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Bounds on Inverse and Double Domination Numbers of Fuzzy Square Graphs

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Abstract: *Let D be a minimum dominating set of a fuzzy graph G^2, if $V(G^2) - D$ contains another dominating set D' of G^2, then D' is called an inverse dominating set with respect to D. The minimum cardinality of vertices in such a set is called an inverse domination number of G^2 and is denoted by $\gamma^{-1}(G^2)$. In this paper, many bounds on $\gamma^{-1}(G^2)$ were obtained in terms of elements of G. Also its relationship with other domination parameters was obtained*

Keywords: Dominating set, Fuzzy graphs, Fuzzy square graphs, Inverse domination number of Fuzzy square graphs

1. Introduction

In 1975, the notion of fuzzy graph and several fuzzy analogues of graph theoretical concepts such as paths cycles and connectedness are introduced by Rosenfeld[4] Bhattacharya[1] has established some connectivity regarding fuzzy cut node and fuzzy bridges. The concept of domination in fuzzy graphs are introduced by A. Somasudaram and S. Somasundaram[6] in 1998. In 2012, Bounds on connected domination in square graph of graph is introduced by M. H. Muddabihal and G.Srinivasa[7]. In this paper We analyze bounds on Inverse domination number of Fuzzy square graphs and proves some results based on fuzzy square graph

2. Preliminaries

Definition 2.1
A fuzzy subset of a nonempty set V is mapping $\sigma: V \rightarrow [0, 1]$ and A fuzzy relation on V is fuzzy subset of V x V. A fuzzy graph is a pair G:(σ,μ) where σ is a fuzzy subset of a set V and μ is a fuzzy relation on σ , where $\mu(u,v) \leq \sigma(x) \wedge \sigma(y)$ $\forall$ x , y $\in$V

Definition 2.2
The fuzzy graph H (V_1, σ', μ') is called a fuzzy sub graph of $G(V,\sigma, \mu)$ if $V_1 \subseteq V, \sigma'(u) \leq \sigma(u)$ for all $u \in V$ and $\mu'(u,v) \leq \mu(u,v)$ for all $u,v \in V$. The fuzzy sub graph $H(V_1, \sigma', \mu')$ is said to be a spanning fuzzy sub graph of G(V, σ, μ) if $\sigma(u) = \sigma'(u)$ for all $u \in V_1$ and $\mu'(u,v) \leq \mu(u,v)$ for all u,v ∈ V.

Definition 2.3
The Strength of the connectedness between two nodes u,v in a fuzzy graph G is $\mu^{\infty}(u,v) = \sup\{\mu^k(u,v); k = 1,2,3..\}$ where $\mu^k(u,v) = \sup\{ \mu(u,u_1) \wedge \mu(u_1,u_2) \wedge \mu(u_2,u_3) \wedge \ldots \mu(u_{k-1},v) \}$. An arc (u,v) is said to be a strong arc if $\mu(u,v) = \mu^{\infty}(u,v)$. If $\mu(u,v) = 0$ for every $v \in V$ then u is called isolated node.

Definition 2.4
Let G (V,σ,μ) be a fuzzy graph and D ⊆ V. D is a dominating set if for every $u \in V-D$ there exist $v \in D$ such that (u,v) is strong arc and $\sigma(u) \leq \sigma(v)$. A dominating set of a fuzzy graph with minimum number of vertices is called a minimum dominating set. The domination number of G is denoted by γ (G). p $=\sum_{v\in V} \sigma(v)$ and number of vertices is denoted by n.

Definition 2.5
Domination number of a fuzzy graph is the sum of membership values of the vertices of a minimum dominating set. Further, if the sub graph < D > is independent, then D is called an independent dominating set of G. The independent domination number of G denoted by $\gamma_i(G)$ is the minimum cardinality of an independent dominating set of G.

Definition 2.6
A dominating set D of a fuzzy graph G = (σ, μ) is connected dominating set if the induced fuzzy sub graph H=<D> is connected. The Minimum fuzzy cardinality of a connected dominating set of G is called the connected dominating number of G and is denoted by $\gamma_c(G)$.

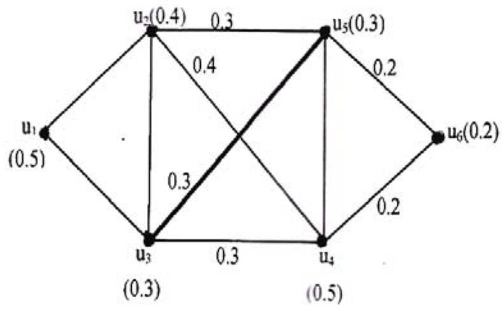

Fig. 2. D= { u_3,u_5}, γ_c(G)=0.6

3. Inverse Domination Number in Square Fuzzy Graphs

In this section, we introduce Inverse domination number in fuzzy square graphs.

