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# (k, d)–Super Root Square Mean Labeling

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**Abstract:** *Let G be a (p, q) graph and $f: V(G) \rightarrow \{k, k+d, k+2d, \dots, k+d(p+q-1)\}$ be an injection. For each edge e = uv, let $f^*(e) = \left\lceil \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rceil$ or $\left\lfloor \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rfloor$, then f is called( k, d)-Super root square mean labeling if $f(V) \cup \{f^*(e): e \in E(G)\} = \{k, k+d, k+2d, \dots, k+d(p+q-1)\}$. A graph that admits a (k, d)-Super root square mean labeling is called (k, d)-Super root square mean graph. In this paper, we investigate( k, d)-Super root square mean labeling of some path related graphs.*

**Keywords:** Super root square mean labeling, Super root square mean graph, k-Super root square mean labeling, k-Super root square mean graph, (k, d)-Super root square mean labeling, (k, d)-Super root square mean graph, path, comb, ladder and triangular snake

## 1. Introduction

We begin with simple, finite, connected and undirected graph G (V, E) with p vertices and q edges. For a detailed survey of graph labeling we refer to Gallian [1]. Terms are not defined here are used in the sense of Harary [2]. S. Somasundram and R. Ponraj introduced mean labeling of graphs in [4]. R.Ponraj and D. Ramya introduced Super mean labeling of graphs in [3]. Root square mean labeling was introduced by S.S. Sandhya, R.Ponraj and S. Anusa [5]. The concept of super root square mean labeling was introduced and studied by K. Thirugnanasambandam et al. [6]. In this paper, I extend k-Super root square mean labeling to (k, d) – Super root square mean labeling and investigate (k, d)-Super root square mean labeling of path, comb, ladder and triangular snake. Throughout this paper k and d denote any integer greater than on equal to 1. For brevity, I use (k, d)-SRSML for (k, d)-Super root square mean labeling.

## 2. Main Results

**Definition 2.1**
Let G be a (p, q) graph and $f: V(G) \rightarrow \{1, 2, \dots, p+q\}$ be an injection. For each edge e = uv, let $f^*(e) = \left\lceil \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rceil$ or $\left\lfloor \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rfloor$, then f is called **Super root square mean labeling** if $f(V) \cup \{f^*(e): e \in E(G)\} = \{1, 2, \dots, p+q\}$. A graph that admits a Super root square mean labeling is called **Super root square mean graph.**

**Definition 2.2**
Let G be a (p, q) graph and $f: V(G) \rightarrow \{k, k+1, k+2, \dots, p+q+k-1\}$ be an injection. For each edge e = uv, let $f^*(e) = \left\lceil \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rceil$ or $\left\lfloor \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rfloor$, then f is called **k-Super root square mean labeling** if $f(V) \cup \{f^*(e): e \in E(G)\} = \{k, k+1, k+2, \dots, p+q+k-1\}$
A graph that admits a k-Super root square mean labeling is called **k-Super root square mean graph.**

**Definition 2.3**
Let G be a (p, q) graph and $f: V(G) \rightarrow \{k, k+d, k+2d, \dots, k+d(p+q-1)\}$ be an injection. For each edge e = uv, let $f^*(e) = \left\lceil \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rceil$ or $\left\lfloor \sqrt{\frac{f(u)^2+f(v)^2}{2}} \right\rfloor$, then f is called **(k,d )-Super root square mean labeling** if $f(V) \cup \{f^*(e): e \in E(G)\} = \{k, k+d, k+2d, \dots, k+d(p+q-1)\}$
A graph that admits a (k, d)-Super root square mean labeling is called **(k, d)-Super root square mean graph.**

**Theorem 2.4**
Any path $P_n$ is a (k, d)-Super root square mean graph.

**Proof:**
Let $V(P_n) = \{v_i; 1 \leq i \leq n\}$ and $E(P_n) = \{e_i = (v_i, v_{i+1}): 1 \leq i \leq n-1\}$ be the vertices and edges of $P_n$ respectively.
Define $f: V(P_n) \rightarrow \{k, k+d, k+2d, \dots, k+d(2n-2)\}$ by $f(v_i) = k + d(2i-2); 1 \leq i \leq n$.
Now the induced edge labels are
$f^*(e_i) = k + d(2i-1); 1 \leq i \leq n-1$.
Here p = n and q = n-1.
Clearly
$f(V) \cup \{f^*(e): e \in E(P_n)\} = \{k, k+d, \dots, k+d(2n-2)\}$.
So f is a (k, d)-Super root square mean labeling. Hence $P_n$ is a (k, d)-Super root square mean graph.

