k–Super Root Square Mean Labeling of Some Graphs

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Abstract: *Let G be a (p, q) graph and $f: V(G) \to \{k, k+1, k+2, \ldots, p+q+k-1\}$ be an injection. For each edge e = uv, let $f^*(e) = \left\lceil \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rceil$ or $\left\lfloor \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rfloor$, then f is called k-Super root square mean labeling if $f(V) \cup \{f^*(e): e \in E(G)\} = \{k, k+1, k+2, \ldots, p+q+k-1\}$. A graph that admits a k-Super root square mean labeling is called k-Super root square mean graph. In this paper, we investigate k-Super root square mean labeling of some path related graphs.*

Keywords: Super root square mean labeling, Super root square mean graph, k-Super root square mean labeling, k-Super root square mean graph, $P_n \odot K_{1,2}$, $P_n \odot K_{1,3}$, $D(T_n)$, $Q_n \odot K_1$.

1. Introduction

All graphs in this paper are finite, simple and undirected. Terms not defined here are used in the sense of Harary [5]. The symbols V(G) and E(G) will denote the vertex set and edge set of a graph G.

Graph labeling was first introduced in the late 1960's. Many studies in graph labeling refer to Rosa's research in 1967 [6]. Labeled graphs serve as useful models for a broad range of applications such as X-ray, crystallography, radar, coding theory, astronomy, circuit design and communication network addressing. Particularly interesting applications of graph labeling can be found in [1-4].

The concept of mean labeling was introduced and studied by S. Somasundaram and R.Ponraj [7].

Root square mean labeling was introduced by S.S. Sandhya, R.Ponraj and S. Anusa [8].

The concept of super root square mean labeling was introduced and studied by K. Thirugnanasambandam et al. [9].

In this paper, I introduce k – Super root square mean labeling and investigate k-Super root square mean labeling of $P_n \odot K_{1,2}$, $P_n \odot K_{1,3}$, $D(T_n)$, $Q_n \odot K_1$. Throughout this paper, k denote any positive integer greater than or equal to 1.

For brevity, we use k-SRSML for k-Super root square mean labeling.

Definition 1.1
Let G be a (p, q) graph and $f: V(G) \to \{1, 2, \ldots, p+q\}$ be an injection. For each edge e = uv, let $f^*(e) = \left\lceil \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rceil$ or $\left\lfloor \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rfloor$, then f is called **Super root square mean labeling** if $f(V) \cup \{f^*(e): e \in E(G)\} = \{1, 2, \ldots, p+q\}$. A graph that admits a Super root square mean labeling is called **Super root square mean graph.**

Definition 1.2
Let G be a (p, q) graph and $f: V(G) \to \{k, k+1, k+2, \ldots, p+q+k-1\}$ be an injection. For each edge e = uv, let $f^*(e) = \left\lceil \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rceil$ or $\left\lfloor \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rfloor$, then f is called **k-Super root square mean labeling** if $f(V) \cup \{f^*(e): e \in E(G)\} = \{k, k+1, k+2, \ldots, p+q+k-1\}$

A graph that admits a k-Super root square mean labeling is called **k-Super root square mean graph.**

Definition 1.3:
If G has order n, the corona of G with H, $G \odot H$ is the graph obtained by taking one copy of G and n copies of H and joining the i th vertex of G with an edge to every vertex in the i th copy of H.

Definition 1.4:
A Double Triangular Snake $D(T_n)$ consists of two triangular snakes that have a commom path.

2. Main Results

Theorem 2.1:
The graph $P_n \odot K_{1,2}$ is a k-Super root square mean graph for $n \geq 2$.

Proof:
Let $V(P_n \odot K_{1,2}) = \{u_i, v_i, w_i; 1 \leq i \leq n\}$ and

$$E(P_n \odot K_{1,2}) = \{e_i = (u_i, u_{i+1}); 1 \leq i \leq n-1\} \cup \{e_i' = (u_i, v_i); 1 \leq i \leq n\} \cup \{e_i'' = (u_i, w_i); 1 \leq i \leq n\}$$

be the vertices and edges of $P_n \odot K_{1,2}$ respectively.

Define
$f: V(P_n \odot K_{1,2}) \to \{k, k+1, k+2, \ldots, k+6n-2\}$ by
$f(u_i) = k + 6i - 4; 1 \le i \le n,$
$f(v_i) = k + 6i - 6;\ 1 \le i \le n,$
$f(w_i) = k + 6i - 2; 1 \le i \le n.$

Now the induced edge labels are as follows:
$f^*(e_i) = k + 6i - 1; 1 \le i \le n - 1,$
$f^*(e_i') = k + 6i - 5; 1 \le i \le n,$
$f^*(e_i'') = k + 6i - 3; 1 \le i \le n.$

Here p = 3n and q = 3n-1.
Clearly, $f(V) \cup \{f^*(e): e \in E(P_n \odot K_{1,2})\} = \{k, k+1, \ldots, k+6n-2\}.$

So f is a k-Super root square mean labeling.

