# Effects of the Direction of a Transverse Magnetic Field on Unsteady MHD Couette Flow with Suction and Injection

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**Abstract:** ***In this paper, the problem of an unsteady magnetohydrodynamic viscous incompressible electrically conducting fluid flow subjected to a constant pressure gradient in the presence of a uniform transverse magnetic field applied parallel to one axis with the plates moving with a time dependent velocity is analyzed. Two cases where the plates are moving (i) in the same direction, (ii) in the opposite direction with different directions of the transverse magnetic field while fluid suction /injection takes place through the walls of the channels with a constant velocity for suction and injection has been investigated. The nonlinear partial differential equation governing the flow are solved numerically using the finite difference method. The results obtained are presented in graphs. The velocity profiles, the effect of the magnetic field direction, time and suction /injection on the flow are discussed. A change on the parameters is observed to either increase, decrease or to have no effect on the velocity profile. The MHD flow between porous plates studied in this work has many important applications in areas such as in controlling boundary layer flow over aircraft wings by injection or suction of fluid out of or into the wing, designing of cooling systems with liquid metals, in petroleum and mineral industries, in underground energy transport, accelerators, MHD generators, pumps, flow meters, purification of crude oil, geothermal reservoirs, and polymer technology.***

**Keywords:** Constant pressure gradient, Injection and Suction

## 1. Introduction

MHD Couette flow is studied by a number of researchers due its varied and wide applications in the areas of geophysics, astrophysics and fluid engineering. The MHD flow between porous plates has many important applications in areas such as the designing of cooling systems with liquid metals, geothermal reservoirs, in petroleum and mineral industries, in underground energy transport, MHD generators, pumps, flow meters, purification of crude oil, polymer technology and in controlling boundary layer flow over aircraft wings by injection or suction of fluid out of or into the wing among many other areas. MHD flows are characterized by a basic phenomenon which is the tendency of magnetic field to suppress vorticity that is perpendicular to itself which is in opposite to the tendency of viscosity to promote vorticity. Researchers have studied unsteady channel or duct flows of a viscous and incompressible fluid with or without magnetic field analyzing different aspects of the problem. Katagiri [1] investigated unsteady hydromagnetic Couette flow of a viscous, incompressible and electrically conducting fluid under the influence of a uniform transverse magnetic field when the fluid flow within the channel is induced due to impulsive movement of one of the plates of the channel. Muhuri [2] considered this fluid flow problem within a porous channel when fluid flow within the channel is induced due to uniformly accelerated motion of one of the plates of the channel. Soundalgekar [3] investigated unsteady MHD Couette flow of a viscous, incompressible and electrically conducting fluid near an accelerated plate of the channel under transverse magnetic field. The effect of induced magnetic field on a flow within a porous channel when fluid flow within the channel is induced due to uniformly accelerated motion of one of the plates of the channel, studied by Muhuri [2]. Mishra and Muduli [4] discussed effect of induced magnetic field on a flow within a porous channel when fluid flow within the channel is induced due to uniformly accelerated motion when one of the plates starts moving with a time dependent velocity. In the above mentioned investigations, magnetic field is fixed relative to the fluid. Singh and Kumar [5] studied MHD Couette flow of a viscous, incompressible and electrically conducting fluid in the presence of a uniform transverse magnetic field when fluid flow within the channel is induced due to time dependent movement of one of the plates of the channel and magnetic field is fixed relative to moving plate. Singh and Kumar [5] considered two particular cases of interest in their study viz. (i) impulsive movement of one of the plates of the channel and (ii) uniformly accelerated movement of one of the plates of the channel and concluded that the magnetic field tends to accelerate fluid velocity when there is impulsive movement of one of the plates of the channel and when there is uniformly accelerated movement of one of the plates of the channel. Katagiri [1] studied the problem when the flow was induced due to impulsive motion of one of the plates while Muhuri [2] studied the problem with accelerated motion of one of the plates. Both had considered that the magnetic lines of force are fixed relative to the fluid. Singh and Kumar [5] considered the problem studied by Katagiri [1] and Muhuri [2] in a non-porous channel with the magnetic lines of force fixed relative to the moving plate. Rudraiah *et al.*, [6] studied the natural convection of an electrically conducting fluid in a rectangular enclosure in the presence of a magnetic field numerically where two vertical side walls are held isothermally at different temperatures, while the horizontal top wall and bottom wall are adiabatic. The numerical results showed that the effect of the magnetic field is to decrease the rate of convective heat transfer while the average Nusselt number decreases with an increase of Hartmann number. Seth *et al.* [7], studied the problem considered by Singh and Kumar [5] when the fluid flow is confined to porous boundaries with suction and injection considering two cases of interest, viz (i) impulsive movement of the lower plate and (ii) uniformly accelerated movement of the lower plate. Seth *et al.* [7] concluded that the suction exerted a retarding influence on the fluid velocity whereas injection has accelerating influence on the flow while the magnetic field, time and injection reduce shear stress at lower plate in both the cases while suction increases shear stress at the lower plate. Ismail *et al.* [8]. MHD flow between two parallel plates through porous medium with one in uniform motion and the other plate at rest and uniform suction at the stationary plate. They used the Similarity transformation method to solve the problem and concluded that the axial velocity of the fluid decreases as density, time, and Hartmann number increases. The Axial velocity of the fluid increases as average entrance velocity increases Transverse velocity of fluid increases as density, Hartmann number and suction increases. Kimeu *et al* [9] considered the steady, two dimensional laminar free convective flow of an electrically conducting viscous incompressible fluid between two infinite parallel porous plates with a transverse magnetic field. They concluded that the magnetic field parameter and the suction parameter when increased, led to the increase in the amplitude of the magnetic field lines. Kandelousi [10] studied the effect of spatially variable magnetic field on Ferro fluid flow and heat transfer considering constant heat flux boundary condition. Their results show that the Nusselt number increases with an increase in magnetic number, Rayleigh number and nanoparticle volume fraction, while it decreases with an increase in the Hartmann number. Heat transfer enhancement increases with an increase in the Hartmann number but it decreases with an increase in Rayleigh number and Magnetic number. Joseph *et al.* [11] studied Unsteady MHD couette flow between two infinite parallel porous plates in an inclined magnetic field with heat transfer with the lower plate considered porous. They concluded it shows that magnetic field has significant effect to the flow of an unsteady MHD couette flow between two infinite parallel porous plates in an inclined magnetic field with heat transfer. Kiema *et al.* [12] considered laminar viscous incompressible fluid between two infinite parallel plates when the upper plate is moving with constant velocity and the lower plate is held stationary under the influence of inclined magnetic field and concluded that the increase in magnetic field strength and magnetic inclination results into decreases in the velocity profiles. Rajesh et al., [13] studied the problem of transient free convection flow and heat transfer of nano fluid past an impulsively started semi-infinite vertical plate in the presence of magnetic field. One of their finding was that as the magnetic parameter increased, the skin friction coefficient and the Nusselt number at the surface decreased for all nanofluids aluminium oxide, copper, titanium oxide and silver. Onyango *et al.* [14] considered magneto hydrodynamic flow between two parallel porous plates with injection and suction in the presence of a uniform transverse magnetic field with the magnetic field lines fixed relative to the moving plate with a constant pressure gradient and concluded that the magnetic field, pressure gradient, time and injection have an accelerating influence on the fluid flow with a constant pressure gradient in the direction of the flow on both cases of suction and injection while viscosity and suction exert a retarding influence. Extensive researches have been done on the flow between parallel plates. Rathod *et al.,* [15] investigated the effect of a magnetic field on peristaltic transport of blood in a non-uniform two-dimensional channels under zero Reynolds number with long wavelength approximation. They concluded that pressure decreases with increase in magnetic field, time-average of flow over one period of wave& couple stress parameter and increases with increasing in amplitude. Pressure with averaged flow rate increases with increase in magnetic field, amplitude & couple stress parameter while the variations of friction force with time and averaged flow rate shows opposite behavior to that of pressure. This study is with consideration when both plates are in motion with the same velocity in the same direction and in opposite directions with different directions for the transverse magnetic field lines fixed relative to the moving plates with suction and injection on the plates.