**Example 2.5**
(25, 1)-Super root square mean labeling of $P_6$ is given in figure 2.1:

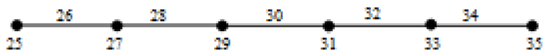


**Figure 2.1:** (25, 1)-SRSML of $P_6$

**Observation 2.6**

If G is a (k, d)-Super root square mean graph, then k and k+2d must be the labels of the adjacent vertices of G since an edge should get label k+d.

**Definition 2.7**

The graph $P_n \odot K_1$ is called a comb.

**Theorem 2.8**

Any comb $P_n \odot K_1$ is a (k, d)-Super root square mean graph.

**Proof:**

Let $V(P_n \odot K_1) = \{v_i, u_i; 1 \le i \le n\}$ and

$E(P_n \odot K_1) = \{e_i = (v_i, v_{i+1}); 1 \le i \le n-1\} \cup \{e_i' = (v_i, u_i); 1 \le i \le n\}$

be the vertices and edges of $P_n \odot K_1$ respectively.

Define

$f: V(P_n \odot K_1) \to \{k, k+d, k+2d \ldots, k+d(4n-2)\}$ by

$$f(u_i) = \begin{cases} k+4d(i-1); 1 \le i \le n, \text{if } i \text{ is odd} \\ k+2d(2i-1); 1 \le i \le n, \text{if } i \text{ is even} \end{cases}$$

$$f(v_i) = \begin{cases} k+4d(i-1); 1 \le i \le n, \text{if } i \text{ is even} \\ k+2d(2i-1); 1 \le i \le n, \text{if } i \text{ is odd} \end{cases}$$

Now the induced edge labels are

$f^*(e_i) = k + d(4i-1); 1 \le i \le n-1$

$f^*(e_i') = k + d(4i-3); 1 \le i \le n$

Here p = 2n and q = 2n-1.

Clearly

$f(V) \cup \{f^*(e): e \in E(P_n \odot K_1)\} = \{k, k+d, k+2d \ldots, k+d(4n-2)\}$

So f is a (k, d)-Super root square mean labeling.

Hence $P_n \odot K_1$ is a (k, d)-Super root square mean graph.

**Example 2.9**

(50, 2)-Super root square mean labeling of $P_6 \odot K_1$ is given in figure 2.2:

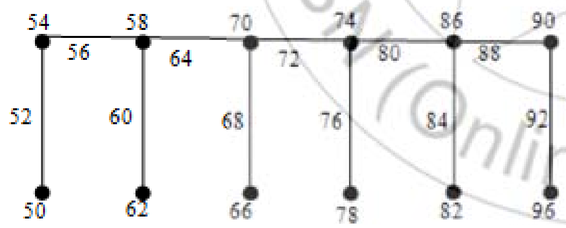


**Figure 2.2:** (50, 2)-SRSML of $P_6 \odot K_1$

**Definition 2.10**

The product graph $P_2 \times P_n$ is called a ladder and it is denoted by $L_n$.

**Theorem 2.11**

Any ladder $L_n$ is a (k, d)-Super root square mean graph.

**Proof:**

Let $V(L_n) = \{v_i, u_i; 1 \le i \le n\}$ and

$E(L_n) = \{e_i = (u_i, u_{i+1}); 1 \le i \le n-1\} \cup \{e_i' = (v_i, v_{i+1}); 1 \le i \le n-1\} \cup \{e_i'' = (u_i, v_i); 1 \le i \le n\}$ be the vertices and edges of $L_n$ respectively.

