

# Optimal Design of Water Distribution Network Using Differential Evolution

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**Abstract:** *A water distribution network (WDN) consists of thousands of nodes with nonlinear hydraulic behaviour, linked by thousands of interconnecting links. The inherent problem associated with cost optimisation in the design of water distribution networks is due to the nonlinear relationship between flow and head loss and availability of the discrete nature of pipe sizes. The importance and huge capital cost of the system leads to considerable attention on seeking the optimal cost design. The present paper is focused on the Differential Evolution (DE) algorithm linked with EPANET software to achieve the goal of optimisation of a specified objective function. A simulation–optimisation model is developed in which the optimization is done by DE. Two well-known benchmark networks were taken for application of the DE algorithm to optimise pipe size and the results prove that the algorithm can perform satisfactorily.*

**Keywords:** Differential Evolution, EPANET, Simulation, Optimization

## 1. Introduction

A water distribution network (WDN) is a collection of many components such as pipes, reservoirs, pumps and valves which are connected to each other to provide water to consumers. Among these components, the interconnecting pipes that transport water from the source node to the demand nodes account for the major fraction of the capital cost. The optimal design of such network is defined as determining the best combination of pipe sizes that gives the minimum cost for the given layout of network such that the constraints on quantities and pressures at the consumer nodes are fulfilled.

Several methods are available to design a water distribution network in which rule of thumb and trial and error are the most popular methods. With the development of high speed digital computers and improved optimisation techniques, the design of water distribution networks was attempted since the 1970s. The complexity of the problem is due to the nonlinear relationship between flow and head loss, the presence of discrete decision variables such as pipe diameter, cost functions for the materials, labour, geographical layout, multiple demand loading patterns, uncertainty in demands, and location of tanks, pumping stations, booster pumps, valves, etc. Numerous literatures exist on the optimization of water distribution networks using linear programming, nonlinear programming, enumeration techniques, and heuristic methods. The evolutionary techniques used for optimal design of water distribution systems includes the genetic algorithm(GA) [12],[14],[16], the modified genetic algorithm [5],[8],[9], the simulated annealing algorithm(SAL)[1], the shuffled leapfrog algorithm(SFLA)[2], ant colony optimization [10],[21], novel cellular automata [6], harmony search(HS)s[3],[4] and the particle swarm algorithm [13],[14] for optimal design of water distribution systems are some of them.

The present work is focused on the Differential Evolution (DE) algorithm linked with EPANET software to achieve the goal of optimisation of a specified objective function. A simulation–optimisation model is developed in which the

optimization is done by DE. Two well-known benchmark networks were taken for application of the DE algorithm to optimise pipe size and the results prove that the algorithm can perform satisfactorily.

## 2. Water Distribution Network Modelling

Water distribution network modeling belongs to a class of large combinatorial non-linear optimization problems, involving complex implicit constraints, such as conservation of mass and energy equations, which are commonly satisfied through the use of hydraulic simulation solvers. Recently, many researchers have shifted the focus from traditional optimization methods to the use of meta-heuristic approaches for handling this complexity. In recent years, evolutionary algorithms are often the preferred choices because of their ability to deal with complex, nonlinear, and discrete optimization problems as well as the ease and generality with which they can be linked to any simulation model.

### 2.1 Simulation tool -EPANET

EPANET is a computer program that performs extended period simulation of hydraulic and water quality behaviour within pressurized pipe networks. A network consists of pipe, node, pump, storage tank or reservoir. EPANET tracks the flow of water in each pipe, the pressure at each node, and the height of water in each tank. EPANET is designed to be used for many different kinds of application in distribution system analysis.

### 2.2 Optimization tool – Differential Evolution

Differential Evolution (DE) algorithm is a branch of evolutionary programming developed by Rainer Storn and Kenneth Price (Price and Storn, 1997) for optimization problems over continuous domains. In DE, each variable's value is represented by a real number. The advantages of DE are its simple structure, ease of use, speed and robustness. DE is one of the best genetic type algorithms for solving problems with the real valued variables. Differential Evolution is a design tool of great utility that is immediately

accessible for practical applications. DE has been used in several science and engineering applications to discover effective solutions to nearly intractable problems without appealing to expert knowledge or complex design algorithms. If a system is amenable to being rationally evaluated, DE can provide the means for extracting the best possible performance from it.