## 2. Mathematical Formulation

This study considers the flow of unsteady viscous incompressible electrically conducting fluid between two parallel porous plates $y = 0$ and $y = h$ of infinite length in $x$ and $z$ directions with a constant pressure gradient in the presence of a uniform transverse magnetic field $H_o$ applied parallel to the $y$ axis in the positive direction of $y$.

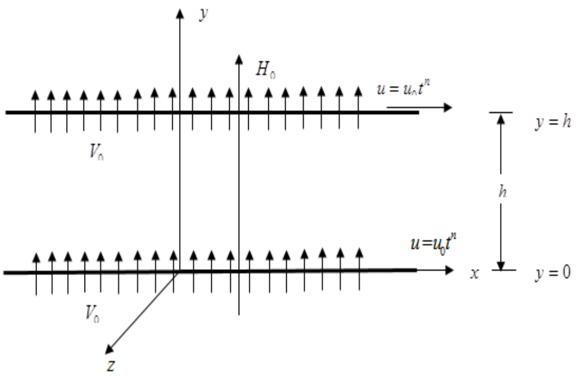



**Figure 1:** Physical model of the problem

Initially (when time $t \le 0$), the fluid and the porous plates of the channel are assumed to be at rest. When time $t > 0$, the lower plate ( $y = 0$) and the upper plate ( $y = h$ ) starts moving with time dependent velocity $u_o t^n$ (where $u_o$ is a constant and $n$ a positive integer) in the $x$ direction with the fluid suction/injection takes place through the walls of the channel with uniform velocity $V_o$ where $V_o > 0$ for suction and $V_o < 0$ for injection.

