homogeneous models whose spatial sections are flat but the expansion or contraction rate is directional dependent. The advantage of theses anisotropic models are that they have a significant role in the description of evolution of early phase of the universe and they help in finding more general cosmological models than the isotropic FRW models. For studying the possible effects of anisotropy in the early universe on present day observations many researchers (Huang 1990;Chimento et al.1997;Lima 1996;Lima and Carvalho1994;Pradhan et al.2004,2006;Saha 2005,2006) have investigated Bianchi-type I models from different point of view. At the present state of evolution the universe is spherically symmetric and the matter distribution in it is on the whole isotropic and homogeneous . But in its early stages of evolution it could have not had such a smoothed out picture close to the big bang singularity, neither the assumbtion of spherical symmetry nor of isotropy can be strictly valid. For simplification and description of the large scale behaviour of the actual universe ,Bianchi-Ispace time have been widely studied. In order to study problems like the formation of galaxies and the process of homogenization and isotropization of the universe, it is necessary to study problems relating to anisotropic space time.

2. The Metric and the Field Equation

We consider homogeneous an axially symmetric Bianchi type-I metric described by the element

$$ds^2 = -dt^2 + A^2 dx^2 + B^2\left(B^2 dy^2 + dz^2\right) \quad (1)$$

Where A and B are function of cosmic time t only. The energy momentum tensor are given by

$$T_{ij} = (\rho + \bar{p})v_i v_j + \bar{p} g_{ij} \tag{2}$$

With

$$\bar{p} = p - \xi v^i{}_{;i} \tag{3}$$

Which satisfies the linear equation of state

$$p = \omega\rho \tag{4}$$

Where p is isotropic pressure, ρ is the energy density of matter, ξ is the coefficient of bulk viscosity and v^i is the flow vector of the fluid satisfying $v_i v^i = -1$. The Einstein's field equation based on Lyra geometry

$$R^i{}_j - \frac{1}{2} g^i{}_j R + \frac{3}{2}\phi_i\phi_j - \frac{3}{4} g^i{}_j \phi_k \phi^k = -8\pi G T^i{}_j \tag{5}$$

The Einstein's field equation (5)for the line element (1) lead to the following system of equations

$$\frac{2\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} = \lambda\rho + \frac{3}{4}\beta^2 \tag{6}$$

$$\frac{2\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} = -\lambda\bar{p} - \frac{3}{4}\beta^2 \tag{7}$$

$$\frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{\ddot{A}}{A} = -\lambda\bar{p} - \frac{3}{4}\beta^2 \tag{8}$$

The energy conservation equation gives

$$\lambda\dot{\rho} + \frac{3}{2}\beta\dot{\beta} + \left[\lambda(\rho + \bar{p}) + \frac{3}{2}\beta^2\right]\left(\frac{\dot{A}}{A} + \frac{2\dot{B}}{B}\right) = 0 \tag{9}$$

The usual energy conservation equation $T^j{}_{i;j} = 0$ gives

$$\dot{\rho} + [\lambda(\rho + \bar{p})]\left(\frac{\dot{A}}{A} + \frac{2\dot{B}}{B}\right) = 0 \tag{10}$$

We define average scale factor

$$a^3 = AB^2 \tag{11}$$

From equations (7),(8) and (10)

$$\frac{\dot{A}}{A} = \frac{\dot{a}}{a} + \frac{2k_1}{3a^3} \tag{12}$$

$$\frac{\dot{B}}{B} = \frac{\dot{a}}{a} - \frac{k_1}{3a^3} \tag{13}$$

From (12) and (13)

$$A = k_2 a \exp\left(\frac{2k_1}{3}\int\frac{dt}{a^3}\right) \tag{14}$$

$$B = k_3 a \exp\left(-\frac{k_1}{3}\int\frac{dt}{a^3}\right) \tag{15}$$

Where k_2 and k_3 are constant of integration such that $k_2 k_3{}^2 = 1$

We know $$H = \frac{\dot{a}}{a} = \frac{1}{3}\left(\frac{\dot{A}}{A} + \frac{2\dot{B}}{B}\right) \tag{16}$$

$$q = -\frac{a\ddot{a}}{\dot{a}^2} \tag{17}$$

$$\theta = v^i{}_{;i} \tag{18}$$

$$\sigma^2 = \frac{1}{2}\sigma_{ij}\sigma^{ij} \tag{19}$$

$$\theta = 3\frac{\dot{a}}{a} = \frac{\dot{A}}{A} + \frac{2\dot{B}}{B} \tag{20}$$