There are three important operators involved in the DE algorithm including the mutation, crossover and selection operators. In DE the mutation operation takes place first and then the crossover operation carries. As evolution progresses, the mutation operator favours exploitation. Hence, DE automatically adapts the mutation increments (i.e., search step) to the best value based on the stage of the evolutionary process. The DE algorithm also uses a uniform crossover that can take child vector parameters from one parent more often than from the other. By using components of existing population members to construct trial vectors, crossover operator efficiently shuffles information about successful combinations, enabling the search for an optimum to focus on the most promising area of the solution space.

### 3. Methodology

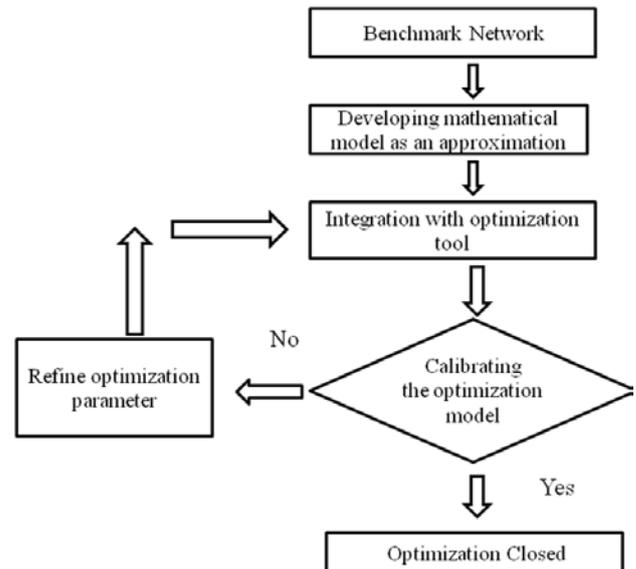
The design of WDN when defined in a mathematical form leads to a non-linear, non-convex and multi-modal problem classified as an NP-hard combinatorial problem. The desired cost to be optimized is fixed and then the mathematical model is developed. In the present study, Differential Evolution algorithm is used as an optimization tool which is integrated with EPANET via the EPANET toolkit. The optimized value is calibrated with the Benchmark networks. If the conditions are satisfied, the simulations get closed else the optimization parameter (Pipe diameter) is refined. The methodology framework has been pictorially represented in Fig .1

#### 3.1 Objective Function

The problem of optimal design of water distribution network usually has an objective of minimizing the total capital cost. The mathematical representation of objective function is mentioned below

$$\text{Min } C = \sum_{i=1}^{np} (D_i L_i)$$

Where C = Cost of the water distribution network  
 D<sub>i</sub> = Diameter of the pipe  
 L<sub>i</sub> = Length of the Pipe  
 np = No. Of pipes



**Figure 1:** Elements of Methodology

#### 3.2 Constraints

Constraints are set of mathematical equations used to find the solution, which impose some conditions that the decision variables should satisfy. The following constraints are used in the optimal design of water distribution network.

##### 3.2.1 Flow continuity constraint

The continuity principle states that the quantity of flow into the node must be equal to the quantity of flow leaving that node. Mathematically it is expressed as

$$\sum Q_{in} - Q_{out} = Q_e$$

Where Q<sub>in</sub>= flow into junction  
 Q<sub>out</sub>= Flow out of junction  
 Q<sub>e</sub>= Demand at junction node

##### 3.2.2 Energy conservation constraint

The total head loss around the closed path (loop) should be equal to zero.

$$\sum_{i \in \text{loop}} hf_i = \Delta H$$

Where hf<sub>i</sub> = Head loss due to friction in pipe i.  
 ΔH = Difference between nodal heads at both ends &  
 ΔH = 0 , if the path is closed.