The velocity and the magnetic fields are given as $q=(u,v_0,0)$ and $\vec{H}\equiv(0,H_0,0)$ respectively.

The magnetic forces $=\sigma\mu_e^{\,2}H_0\times Velocity$

From the Navier Stokes equation

$$\rho\frac{\partial u}{\partial t}+\rho u\nabla u=-\nabla P+\mu\nabla^2 u+F \quad (1)$$

$$\rho\frac{\partial u}{\partial t}+\rho u\nabla u=-\nabla P+\mu\nabla^2 u+J\times B \quad (2)$$

The flow is incompressible (the density $\rho$, is considered a constant) and is considered in one dimension along the x-axis hence the Navier stokes equation along the x-axis is given as

$$\rho\frac{\partial u}{\partial t}+\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial P}{\partial x}+\mu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)+J\times B \quad (3)$$

For a Couette flow $-\frac{\partial P}{\partial x}=0$ but for the analysis $-\frac{\partial P}{\partial x}=$ a constant $\beta^*$. The two plates are infinite in length hence $\frac{\partial u}{\partial x}=0$. The fluid is injected on the lower plate with a constant velocity $V_0$ and is also sucked from the upper plate at the same constant velocity $V_0$. The general equation governing the flow reduces to

$$\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}=\frac{\beta^*}{\rho}+\frac{\mu}{\rho}\frac{\partial^2 u}{\partial y^2}+\frac{(-\sigma\mu_e^{\,2}H_0^{\,2}u)}{\rho} \quad (4)$$

Where $\beta=\frac{\beta^*}{\rho}$, and

**Non-Dimensionalization of the Equations**

The non dimensionalization of the governing equation is performed by selecting characteristic dimensionless quantities. The dimensionless quantities used in non dimensionalization of the governing equation (6) and the boundary condition (8) are

$$y^*=\frac{y}{h},\qquad u^*=\frac{uh}{\upsilon}\quad\text{and}\quad t^*=\frac{t\upsilon}{h^2} \quad (9)$$

$$\frac{\partial u}{\partial t}=\frac{\partial u}{\partial y^*}\frac{\partial y^*}{\partial t^*}\frac{\partial t^*}{\partial t}=\frac{\upsilon}{h}\frac{\partial u^*}{\partial t^*}\frac{\upsilon}{h^2}=\frac{\upsilon^2}{h^3}\frac{\partial u^*}{\partial t^*} \quad (10)$$

$$\frac{\partial u}{\partial y}=\frac{\partial u}{\partial u^*}\frac{\partial u^*}{\partial y^*}\frac{\partial y^*}{\partial y}=\frac{\upsilon}{h}\frac{\partial u^*}{\partial y^*}\frac{1}{h}=\frac{\upsilon}{h^2}\frac{\partial u^*}{\partial y^*} \quad (11)$$

$$\frac{\partial^2 u}{\partial y^2}=\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)=\frac{\partial}{\partial y}\left(\frac{\upsilon}{h^2}\frac{\partial u^*}{\partial y^*}\right)=\frac{\partial}{\partial y^*}\left(\frac{\upsilon}{h^2}\frac{\partial u^*}{\partial y^*}\right)\frac{\partial y^*}{\partial y}=\frac{\upsilon}{h^3}\frac{\partial^2 u^*}{\partial y^{*2}} \quad (12)$$

Replacing on the governing equation (7)

$$\frac{\upsilon^2}{h^3}\frac{\partial u^*}{\partial t^*}+V_0.\frac{\upsilon}{h^2}\frac{\partial u^*}{\partial y^*}=\beta+\upsilon.\frac{\upsilon}{h^3}\frac{\partial^2 u^*}{\partial y^{*2}}-\frac{\sigma\mu_e^{\,2}H_0^{\,2}(u-u_0 t)}{\rho} \quad (13)$$

$$\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}=\beta+\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\sigma\mu_e^{\,2}H_0^{\,2}u}{\rho} \quad (5)$$

where $\upsilon=\frac{\mu}{\rho}$

The magnetic field lines are fixed relative the moving plates (The upper plate and the lower are accelerating uniformly–a function of time) hence the velocity is considered as a relative velocity and reflects how fast the fluid is moving relative to the moving plates. The general equation governing the flow

$$\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}=\beta+\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\sigma\mu_e^{\,2}H_0^{\,2}\left(u-u_0 t^n\right)}{\rho} \quad (6)$$

For consideration of the two cases of interest viz. (i) movement of the plates in the same direction (i.e. $n=1$) and (ii) movement of the plates in the opposite direction (i.e. $n=1$).