From $\sigma^2 = \frac{1}{2}\left[\sum_{i=1}^{3} H_i{}^2 - 3H^2\right]$ we get

$$\sigma = \frac{1}{\sqrt{3}}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) \tag{21}$$

From this we get

$$\dot{\sigma} = -3\sigma H \tag{22}$$

Equation (6),(7),(8) and (10) can be written in terms of H σ and q as

$$8\pi G\rho + \Lambda = 3H^2 - \sigma^2 \tag{23}$$

$$8\pi G\bar{p} - \Lambda = (2q - 1)H^2 - \sigma^2 \tag{24}$$

$$\dot{\rho} + 3(\rho + \bar{p})H = 0 \tag{25}$$

3. Basic Assumption:

According to Berman

$$H = la^{-n} = (AB^2)^{-\frac{n}{3}} \tag{26}$$

where $l \rangle 0$ and $n \geq 0$ are constant.

From (16) and (26) we get

$$\dot{a} = la^{-n+1} \tag{27}$$

$$\ddot{a} = -l^2(n-1)a^{-2n+1} \tag{28}$$

From Equation (17) we get

$$q = n - 1 \tag{29}$$

For $n \neq 0$ (27) gives $$a = (\ln t + c_1)^{\frac{1}{n}} \tag{30}$$

For $n = 0$ (27) gives $$a = \exp\{l(t - c_2)\} \tag{31}$$

And

$$H = l(\ln t + c_1)^{-1} \tag{32}$$

Meng and Ma 2012 i,e

$$\xi = \xi_0 + \xi_1 H + \xi_2 H^2 \tag{33}$$

Where ξ, ξ_1, ξ_2 are constant

From Equation (25) gives

$$\dot{\rho}+3(1+w)\rho H=\rho\xi H^{2} \quad (34)$$

Solution of field Equations:

Cosmology for $n \neq 0$

$$a=(\ln t+c_1)^{\frac{1}{n}}$$

$$\dot{a}=l(\ln t+c_1)^{\frac{1}{n}-1}$$

$$\ddot{a}=l^2(1-n)(\ln t+c_1)^{\frac{1}{n}-2}$$

$$A=k_3(\ln t+c_1)^{\frac{1}{n}}\exp\left[\frac{2k_1}{3l(n-3)}(\ln t+c_1)^{\frac{n-3}{n}}\right] \quad (35)$$

$$B=k_4(\ln t+c_1)^{\frac{1}{n}}\exp\left[\frac{k_1}{3l(3-n)}(\ln t+c_1)^{\frac{n-3}{n}}\right] \quad (36)$$

$$H=l(\ln t+c_1)^{-1} \quad (37)$$

From (33),(34) and (37),

$$\rho=\frac{q\xi_0}{n\{3(1+w)-n\}t}+\frac{q\xi_1}{n^2\{3(1+w)-2n\}t^2}+\frac{q\xi_2}{n^3\{3(1+w)-2n\}t^3}+k_5(\ln t+c_1)^{\frac{-3(1+w)}{n}} \quad (38)$$