##### 3.2.3 Minimum head constraint

The pressure head in all nodes should be greater than the prescribed minimum pressure head

$$H_j^{avl} \geq H_j^{\min}$$

Where H<sub>j</sub><sup>avl</sup> = pressure head at node  
 H<sub>j</sub><sup>min</sup> = minimum pressure head

#### 3.3 Benchmark network

Validation of the developed optimization model is carried out using two Benchmark water distribution networks. These networks are frequently used for verifying the model accuracy developed for the design of WDN.

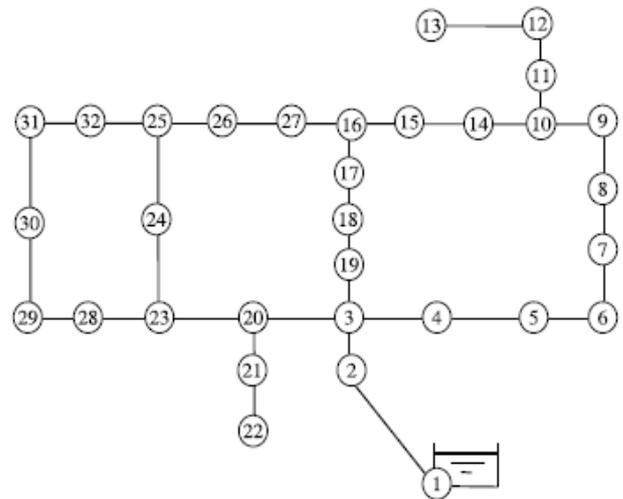
**3.3.1 Two loop network**

The two-loop network, shown in Fig.2, was originally presented by Alperovits and Shamir (1977). The network consists of eight links, six demand nodes and a reservoir with two loops, and is fed by gravity from a reservoir with a 210m fixed head. The pipes are all 1000 m long with the assumed Hazen–Williams coefficient of 130. The minimum pressure limitation at all demand nodes is 30 m above ground level.

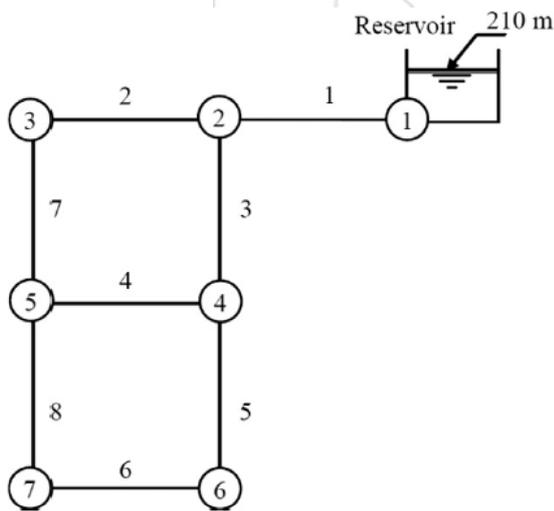
Numerous researchers have examined the chosen case study (Savic & Walter 1997; Cunha & Sousa 1999; Eusuff & Lansey 2003). The same existing pipe input data including discrete set of available diameters, and minimum head and demand at each node are used in this study. There are 14 commercial diameters to be selected; thus the problem search space consists of  $14^8 = 1.48 \times 10^9$  different network designs.

**3.3.2 Hanoi network**

The Hanoi network shown in Fig.3 was first presented by Fujiwara and Khang(1987). The network consists of 32 nodes and 34 links arranged in three loops, fed from a single fixed head source providing a head of 100m. The input data remains the same as used by numerous authors (Savic and Walters 1997; Cunha and Sousa 1999; Eusuff and Lansey 2003). The minimum required head for all the nodes is 30m. There are six available pipe diameters to be selected for each new pipe; thus the total search space consists of  $6^{34} = 2.865 \times 10^{26}$  possible designs.