**Case I. Movement of the plates in the same direction with magnetic field in the positive direction of the y-axis (i.e. *n* = 1)**

Taking $n=1$, for a case of uniform acceleration, the governing equation for the flow becomes

$$\frac{\partial u}{\partial t}+v\frac{\partial u}{\partial y}=\beta+\upsilon\frac{\partial^2 u}{\partial y^2}-\frac{\sigma\mu_e^{\,2}H_0^{\,2}\left(u-u_0 t\right)}{\rho} \quad (7)$$

With the boundary conditions defined as;

$$u=0 \qquad 0\le y\le h \quad t\le 0$$
$$u=u_0 t^n \qquad \text{at}\quad y=h \quad t>0 \qquad (8)$$
$$u=u_0 t^n \qquad \text{at}\quad y=0 \quad t>0$$

## 3. Numerical Computation

Non dimensionalizing the relative velocity in equation (2.2) by setting $u^*=\frac{u}{\upsilon}h\Rightarrow u=\frac{\upsilon u^*}{h}$ and $t^*=\frac{t\upsilon}{h^2}\Rightarrow t=\frac{t^*h^2}{\upsilon}$

Substituting in (13) to non-dimensionalize the relative velocity

$$\frac{\upsilon^2}{h^3}\frac{\partial u^*}{\partial t^*}+V_0.\frac{\upsilon}{h^2}\frac{\partial u^*}{\partial y^*}=\beta+\upsilon.\frac{\upsilon}{h^3}\frac{\partial^2 u^*}{\partial y^{*2}}-\frac{\sigma\mu_e^{\,2}H_0^{\,2}}{\rho}\left(\frac{\upsilon u^*}{h}-u_0\frac{t^*h^2}{\upsilon}\right) \quad (14)$$

and multiplying the equation by $\frac{h^3}{\upsilon^2}$ gives

$$\frac{h^3}{\upsilon^2}.\frac{\upsilon^2}{h^3}\frac{\partial u^*}{\partial t^*}+\frac{h^3}{\upsilon^2}.V_0.\frac{\upsilon}{h^2}\frac{\partial u^*}{\partial y^*}=\frac{h^3}{\upsilon^2}\beta+\frac{h^3}{\upsilon^2}.\upsilon.\frac{\upsilon}{h^3}\frac{\partial^2 u^*}{\partial y^{*2}}-\frac{h^3}{\upsilon^2}\frac{\sigma\mu_e^{\,2}H_0^{\,2}}{\rho}\left(\frac{\upsilon u^*}{h}-u_0\frac{t^*h^2}{\upsilon}\right) \quad (15)$$

$$\frac{\partial u^*}{\partial t^*}+\frac{V_0 h}{\upsilon}\frac{\partial u^*}{\partial y^*}=\frac{h^3}{\upsilon^2}\beta+\frac{\partial^2 u^*}{\partial y^{*2}}-\frac{h^3}{\upsilon^2}\frac{\sigma\mu_e^{\,2}H_0^{\,2}}{\rho}\left(\frac{\upsilon u^*}{h}-u_0\frac{t^*h^2}{\upsilon}\right) \quad (16)$$

$$\frac{\partial u^*}{\partial t^*}+\frac{V_0 h}{\upsilon}\frac{\partial u^*}{\partial y^*}=\frac{h^3}{\upsilon^2}\beta+\frac{\partial^2 u^*}{\partial y^{*2}}-\frac{h^3}{\upsilon^2}\frac{\sigma\mu_e^{\,2}H_0^{\,2}}{\rho}.\frac{1}{h}\left(\upsilon u^*-u_0\frac{t^*h^3}{\upsilon}\right)$$

$$\frac{\partial u^*}{\partial t^*}+\frac{V_0 h}{\upsilon}\frac{\partial u^*}{\partial y^*}=\frac{h^3}{\upsilon^2}\beta+\frac{\partial^2 u^*}{\partial y^{*2}}-\frac{\sigma\mu_e^2 H_0^2 h^2}{\rho\upsilon^2}\left(\upsilon u^*-u_0\frac{t^* h^3}{\upsilon}\right) \quad (17)$$

$$\frac{\partial u^*}{\partial t^*}+\frac{V_0 h}{\upsilon}\frac{\partial u^*}{\partial y^*}=\frac{h^3}{\upsilon^2}\beta+\frac{\partial^2 u^*}{\partial y^{*2}}-\frac{\sigma\mu_e^2 H_0^2 h^2}{\rho\upsilon^2}.\upsilon\left(u^*-u_0\frac{t^* h^3}{\upsilon^2}\right) \quad (18)$$

$$\frac{\partial u^*}{\partial t^*}+\frac{V_0 h}{\upsilon}\frac{\partial u^*}{\partial y^*}=\frac{h^3}{\upsilon^2}\beta+\frac{\partial^2 u^*}{\partial y^{*2}}-\frac{\sigma\mu_e^2 H_0^2 h^2}{\rho\upsilon}\left(u^*-u_0\frac{t^* h^3}{\upsilon^2}\right) \quad (19)$$

(20)

