2. Some new Bi- Measures of fuzzy entropy

Consider the measure

$$F_{a,b,k}(A)= -\sum_{i=1}^{n}\left(\mu_A(x_i)\ln\mu_A(x_i)+(1-\mu_A(x_i))\ln(1-\mu_A(x_i))\right) +$$

$$\frac{b}{a^k}\sum_{i=1}^{n}\left((1+a\mu_A(x_i))\ln(1+a\mu_A(x_i))+(1+a(1-\mu_A(x_i)))\ln(1+a(1-\mu_A(x_i)))\right)$$

$$-2\frac{b}{a^k}(1+a)\ln(1+a)$$

where a, b, k are parameters.

Here $F_{a,b,k}(A)$ is defined for all $\mu_A(x_i)$, in the range $0 \le \mu_A(x_i) \le 1,\ i = 1,2,\ldots..,\ n$ and it should be continuous in this range.

$$F_{a,b,k}(A) = 0,\ when\ \mu_A(x_i) = 0\ or\ \mu_A(x_i) = 1.$$

$$F_{a,b,k}(\mu_A(x_i)) = F_{a,b,k}(1-\mu_A(x_i)).$$

Now $F_{a,b,k}(A)$ can be written as $\sum_{i=1}^{n} f(x_i)$, where

$$f(x)= -x\ln x+(1-x)\ln(1-x)+\frac{b}{a^k}\left((1+ax)\ln(1+ax)+(1+a(1-x))\ln(1+a(1-x))\right)$$

$$-2\frac{b}{a^k}(1+a)\ln(1+a)$$

Then

$$f'(x)=-\ln x+\ln(1-x)+\frac{b}{a^k}\left(a\ln(1+ax)-a\ln(1+a(1-x))\right)\ and$$

$$f''(x)=-\frac{1+(a-c)x}{x(1+ax)}-\frac{1+(a-c)(1-x)}{(1-x)(1-a(1-x))};\ c=\frac{b}{a^k}.$$

Hence $F_{a,b,k}(A)$ will be concave, if $1+(a-c)x\ge 0\ \ and\ 1+(a-c)(1-x)\ge 0$

But it is true when

$1\ge c\ or\ a\ge c\ \ i.e.\ when\ a^{k-2}\ge b\ or\ a^{k-1}\ge b.$ ....(1)

$(k-2)\ln a\ge 0\ \ or\ \ (k-1)\ln a\ge 0.$ ....(2)

$But\ \ (k-2)\ln a\ge 0,\ if\ a\ge 1 or\ k\ge 2\ or\ 0<a<1,\ k<2;$

$(k-1)\ln a\ge 0,\ if\ a\ge 1 or\ k\ge 1\ or\ 0<a<1,\ k<1.$

3. Special Cases:

1) When $a^{k-2}\ge b$ or $a^{k-1}\ge b$, $F_{a,b,k}(A)$ represent a three parametric bi- measure of fuzzy entropy.
   When $b=1, k=1,$
   $F_{a,b,k}(A) = F_4(A).$
2) When $b=1, k=2,$
   $F_{a,b,k}(A) = F_5(A).$
3)
4) When $0<b\le 1$, then (1) will be hold if
5) When $b>1$, we can find suitable value of a, k using (2). Thus in any case, the family of bi-measure of fuzzy entropy is much larger than $F_4(A)\ and\ F_5(A).$

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