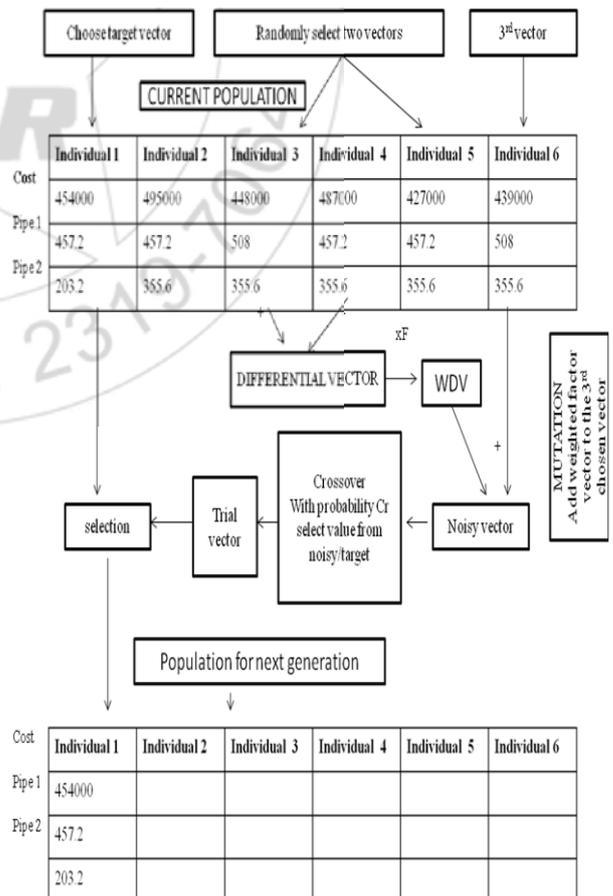
**Figure 3:** Layout of Hanoi network



**Figure 2:** Layout of two loop network

**4. Implementation Strategy**

In the present study, a combined simulation–optimisation model is developed and used. The optimisation model is an outer driven whereas the simulation is an inner driven one. The computer programming code was written for DE using Matlab and EPANET (Rossman 2000) is linked via the EPANET Toolkit. The complete programme performs a hydraulic network analysis at each function evaluation to determine the pressure head at the nodes. The algorithm is applied to two well-known benchmark network.



**Figure 4:** Computational module for differential evolution algorithm

The implementation strategy of DE for optimal design of the water distribution network is presented in Fig. 4. It can be seen from Fig. 4. that all the initial solution vectors consist of discrete pipe sizes. From the initial population, two solution vectors are randomly selected and the difference between each of the parameters is determined. The weighted vector is obtained by multiplying by the mutation constant and it is added to the third randomly selected vector from the initial population to get a noisy vector. Furthermore, a new solution vector is obtained by performing crossover, which basically selects the pipe diameter either from the noisy vector or target vector, according to the selection probability. The selection probability is randomly generated in order to compare with the crossover constant. If it is less than or equal to the crossover constant, the pipe diameter is selected from the noisy vector, otherwise from the target vector. The overall cost of the new solution (Called a trial vector) is calculated after converting the pipe diameters selected from the noisy vector to the nearest commercial size. As this conversion of continuous diameter to discrete diameter occurs within the optimisation (i.e. before the selection of the vector for the next generation), this does not affect the goal of optimisation.

## 5. Results and Discussion

### 5.1 Two loop network

The Differential Evolution technique is applied to solve the two-loop network problem. Different trial runs are performed with different initial random seeds, for each set of selected operator constants by setting a population size as 20. The mutation constant is varied from 0.6–0.9 in 0.1 increments and similarly the crossover constant is varied from 0.3–0.5 in increments of 0.1. The termination criterion for the optimisation is arbitrarily set to 500 generations. As the population size is set to twenty, each generation consists of 20 function evaluation. From the trials, the least cost of \$4,19,000 is found out which coincides with global optimal solution reported in the literature. The optimal diameters for links 1-8 are found and listed in Table 1.