Define $f: V(L_n) \to \{k, k+d, k+2d, \ldots, k+d(5n-3)\}$ by

$$f(u_i) = \begin{cases} k+5d(i-1); 1 \le i \le n, \text{if } i \text{ is odd} \\ k+d(5i-3); 1 \le i \le n, \text{if } i \text{ is even} \end{cases}$$

$$f(v_i) = \begin{cases} k+5d(i-1); 1 \le i \le n, \text{if } i \text{ is even} \\ k+d(5i-3); 1 \le i \le n, \text{if } i \text{ is odd} \end{cases}$$

Now the induced edge labels are

$$f^*(e_i) = \begin{cases} k+d(5i-1); 1 \le i \le n-1, \text{if } i \text{ is odd} \\ k+d(5i-2); 1 \le i \le n-1, \text{if } i \text{ is even} \end{cases}$$

$$f^*(e_i') = \begin{cases} k+d(5i-1); 1 \le i \le n-1, \text{if } i \text{ is even} \\ k+d(5i-2); 1 \le i \le n-1, \text{if } i \text{ is odd} \end{cases}$$

$f^*(e_i'') = k + d(5i-4); 1 \le i \le n$

Here p = 2n and q = 3n-2.

Clearly

$f(V) \cup \{f^*(e): e \in E(L_n)\} = \{k, k+d, k+2d, \ldots, k+d(5n-3)\}$

So f is a (k, d)-Super root square mean labeling.

Hence $L_n$ is a (k, d)-Super root square mean graph.

**Example 2.12**

(50, 2)-Super root square mean labeling of $L_6$ is given in figure 2.3:

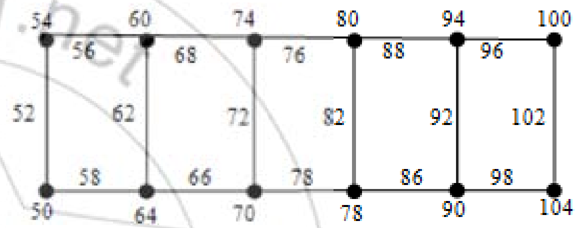


**Figure 2.3:** (50, 2)-SRSML of $L_6$

**Definition 2.13**

A triangular snake ($T_n$) is obtained from a path by identifying each edge of the path with an edge of the cycle $C_3$.

**Theorem 2.14**

Triangular snake, $T_n$ is a (k, d)-Super root square mean graph.

**Proof:**

Let $V(T_n) = \{v_i; 1 \le i \le n-1\} \cup \{u_i; 1 \le i \le n\}$ and

$E(T_n) = \{e_i = (u_i, u_{i+1}); 1 \le i \le n-1\} \cup \{e_i' = (v_i, u_i); 1 \le i \le n-1\} \cup \{e_i'' = (u_{i+1}, v_i); 1 \le i \le n-1\}$ be the vertices and edges of $T_n$ respectively.

Define $f: V(T_n) \to \{k, k+1, k+2, \ldots, 5n+k-5\}$ by

$$f(u_i) = \begin{cases} k+2d \ ; i = 1 \\ k+d(5i-5); 2 \le i \le n \end{cases}$$

$$f(v_i) = \begin{cases} k \ ; i = 1 \\ k+d(5i-2); 2 \le i \le n-1 \end{cases}$$

Now the induced edge labels are

$$f^*(e_i) = \begin{cases} k+4d \ ; i = 1 \\ k+d(5i-3); 2 \le i \le n-1 \end{cases}$$

$f^*(e_i') = k + d(5i-4); 1 \le i \le n-1$

$$f^*(e_i'') = \begin{cases} k+3d \ ; i = 1 \\ k+d(5i-1); 2 \le i \le n-1 \end{cases}$$

Here p = 2n-1 and q = 3n-3.

Clearly

$f(V) \cup \{f^*(e): e \in E(T_n)\} = \{k, k+d, k+2d, \ldots, k+d(5n-5)\}$

So f is a (k, d)-Super root square mean labeling.

Hence $T_n$ is a (k, d)-Super root square mean graph.

**Example 2.15**
(500, 3)-Super root square mean labeling of $L_6$ is given in figure 2.3:

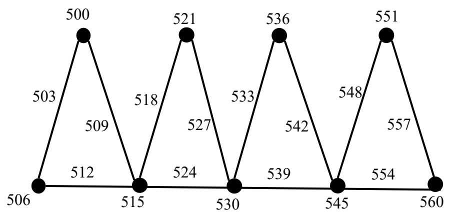


**Figure 2.4: (500, 3)-SRSML of $T_5$**

## 3. Conclusion

- Every Super root square mean labeling is a k- Super root square mean labeling.
- (k, d)- Super root square mean labeling is a Super root square mean labeling if k = 1and d = 1.

## References

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