Definition 3.1
The square fuzzy graph G denoted by G^2 has the same vertices as in G and the vertices u and v are joined in G^2 if and only if they are joined in G by a path of length is less than or equal to two.(ie, path of vertices is one or two)

Definition 3.2
A set D $\subseteq$ V is said to be a dominating set of G^2 if for every vertex u$\in$ V-D is adjacent to some vertex in D. A dominating set of a fuzzy square graph with minimum number of vertices is called a minimum dominating set. The domination number of G^2 is denoted by γ (G^2). Domination number of a fuzzy square graph is the sum of membership values of the vertices of a minimum dominating set.

Definition 3.3
The square fuzzy graph G is G^2 =(V,E(G^2)). A set D of G^2 is said to be connected dominating set of G^2 if every vertex not in D is adjacent to at least one vertex in D and the sub graph H = <D> is connected dominating set of G^2 is called the connected domination number of square fuzzy graph G^2 and denoted by γ_c (G^2).

Definition 3.4
Let D be a minimum dominating set of a fuzzy graph G^2 If V(G^2) − D contains another dominating set D' of G^2 , then D' is called an inverse dominating set with respect to D. The minimum cardinality of vertices in such a set is called an inverse domination number of $G^2$and is denoted by γ^{-1} (G^2).

Theorem 3.5
Let D, D' be the dominating set of G and G^2. If the dominating vertices of G and G^2 are same then each vertex in D is maximum adjacent to the vertices.

Proof: For n=2, the result is obvious. Let n$\geq$ 3. Since, V(G)=V(G^2) such that G^2 does not contain any end vertex. let D = {v_1,v_2.v_3,…v_n} be a dominating set of G^2 If there exists a vertex v $\in$ D such that v is adjacent to some vertices in V(G^2) –D. Then every vertex w $\in$D-{v} is an end vertex in < D>. Further, if w is adjacent to a vertex u $\in$V(G^2) –D .Then D' = D-{v,w} $\cup$ {u} is a dominating set of G^2, a contradiction. Hence each vertex in D is of maximum adjacent to the vertices

Theorem 3.6: For any connected fuzzy graph G, The inverse dominating vertex of G^2 is one, if and only if G^2 has at least two vertices, that vertices are adjacent to (n-1) vertices.

Proof: To prove this result, we consider the following two cases.
Case1. Suppose G^2 has exactly one vertex v is are adjacent to (p-1) vertices. Then in this case D = {v} is a dominating set of G2.Clearly, V-D =V –{v}.Further, if D_1= {u}$\in$N(v) in V(G^2) –D, the vertex u is adjacent to (n-1) vertices. Then there exists at least one vertex w $\in$ N(u) in G^2 such that D'= D $\cup$ {w} forms an inverse dominating set of G^2, a contradiction

Case 2. Suppose G^2 has at least two vertices u and v are adjacent to (p-1) vertices such that u and v are not adjacent. Then D = {u} dominates G^2 since the vertex u is adjacent to (p-1) vertices and V(G^2) –D = V(G^2) –{u}. Further, since u and v are not adjacent, D'= {v} $\cup$ V' where V' $\subseteq$ D forms a inverse dominating set of G^2, a contradiction. Conversely, suppose the vertex u is adjacent to (n-1) vertices, such that u and v are adjacent to all the vertices in G^2, D' ={v}$\in$N(u) where {v} $\subseteq$V(G^2) –D and vice-versa. In any case, we obtain the dominating vertex of D' is one. Therefore The inverse dominating vertex of G^2 is one.

Theorem 3.7: For any connected fuzzy graph G without isolates, then γ (G^2) + γ^{-1} (G^2)$\leq$ p
Proof: Suppose D= {v_1,v_2,..v_n} $\subseteq$V(G^2) be the dominating set of G^2, the D' = {v_1,v_2,..v_n} $\subseteq$V(G^2)-D forms a minimal inverse dominating set of G^2 .Since |D| $\leq \left[\frac{p}{4}\right]$ and | D'| $\leq \left[\frac{p}{3}\right]$,it follows that, |D| $\cup$ | D'| $\leq$ p. Therefore,γ (G^2) + γ^{-1} (G^2)$\leq$ p. Suppose V-D is not independent, then there exists at least one vertex _u$\in D'$ such that N(u) $\subseteq$V-D Clearly, | D' | =|{v-D} − {u}| and hence, D| $\cup$ | D' | $\leq$ p, a contradiction. Conversely, if V-D is independent. Then in this case,| D'| = |V-D | in G^2. Clearly, it follows that |D| $\cup$ | D'| = p. Hence, γ (G^2) + γ^{-1} (G^2) =p

Theorem 3.8: For any connected fuzzy graph G with n $\geq$ 3 vertices, the sum of inverse dominating vertices of G^2 and Independent dominating vertices of G is less than or equal to n-1
Proof: For n $\leq$ 2, the sum of inverse dominating vertices of G^2 and independent dominating vertices of G is not less than or equal to n-1.Consider n $\geq$ 3, let F ={v_1,v_2,..v_m} be the minimum set of vertices such that for every two vertices u,v $\in$ F, N(u) $\cap$N(v) $\in$V(G)-F. Suppose there exists a vertex S ={ v_1,v_2,..v_k } $\subseteq$ F which covers all the vertices in G and if the sub graph < S> is totally disconnected. Then S forms the minimal independent dominating set of G. Now in G^2 , since V(G) = V(G^2) and distance between two vertices is at most two in G^2, there exists a vertex set D={v_1,v_2,..v_j}$\subseteq$ S, which forms minimal dominating set of G^2 Then the complementary set V(G^2) –D contains another set D' such that N(D')= V(G^2) . Clearly, D' forms an inverse dominating set of $G^2$and it follows that | D' | $\cup$ |S| $\leq$ n-1.Therefore, the sum of inverse dominating vertices of G^2 and independent dominating vertices of G is less than or equal to n-1