#### 5.1.1 Mutation and Crossover Probability

Different combinations of constants are considered for the mutation and crossover probability. Table 2 provides the results of an average of different trial runs for each combination of constants. In the evaluation process, one of the trials having a weighting factor (mutation probability) of 0.6 and crossover constant of 0.5 has provided an optimal solution of \$4,19,000 at the expense of 5,300 function evaluations.

**Table 1:** Optimal diameters for two-loop network

Pipe no.	Diameter(mm)
1	457.2
2	254
3	406.4
4	101.6
5	406.4
6	254
7	254
8	25.4

### 5.1.2 Function evaluation

The proposed DE algorithm yielded the best solution as \$419,000 with an average function evaluation of 5,526. Table.6 compares the results with those obtained using the earlier techniques with respect to the optimal solution obtained and the average number of function evaluations taken to get the global optimum. Table.3 shows that the Differential Evolution algorithm performed well in finding the optimal solution more quickly than previous techniques. Figure.5 shows the evolution process for the two loop network corresponding to the least function evaluations obtained in the trial runs.

**Table 2:** Results of the trial runs for two-loop network

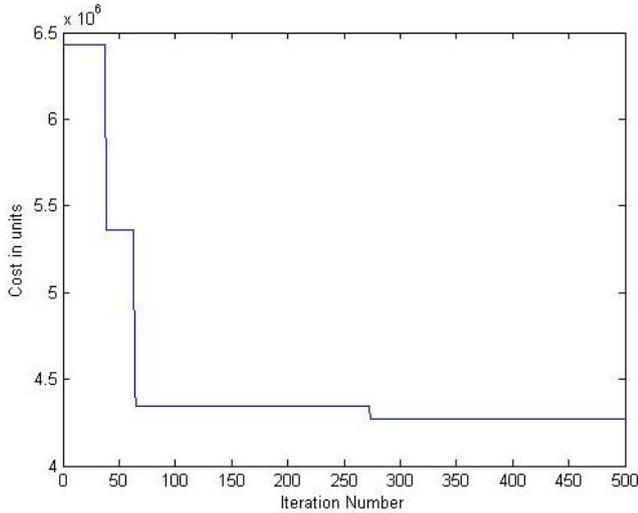
Mutation rate	Crossover probability	Average number of function evaluation in getting least cost solution
0.6	0.3	5,350
0.6	0.4	5,780
<b>0.6</b>	<b>0.5</b>	<b>5,300</b>
0.7	0.3	5,420
0.7	0.4	5,428
0.7	0.5	6,008
0.8	0.3	5,560
0.8	0.4	5,715
0.8	0.5	5,564
0.9	0.4	5,323
0.9	0.5	5,600

**Table 3:** Solution for two loop network

Author	Technique used	Cost (\$)	Average no. of function evaluation
Savic & Walter (1997)	GA	4,19,000	65,000
Cunha & sousa (1999)	SA	4,19,000	25,000
Eusuff & Lansey (2003)	SFLA	4,19,000	11,155
Present work	DE	4,19,000	5,300

### 5.2 Hanoi network

Similar to the previous case study, 300 trial runs are performed by keeping the population size as 100 with weighting factors ranging from 0.6–0.9 (mutation rate) and crossover constant ranging from 0.3–0.5. The termination criterion for the algorithm is arbitrarily set to 500 generations. In each combination of constants, 30 trials are performed with different initial random seeds. The network solution having a least cost of \$60,81,087 was obtained. The optimal diameter and the nodal pressure heads for the solution having cost of \$6.081 million while analysing using EPANET version 2 are listed in the Table4.