The expression $\frac{\sigma\mu_e^2 H_0^2 h^2}{\rho\upsilon}=M^2$ is the Hartmann number squared, and $\frac{u_0 h}{\upsilon}$ is the Reynolds number Re and hence substituting in Equation 20, this gives

$$\frac{\partial u^*}{\partial t^*}+\frac{V_0 h}{\upsilon}\frac{\partial u^*}{\partial y^*}=\frac{h^3}{\upsilon^2}\beta+\frac{\partial^2 u^*}{\partial y^{*2}}-M^2\left(u^*-u_0\frac{t^* h^3}{\upsilon^2}\right) \quad (21)$$

$$\frac{\partial u^*}{\partial t^*}+\frac{V_0 h}{\upsilon}\frac{\partial u^*}{\partial y^*}=\frac{h^3}{\upsilon^2}\beta+\frac{\partial^2 u^*}{\partial y^{*2}}-M^2\left(u^*-\frac{\mathrm{Re}\,h}{\upsilon}t^*\right) \quad (22)$$

Equation (22) is the governing equation in non-dimensional form.

Dimensionalizing the boundary conditions from (8) using the non-dimensional parameters from equations (10), (11) and (12) are obtained as

$$u^*=0 \qquad 0\le y\le 1 \text{ and } t^*\le 0$$
$$u^*=\frac{t^* h}{\upsilon}\mathrm{Re} \quad \text{at } y^*=1; \quad t^*>0 \qquad (23)$$
$$u^*=\frac{t^* h}{\upsilon}\mathrm{Re} \quad \text{at } y^*=0; \quad t^*>0$$

**Case II. Movement of the plates in the opposite direction (i.e. *n* = 1)**

For case (ii) where the parallel porous plates of the channel are in motion in the opposite directions.

The governing equation is given by

$$\frac{\partial u^*}{\partial t^*}+\frac{V_0 h}{\upsilon}\frac{\partial u^*}{\partial y^*}=\frac{h^3}{\upsilon^2}\beta+\frac{\partial^2 u^*}{\partial y^{*2}}-M^2\left(u^*-\frac{\mathrm{Re}\,h}{\upsilon}t^*\right) \quad (24)$$

The boundary conditions are as follows

$$u^*=0 \qquad 0\le y\le 1 \text{ and } t^*\le 0$$
$$u^*=\frac{t^* h}{\upsilon}\mathrm{Re} \quad \text{at } y^*=1; \quad t^*>0 \qquad (25)$$
$$u^*=-\frac{t^* h}{\upsilon}\mathrm{Re} \quad \text{at } y^*=0; \quad t^*>0$$

The governing equations in non-dimensional form together with the boundary conditions for both cases will be presented in their finite difference forms consistent with the method of solution.

## 4. Governing Equation in Finite Difference Form

The finite difference analogues of the PDEs arising from the equation governing this flow are obtained by replacing the derivatives in the governing equations by their corresponding difference approximation .The following substitutions are done for the derivatives for the Crank Nicolson, we have the proposed averages as

$$u^*=\frac{u_{i,j+1}+u_{i,j}}{2} \qquad (26)$$

$$\frac{\partial u^*}{\partial t^*}=\frac{u_{i,j+1}-u_{i,j}}{\Delta t} \qquad (27)$$

$$\frac{\partial u^*}{\partial y^*}=\frac{u_{i+1,j+1}-u_{i-1,j+1}+u_{i+1,j}-u_{i-1,j}}{4(\Delta y)} \qquad (28)$$

$$\frac{\partial^2 u^*}{\partial y^{*2}}=\frac{u_{i-1,j}-2u_{i,j}+u_{i+1,j}}{(\Delta y)^2} \qquad (29)$$

$$\frac{\partial^2 u^*}{\partial y^{*2}}=\frac{1}{2}\left(\left\{\frac{u_{i-1,j}-2u_{i,j}+u_{i+1,j}}{(\Delta y)^2}\right\}+\left\{\frac{u_{i-1,j+1}-2u_{i,j+1}+u_{i+1,j+1}}{(\Delta y)^2}\right\}\right) \qquad (3.8)$$

Replacing in the governing equation, Multiplying through by $\Delta t$ and rearranging (24) gives

$$\left(u_{i,j+1}-u_{i,j}\right)+\frac{V_0 h\Delta t}{4\upsilon(\Delta y)}\left(u_{i+1,j+1}-u_{i-1,j+1}+u_{i+1,j}-u_{i-1,j}\right)=\frac{h^3\Delta t}{\upsilon^2}\beta+$$
$$\frac{1\Delta t}{2(\Delta y)^2}\left(u_{i+1,j+1}-2u_{i,j+1}+u_{i-1,j+1}+u_{i+1,j}-2u_{i,j}+u_{i-1,j}\right)-\frac{M^2\Delta t}{2}\left(u_{i,j+1}+u_{i,j}\right)+$$
$$M^2\Delta t\frac{\mathrm{Re}\,h}{\upsilon}t_j \qquad (25)$$

Letting $\gamma=-\frac{V_0 h\Delta t}{4\upsilon(\Delta y)}$, $\zeta=\frac{h^3\Delta t}{\upsilon^2}\beta$, $\varsigma=\frac{1\Delta t}{2(\Delta y)^2}$, $\eta=\frac{M^2\Delta t}{2}$, $\vartheta=M^2\Delta t\frac{\mathrm{Re}\,h}{\upsilon}$ and the suction/ injection parameter $S=\frac{V_0 h}{\upsilon}$.