Hence $P_n \odot K_{1,2}$ is a k-Super root square mean graph.

Example 2.2:
100- SRSML of $P_3 \odot K_{1,2}$ is given in figure 2.1:

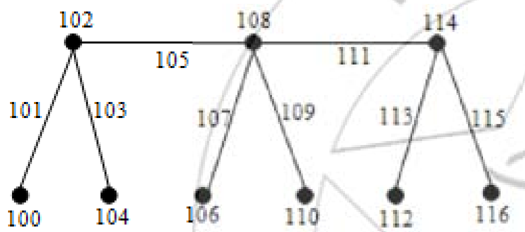

Figure 2.1: 100-SRSML of $P_3 \odot K_{1,2}$

Theorem 2.3:
The graph $P_n \odot K_{1,3}$ is a k-Super root square mean graph for $n \ge 2$.

Proof:
Let $V(P_n \odot K_{1,3}) = \{u_i, v_i, w_i, s_i: 1 \le i \le n\}$ and
$E(P_n \odot K_{1,3}) = \{e_i = (u_i, u_{i+1}): 1 \le i \le n-1\} \cup$
$\{e_i' = (u_i, v_i): 1 \le i \le n\} \cup$
$\{e_i'' = (u_i, w_i): 1 \le i \le n\} \cup$
$\{e_i'' = (u_i, w_i): 1 \le i \le n\}$ be the vertices and edges of $P_n \odot K_{1,3}$ respectively.

Define
$f: V(P_n \odot K_{1,3}) \to \{k, k+1, k+2, \ldots, k+8n-2\}$ by
$f(u_i) = k + 8i - 6; 1 \le i \le n,$
$f(v_i) = k + 8i - 8;\ 1 \le i \le n,$
$f(w_i) = k + 8i - 4; 1 \le i \le n.$
$f(s_i) = k + 8i - 2; 1 \le i \le n$

Now the induced edge labels are as follows:
$f^*(e_i) = k + 8i - 1; 1 \le i \le n - 1,$
$f^*(e_i') = k + 8i - 7; 1 \le i \le n,$
$f^*(e_i'') = k + 8i - 5; 1 \le i \le n$
$f^*(e_i''') = k + 8i - 3; 1 \le i \le n.$
Here p = 4n and q = 4n-1.
Clearly, $f(V) \cup \{f^*(e): e \in E(P_n \odot K_{1,3})\} =$
$\{k, k+1, \ldots, k+8n-2\}.$
So f is a k-Super root square mean labeling.
Hence $P_n \odot K_{1,3}$ is a k-Super root square mean graph.

Example 2.4:
200- SRSML of $P_3 \odot K_{1,3}$ is given in figure 2.2:

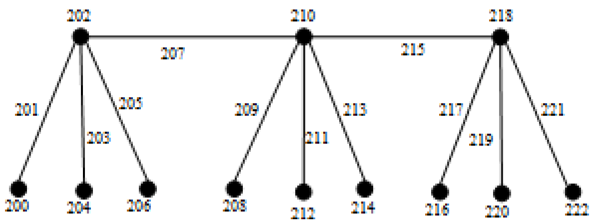

Figure 2.2: 200-SRSML of $P_3 \odot K_{1,3}$

Theorem 2.5:
The graph $Q_n \odot K_1$ is a super root square mean graph for $n \ge 2$.

Proof:
Let $V(Q_n \odot K_1) = \{u_i, u_i': 1 \le i \le n\} \cup$
$\{v_i, v_i', w_i, w_i': 1 \le i \le n-1\}$ and
$E(T_n \odot K_1) = \{e_i = (u_i, u_{i+1}): 1 \le i \le n-1\} \cup$
$\{e_i' = (u_i, u_i'): 1 \le i \le n\} \cup$
$\{e_i'' = (u_i, v_i): 1 \le i \le n-1\}$
$\{e_i''' = (u_{i+1}, w_i): 1 \le i \le n-1\} \cup$
$\{e_i^{iv} = (u_i, w_i): 1 \le i \le n-1\} \cup$
$\{e_i^{v} = (v_i, v_i'): 1 \le i \le n-1\} \cup$
$\{e_i^{vi} = (w_i, w_i'): 1 \le i \le n-1\}$
be the vertices and edges of $Q_n \odot K_1$ respectively.