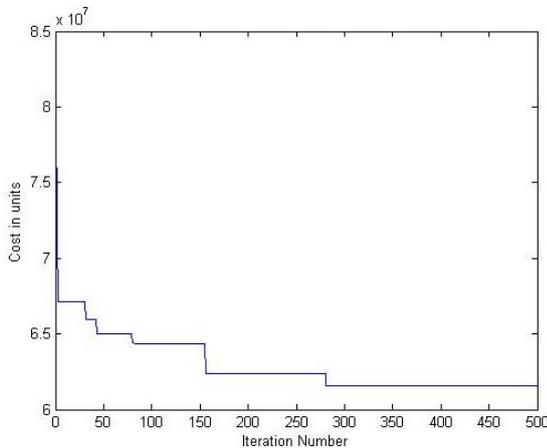
**Figure 5:** Evolution process of two loop network

**5.2.1 Mutation and Crossover Probability**

Different combinations of constants are considered for the mutation and crossover probability. Table.5. provides the results of an average of different trial runs for each combination of constants. The average number of function evaluations corresponding to the least cost is determined as 49,550. In the evaluation process, one of the trials having a weighting factor (mutation probability) of 0.6 and crossover constant of 0.4 has provided an optimal solution of \$60,81,087 at the expense of 28,000 function evaluations

**5.2.2 Function evaluation**

The results obtained using DE algorithm and those previously reported in literature are shown in Table.6. The table shows that the Differential Evolution algorithm performed well in finding the optimal solution more quickly than previous techniques. Fig.6 shows the evolution process for the Hanoi network corresponding to the least function evaluations obtained in the trial runs.



**Figure 6:** Evolution process of Hanoi network

**Table 4:** Pipe diameter and nodal pressure heads for solution of Hanoi network obtained using EPANET.

Pipe / Node no.	Diameter (mm)	Pressure (m)
1	1016	100
2	1016	97.14
3	1016	61.67
4	1016	56.92
5	1016	51.02

6	1016	44.81
7	1016	43.35
8	1016	41.61
9	1016	40.23
10	762	39.20
11	609.6	37.64
12	609.6	34.21
13	508	30.01
14	406.4	35.52
15	304.8	33.72
16	304.8	31.30
17	406.4	33.41
18	609.6	49.93
19	508	55.09
20	1016	50.61
21	508	41.26
22	304.8	36.10
23	1016	44.52
24	762	38.93
25	762	35.34
26	508	31.70
27	304.8	30.76
28	304.8	38.93
29	406.4	30.13
30	304.8	30.42
31	304.8	30.70
32	406.4	33.18
33	406.4	
34	609.6	

**Table 5:** Results of the trial run for Hanoi network

Mutation rate	Crossover probability	Average number of function evaluation in getting least cost solution
0.6	0.3	30,718
<b>0.6</b>	<b>0.4</b>	<b>28,000</b>
0.6	0.5	53,150
0.7	0.4	42,300
0.7	0.5	63,532
0.8	0.3	40,450
0.8	0.4	50,840
0.8	0.5	56,680
0.9	0.3	46,632
0.9	0.4	52,800
0.9	0.5	61,120

**Table 6:** Solutions for Hanoi network

Author	Technique used	Cost (\$)	Average no. of function evaluation
Savic & Walter (1997)	GA	6,073,000	1,000,000
Cunha & sousa (1999)	SA	6,056,000	53,000
Geem et al. (2002)	HS	6,056,000	200,000
Present work	DE	6,081,087	49,550

**6. Conclusions**

The developed simulation optimization model gives better performance to the optimization problem. The following are the conclusions derived from the present work.

- 1) For the two loop network the global optimal solution of \$4,19,000 is obtained with an average function evaluation of 5,300 for the mutation probability of 0.6 and crossover probability of 0.4.
- 2) In the Hanoi network the least cost solution of \$60,81,087 is obtained with an average function

evaluation of 28,000 for the mutation probability of 0.6 and crossover probability of 0.4.

- 3) The DE obtained best known solution in fewer evaluations than other optimization algorithms, including GA, SA, SFLA, and HS. The result shows that the proposed DE algorithm can be effectively used to solve complex WDN design problems with better efficiency with least function evaluation.