Substituting the values of $\gamma$, $\zeta$, $\varsigma$, $\eta$, $\vartheta$ and S in (25) gives

$$\left(u_{i,j+1}-u_{i,j}\right)-\gamma\left(u_{i+1,j+1}-u_{i-1,j+1}+u_{i+1,j}-u_{i-1,j}\right)=\zeta+$$
$$\varsigma\left(u_{i+1,j+1}-2u_{i,j+1}+u_{i-1,j+1}+u_{i+1,j}-2u_{i,j}+u_{i-1,j}\right)-\eta\left(u_{i,j+1}+u_{i,j}\right)+$$
$$\vartheta t_j \qquad (26)$$



Paper ID: ART20171276 DOI: 10.21275/ART20171276

Rearranging (26) gives

$$u_{i,j+1}-u_{i,j}-\gamma u_{i+1,j+1}+\gamma u_{i-1,j+1}-\gamma u_{i+1,j}+\gamma u_{i-1,j}=\zeta+ \varsigma u_{i+1,j+1}-2\varsigma u_{i,j+1}+\varsigma u_{i-1,j+1}+\varsigma u_{i+1,j}-2\varsigma u_{i,j}+\varsigma u_{i-1,j} -\eta u_{i,j+1}-\eta u_{i,j}+\vartheta t_j \quad (27)$$

Rearranging equation (27) gives

$$u_{i,j+1}-\gamma u_{i+1,j+1}+\gamma u_{i-1,j+1}-\varsigma u_{i+1,j+1}+\varsigma u_{i-1,j+1}+2\varsigma u_{i,j+1}+\varsigma u_{i,j+1}=\zeta+ u_{i,j}-\gamma u_{i-1,j}+\gamma u_{i+1,j}+\varsigma u_{i+1,j}-2\varsigma u_{i,j}+\varsigma u_{i-1,j}-\eta u_{i,j}+\vartheta t_j \quad (28)$$

Collecting the like terms form equation (28) gives

$$(1+2\varsigma+\eta)u_{i,j+1}-(\gamma+\varsigma)u_{i+1,j+1}+(\gamma+\varsigma)u_{i-1,j+1} =\zeta+(1-2\varsigma+\eta)u_{i,j}+\varsigma u_{i+1,j}+(\varsigma-\gamma)u_{i-1,j}+\vartheta t_j \quad (29)$$

Rearranging equation (29)

$$-(\gamma+\varsigma)u_{i+1,j+1}+(1+2\varsigma+\eta)u_{i,j+1}+(\gamma+\varsigma)u_{i-1,j+1}= \varsigma u_{i+1,j}+(1-2\varsigma+\eta)u_{i,j}+(\varsigma-\gamma)u_{i-1,j}+\vartheta t_j+\zeta \quad (30)$$

The finite difference equations obtained at any space node, say, $i$ at the time level $t_{j+1}$ has only three unknown coefficients involving space nodes at $i-1, i$ and $i+i$ at $t_{j+1}$. In matrix notation, these equations can be expressed as $AU=B$ where $U$ is the unknown vector of order $(N-1)$ at any time level $t_{j+1}$. $B$ is the known vector of order $(N-1)$ which has the value of $U$ at the $n^{th}$ time level and $A$ is the coefficient square matrix of order $(N-1)\times(N-1)$ which is a tridiagonal structure.

The coefficients of the interior nodes will be represented as:

$$a_j=-(\gamma+\varsigma) \qquad d_j=(\varsigma-\gamma)u_{i-1,j} \qquad g_j=\vartheta t_j$$
$$b_j=(1+2\varsigma+\eta) \qquad e_j=(1-2\varsigma-\eta)u_{i,j} \qquad h=\zeta$$
$$c_j=(\gamma+\varsigma) \qquad f_j=\varsigma u_{i+1,j} \quad (31)$$