Define
$f: V(Q_n \odot K_1) \to \{k, k+1, k+2, \ldots, k+13n-11\}$ by
$f(u_i) = k + 13i - 11; 1 \le i \le n,$
$f(v_i) = k + 13i - 9;\ 1 \le i \le n-1,$
$f(w_1) = k + 10,$
$f(w_i) = k + 13i - 2; 2 \le i \le n-1,$
$f(u_1') = k,$
$f(u_i') = k + 13i - 14; 2 \le i \le n,$
$f(v_i') = k + 13i - 7; 1 \le i \le n-1,$
$f(w_i') = k + 13i - 5; 1 \le i \le n-1.$

Now the induced edge labels are as follows:
$f^*(e_1) = k + 11,$
$f^*(e_i) = k + 13i - 3; 2 \le i \le n-1,$
$f^*(e_i') = k + 13i - 12; 1 \le i \le n,$
$f^*(e_i'') = k + 13i - 10; 1 \le i \le n-1,$
$f^*(e_i''') = k + 13i; 1 \le i \le n-1,$
$f^*(e_i^{iv}) = k + 13i - 6; 1 \le i \le n-1,$
$f^*(e_i^{v}) = k + 13i - 8; 1 \le i \le n-1,$
$f^*(e_i^{vi}) = k + 13i - 4; 1 \le i \le n-1.$
Here p = 6n-4 and q = 7n-6.
Clearly, $f(V) \cup \{f^*(e): e \in E(Q_n \odot K_1)\} =$
$\{k, k+1, k+2, \ldots, k+13n-11\}.$
So f is a k-Super root square mean labeling.

Hence $Q_n \odot K_1$ is a k-Super root square mean graph.

Example 2.6:
250- SRSML of $Q_4 \odot K_1$ is given in figure 2.3:

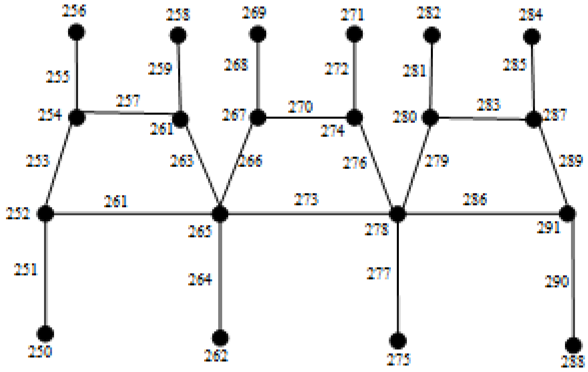

Figure 2.3: 250-SRSML of $\boldsymbol{Q_4 \odot K_1}$

Theorem 2.7:
Any Double Triangular Snake $D(T_n)$ is a k-Super root square mean labeling.

Proof:
Let $V(D(T_n)) = \{u_i; 1 \leq i \leq n\} \cup \{v_i, w_i; 1 \leq i \leq n-1\}$
and

$$E(D(T_n)) = \{e_i = (u_i, u_{i+1}); 1 \leq i \leq n-1\} \cup \{e_i' = (u_i, v_i); 1 \leq i \leq n-1\} \cup \{e_i'' = (u_{i+1}, v_i); 1 \leq i \leq n-1\} \cup \{e_i''' = (u_i, w_i); 1 \leq i \leq n-1\} \cup \{e_i^{iv} = (u_{i+1}, w_i); 1 \leq i \leq n-1\}$$

be the vertices and edges of $D(T_n)$ respectively.
Define $f: V(D(T_n)) \to \{k, k+1, k+2, \ldots, k+8n-8\}$ by
$f(u_i) = k + 8i - 8; 1 \leq i \leq n,$
$f(v_i) = k + 8i - 6;\ 1 \leq i \leq n-1,$
$f(w_i) = k + 8i - 4;\ 1 \leq i \leq n-1$
Now the induced edge labels are
$f^*(e_i) = k + 8i - 3; 1 \leq i \leq n-1,$
$f^*(e_i') = k + 8i - 7; 1 \leq i \leq n-1,$
$f^*(e_i'') = k + 8i - 2; 1 \leq i \leq n-1,$
$f^*(e_i''') = k + 8i - 5; 1 \leq i \leq n-1,$
$f^*(e_i^{iv}) = k + 8i - 1; 1 \leq i \leq n-1$
Here p = 3n-2 and q = 5n-5.
Clearly, $f(V) \cup \{f^*(e): e \in E(D(T_n))\} = \{k, k+1, k+2, \ldots, k+8n-8\}.$
So f is a k-Super root square mean labeling.
Hence $D(T_n)$ is a k-Super root square mean graph.

Example 2.8:
10- SRSML of $D(T_5)$ is given in figure 2.4:

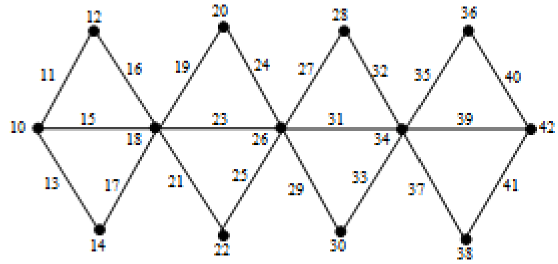

Figure 2.4: 10-SRSML of $\boldsymbol{D(T_5)}$

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