And,

$$G=\frac{1}{12\pi(\ln)^{\frac{6}{n}}}\left[\frac{3(\ln t+c_1)^{\frac{6}{n}}-k_2{}^2n(\ln t+c_1)^2}{\left(\frac{3\xi_0}{3+3w-n}\right)(\ln t+c_1)^{\frac{6+n}{n}}+\frac{6\xi_1}{n(3+3w-2n)}(\ln t+c_1)^{\frac{6}{n}}+\frac{9\xi_2}{n^2(3+3w-2n)}(\ln t+c_1)^{\frac{6-n}{n}}+k_5(\ln t+c_1)^{\frac{2n+3(1-w)}{n}}}\right] \quad (39)$$

$$\Lambda=\frac{1}{3(\ln)^{\frac{6}{n}}}\left[\frac{k_2{}^2n(\ln t+c_1)^2-3(\ln t+c_1)^{\frac{6}{n}}}{\left(\frac{3\xi_0}{3+3w-n}\right)(\ln t+c_1)^{\frac{6+n}{n}}+\frac{6\xi_1}{n(3+3w-2n)}(\ln t+c_1)^{\frac{6}{n}}+\frac{9\xi_2}{n^2(3+3w-2n)}(\ln t+c_1)^{\frac{6-n}{n}}+k_5(\ln t+c_1)^{\frac{2n+3(1-w)}{n}}}\right]$$
$$\times\left\{\frac{9\xi_0}{3(3+3w-n)}+\frac{9\xi_1}{n^2(3+3w-2n)(\ln t+c_1)^2}+\frac{9\xi_2}{n^3(3+3w-2n)(\ln t+c_1)^3}+\frac{k_5}{(\ln t+c_1)^{\frac{3(1+w)}{n}}}\right\} \quad (40)$$
$$+\frac{1}{n^2(\ln t+c_1)^2}-\frac{k_2{}^2}{3(\ln t+c_1)^{\frac{6}{n}}}$$

Cosmology for $n=0$

From $\frac{\dot{a}}{a}=la^{-n}$ we get

$$a=\exp\{l(t-c_2)\}$$
$$=e^{lT}$$

Where $l(t-c_2)=lT$

$$\dot{a}=le^{lT}$$

$$\frac{\dot{a}}{a}=l$$
$$H=l \quad (41)$$

From (33) we get

$$\xi=\xi_0+\xi_1 l+\xi_2 l^2 \quad (42)$$

Equation (34) implies

$$\dot{\rho}+3(1+w)\rho l=q\{(\xi_0+\xi_1 l+\xi_2 l^2)l^2\}$$

So,

$$\rho=\frac{q(\xi_0+\xi_1 l+\xi_2 l^2)}{3(1+w)l}+k_5e^{-3(1+w)lT} \quad (43)$$

$$A=\alpha\exp\left\{l(t-c_2)-\frac{2k_1}{9l}e^{-3l(t-c_2)}\right\} \quad (44)$$

$$B=\beta\exp\left\{l(t-c_2)-\frac{2k_1}{9l}e^{-3l(t-c_2)}\right\} \quad (45)$$

$$G=\frac{1}{\frac{6}{n}}\left[\frac{3(\ln t+c_1)^{\frac{6}{n}}-k_2{}^2 n(\ln t+c_1)^2}{\left(\frac{3\xi_0}{3+3w-n}\right)(\ln t+c_1)^{\frac{6+n}{n}}+\frac{6\xi_1}{n(3+3w-2n)}(\ln t+c_1)^{\frac{6}{n}}+\frac{9\xi_2}{n^2(3+3w-2n)}(\ln t+c_1)^{\frac{6-n}{n}}+k_5(\ln t+c_1)^{\frac{2n+3(1-w)}{n}}}\right]$$

4. Conclusion

Generally, the models are expanding, shearing and rotating. In all these models, we observe that they do not approach isotropy for large values of time T in the presence of magnetic field. It is seen that the solutions obtained by Bali and Meena and Pradhan and Rai are particular cases of our solutions. The coefficient of bulk viscosity is assumed to be a power function of mass density. The effect of bulk viscosity is to introduce a change in the perfect fluid model. We also observe here that the conclusion of Murphy about the absence of a big bang type of singularity in the finite past in models with bulk viscous fluid is, in general, not true. The cosmological constant in all models are decreasing function of time and they all approach a small positive value at late time. These results are supported by the results from recent supernovae Ia time. These results are supported by the results from recent supernovae Ia observations recently obtained by High - Z Supernova Team and Supernova Cosmological Project .We have also discussed the cosmology using Berman's law.

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