For $j=2,3,4....(N-1)$, then the equation (30) becomes

$$a_j u_{i+1,j+1}+b_j u_{i,j+1}+c_j u_{i-1,j+1}=d_j+e_j+f_j+g_j+h \quad (32)$$

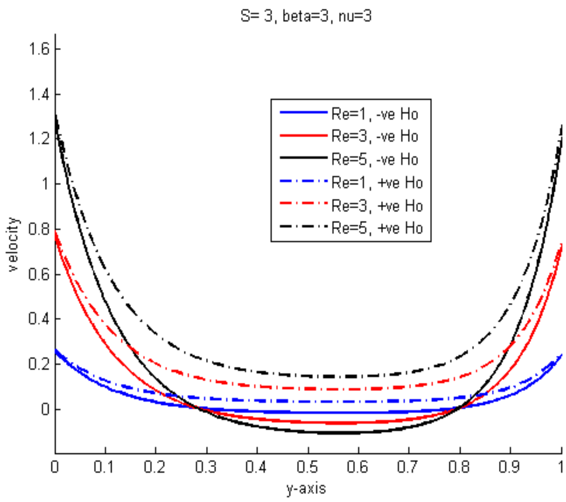
The system of equations resulting from equation (32) are represented in a tridiagonal matrix form as

$$\begin{bmatrix} a_2 & b_2 & c_2 & 0 & 0 & 0 & 0 \\ 0 & a_3 & b_3 & c_3 & \ddots & 0 & 0 \\ 0 & 0 & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & 0 & 0 & \ddots & \ddots & \ddots & \ddots \\ 0 & 0 & 0 & 0 & a_{N-1} & b_{N-1} & c_{N-1} \end{bmatrix} \begin{bmatrix} u_{1,j+1} \\ u_{2,j+1} \\ \vdots \\ u_{3,j+1} \end{bmatrix} = \begin{bmatrix} d_2 \\ d_3 \\ \vdots \\ d_{N-1} \end{bmatrix} + \begin{bmatrix} e_2 \\ e_3 \\ \vdots \\ e_{N-1} \end{bmatrix} + \begin{bmatrix} f_2 \\ f_2 \\ \vdots \\ f_{N-1} \end{bmatrix} + \begin{bmatrix} g_2 \\ g_3 \\ \vdots \\ g_{N-1} \end{bmatrix} + \begin{bmatrix} h \\ h \\ \vdots \\ h \end{bmatrix} \quad (33)$$

Equation (33) is implemented in matlab to obtain the results

## 5. Results and Discussions

The effects of various flow parameters on the flow regime are depicted graphically and discussed. The simulations are curried out using ISO FLUIDS 3448 which are industrial oils whose kinematic viscosities range between 2 and 10.