## 7. Acknowledgement

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## References

- [1] Cunha, Maria da Conceicao, and Joaquim Sousa. "Water distribution network design optimization: simulated annealing approach." *Journal of Water Resources Planning and Management* 125.4 (1999): 215-221.
- [2] Eusuff, Muzaffar M., and Kevin E. Lansey. "Optimization of water distribution network design using the shuffled frog leaping algorithm." *Journal of Water Resources Planning and Management* 129.3 (2003): 210-225.
- [3] Geem, Zong Woo, and Yoon-Ho Cho. "Optimal design of water distribution networks using parameter-setting-free harmony search for two major parameters." *Journal of Water Resources Planning and Management* 137.4 (2010): 377-380.
- [4] Geem, Zong Woo. "Multiobjective optimization of water distribution networks using fuzzy theory and harmony search." *Water* 7.7 (2015): 3613-3625.
- [5] Kadu, Mahendra S., Rajesh Gupta, and Pramod R. Bhawe. "Optimal design of water networks using a modified genetic algorithm with reduction in search space." *Journal of Water Resources Planning and Management* 134.2 (2008): 147-160.
- [6] Keedwell, Edward, and Soon-Thiam Khu. "Novel cellular automata approach to optimal water distribution network design." *Journal of Computing in Civil Engineering* 20.1 (2006): 49-56.
- [7] Mohan, S., and KS Jinesh Babu. "Optimal water distribution network design with honey-bee mating optimization." *Journal of Computing in Civil Engineering* 24.1 (2009): 117-126.
- [8] Montesinos, Pilar, Adela Garcia-Guzman, and Jose Luis Ayuso. "Water distribution network optimization using a modified genetic algorithm." *Water Resources Research* 35.11 (1999): 3467-3473.
- [9] Neelakantan, T. R., and C. R. Suribabu. "Optimal design of water distribution networks by a modified genetic algorithm." *Journal of Civil & Environmental Engineering* 1 (2005): 20-34.
- [10] Ostfeld, Avi, and Ariel Tubaltzev. "Ant colony optimization for least-cost design and operation of pumping water distribution systems." *Journal of Water Resources Planning and Management* 134.2 (2008): 107-118.
- [11] Saminu, A., et al. "Modifications of Optinet work Software for the Implementation of Advance Genetic Algorithm on Existing Water Distribution Network."
- [12] Savic, Dragan A., and Godfrey A. Walters. "Genetic algorithms for least-cost design of water distribution networks." *Journal of water resources planning and management* 123.2 (1997): 67-77.
- [13] Sedki, A., and Driss Ouazar. "Hybrid particle swarm optimization and differential evolution for optimal design of water distribution systems." *Advanced Engineering Informatics* 26.3 (2012): 582-591.
- [14] Suribabu, C. R., and T. R. Neelakantan. "Design of water distribution networks using particle swarm optimization." *Urban Water Journal* 3.2 (2006): 111-120.
- [15] Suribabu, C. R. "Differential evolution algorithm for optimal design of water distribution networks." *Journal of Hydroinformatics* 12.1 (2010): 66-82
- [16] Suribabu, C. R., and T. R. Neelakantan. "Particle swarm optimization compared to other heuristic search techniques for pipe sizing." *Journal of Environmental Informatics* 8.1 (2006): 1-9.
- [17] Vasan, A., and Slobodan P. Simonovic. "Optimization of water distribution network design using differential evolution." *Journal of Water Resources Planning and Management* 136.2 (2010): 279-287.
- [18] Vairavamoorthy, Kalanithy, and Mohammed Ali. "Pipe index vector: A method to improve genetic-algorithm-based pipe optimization." *Journal of Hydraulic Engineering* 131.12 (2005): 1117-1125.
- [19] van Thienen, Peter, and Ina Vertommen. "Gondwana: A Generic Optimization Tool for Drinking Water Distribution Systems Design and Operation." *Procedia Engineering* 119 (2015): 1212-1220.
- [20] Wu, Zheng Yi, and Angus R. Simpson. "Competent genetic-evolutionary optimization of water distribution systems." *Journal of Computing in Civil Engineering* 15.2 (2001): 89-101.
- [21] Zecchin, Aaron C., et al. "Max-min ant system applied to water distribution system optimization." *Proc. Int. Congr. Modeling Simulation (MODSIM) 2* (2003): 795-800.