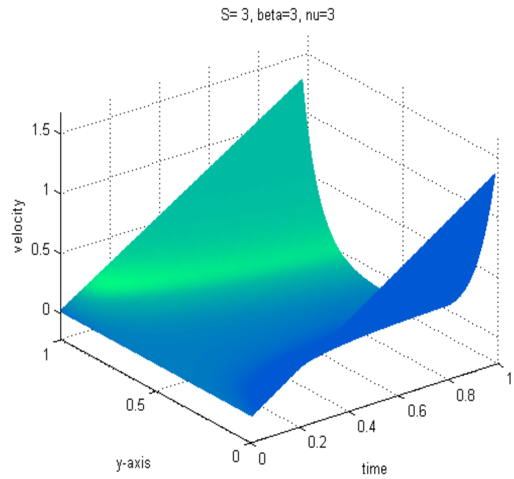




**Figure 2:** Varying the Reynolds number for both ((+ $H_0$ ) and (- $H_0$ )) with plates moving in the same direction.

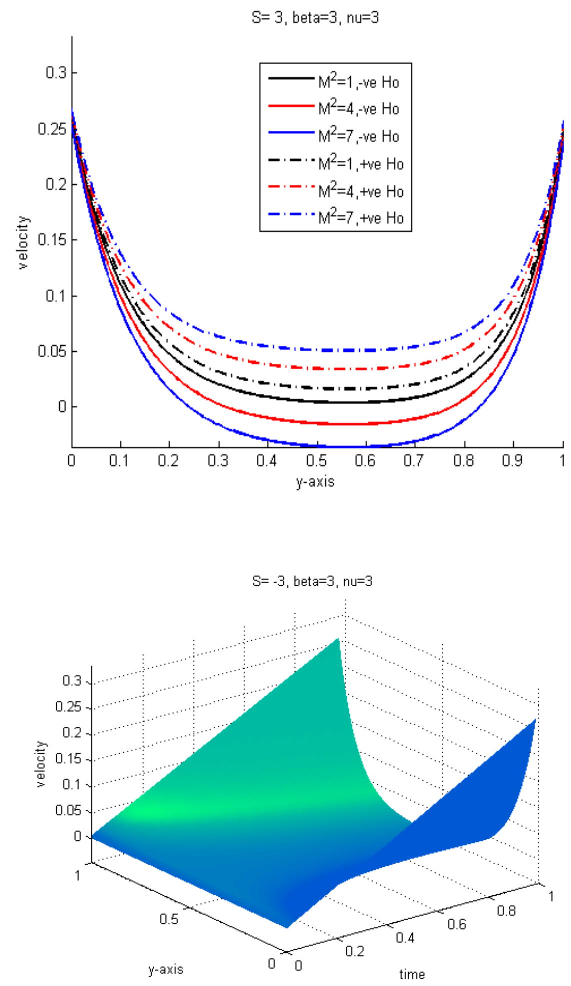



**Figure 3:** Varying the Magnetic number for both (($+H_0$) and ($-H_0$)) with plates moving in the same direction.

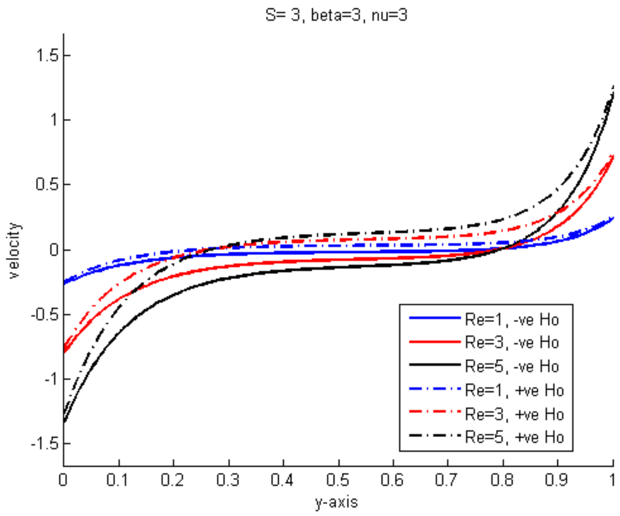



**Figure 4:** Varying the Reynolds number for both (($+H_0$) and ($-H_0$)) with plates moving in the opposite direction

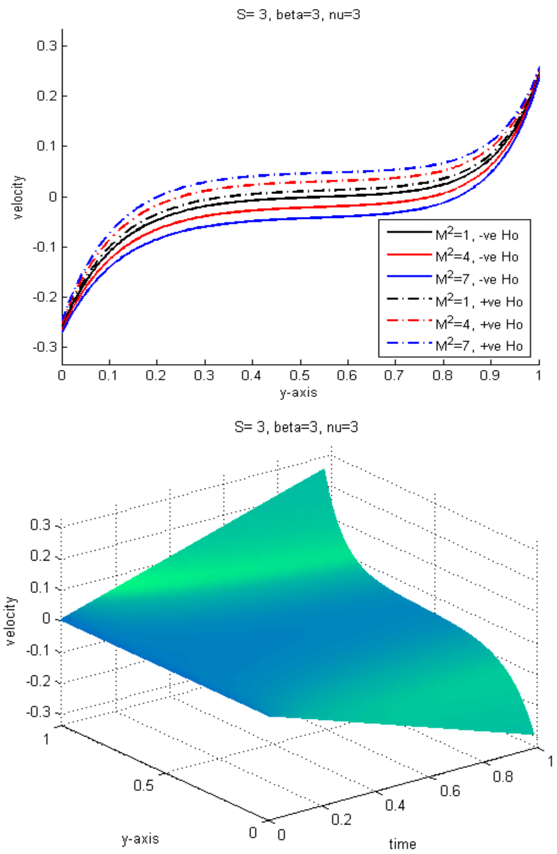



**Figure 5:** Varying the Magnetic number for both (($+H_0$) and ($-H_0$)) with plates moving in the opposite direction.

From fig 2. And fig 4. The velocity profiles for both cases of (($+H_0$) and ($-H_0$) where the transverse magnetic field is in the direction of the y axis and in the opposite direction increase with increase in the Reynolds number but the velocities of the case of $+H_0$ are greater than the velocities of $-H_0$.The direction of the transverse magnetic field leads to increased velocities of the fluid or decreased velocities of the fluid since the direction determines the direction of the induced transverse Lorentz force into the fluid. If the force is in the direction of the motion of the plate, the velocities are increased and vice versa. An increase in the Reynolds number leads to an increase in the velocity of the fluid due to decreased viscous drag on the fluid.

From fig 3. And fig 5. The velocity profiles increase with increase in the magnetic number in the ($+H_0$) transverse magnetic field and decreases with the increase in the ($-H_0$) transverse magnetic field. The velocity of the fluid near the plate with injection are greater than the velocity of the fluid near the plate with suction since injection of the fluid through the plate destabilizes the boundary, increasing the pressure and leading to a decrease in the viscous forces hence increase in the motion of the fluid.

## 6. Conclusions

This study leads to a conclusion that the direction of the transverse magnetic field is important as it leads to increased or decreased velocity of the fluid between the parallel plates due to the direction of the induced magnetic field. The magnetic field, pressure gradient and injection have an accelerating influence on the fluid flow with a constant pressure gradient on both cases of suction and injection. The injection and suction of fluid from either of the plates has a significant effect on the velocity profiles with injection leading to increased velocities and suction leading to decreased velocities of the fluid.

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