Theorem 3.9 : For any connected fuzzy graph G, γ^{-1} (G^2) +γ_c(G) $\leq$ p+γ (G)
Proof: For n $\leq$ 5, the result follows immediately. Let n $\geq$ 6, suppose D = {v_1,v_2.v_3,…v_n} the vertex vi is adjacent to one or two vertices, 1 $\leq$i $\leq$ n be a minimal dominating set of G. Now we construct a connected dominating set D_c from D by adding in every step at most two components of D forms a connected component in D. Thus we get a connected dominating set D_c after at most D− 1 steps. Now in G^2 , V(G) = V(G^2) .Suppose $\subseteq$ D be minimal dominating set of G^2 'Then there exists a vertex set D' ={v_1,v_2.v_3,…v_k}$\subseteq$ V(G^2)-D_1,such that the path (u,v) of vertices is one or two, u,v$\in D'$, which covers all the vertices in G^2,Clearly, D' forms a minimal inverse dominating set of G^2 ,Hence it follows that | D'| $\cup$ | D_c | $\leq$ p$\cup$ | D | Therefore, γ^{-1} (G^2) +γ_c(G) $\leq$ p+γ (G)

4. Double Domination Number in Square Fuzzy Graphs

In this section, we introduce Double domination number in fuzzy square graphs .

Definition 4.1

A set $D \subseteq V$ is said to be a double dominating set of G , if every vertex of G is dominated by at least two vertices of D. The double domination number of G is denoted by γ_d (G) and is the minimum cardinality of a double D' dominating set of G.

Definition 4.2

A subset $D \subseteq V(G^2)$ is said to be double dominating set of G^2 , if every vertex in G^2 is dominated by at least two vertices of D. The double domination number of G^2, denoted by $\gamma_d(G^2)$, is the minimum cardinality of a double dominating set of G^2.

Definition 4.3

A dominating set D of G^2 is said to be total dominating set of G^2, if for every vertex, $v \in V(G^2)$ there exists a vertex such that $u \in D$ is adjacent to v or if the sub graph $\langle D \rangle$ has no isolated vertex. The total domination number of G^2, denoted by $\gamma_t(G^2)$ is the minimum cardinality of total dominating set of G^2.

Theorem 4.4: For any connected fuzzy graph G with $n \geq 3$ vertices, $\gamma_d(G^2) + \gamma(G^2) \leq p$

Proof: Let $D = \{v_1, v_2. v_3, \ldots v_k\}$ be the minimal set of vertices which covers all the vertices in G^2 . Clearly, D forms a dominating set of G^2 . Further, if there exists a vertex set $V(G^2)–D = V_1$ in G^2 . Then $D \cup V' = S$, where $V' \subseteq V_1$ in G^2 be the set of vertices such that $\forall v \in V(G^2)$ there exists two vertices in $D \cup V' = S$. Further, since every vertex of G^2 are adjacent to at least two vertices of G^2, clearly S forms a double dominating set of G^2. Therefore, Hence

$\gamma_d(G^2) + \gamma(G^2) \leq p$.

Theorem 4.5: For any connected fuzzy graph G , $\gamma(G) \leq \gamma_d(G^2)$.

Proof: If $V_1 = \{v_1, v_2. v_3, \ldots v_n\} \subseteq V(G)$ be the set of vertices and vertex v_i is adjacent to one or two vertices, $1 \leq i \leq n$, $1 \leq i \leq n$. Then $S = \{v_1, v_2. v_3, \ldots v_n\} \subseteq V_1$ forms a minimal dominating set of G . Now without loss of generality in G^2 since $V(G) = V(G^2)$ If $V_2 = \{v_1, v_2. v_3, \ldots v_k\}$ be the set of vertices and the vertex v_k is adjacent to more than two vertices .If $V_2 \in V(G)$, then the vertices which are at a distance at least two are adjacent to each vertex of V_2 in $G2$. Hence $S_1 \cup V_2 = D$ where $S_1 \subseteq S$ forms a minimal double dominating set of G^2. If $V_2 = \phi$, then $S \cup V_3 = D$ where $V_3 \subseteq V_1$ forms a minimal double dominating set of G^2 .Further, since every vertex in G^2 is adjacent to at least two vertices of D , it follows that $\gamma(G) \leq \gamma_d(G^2)$.

Theorem 4.6: For any connected fuzzy graph G , $\gamma_d(G^2) \leq \gamma_t(G)$

Proof: Let $K = \{v_1, v_2. v_3, \ldots v_n\} \subseteq V(G)$ be the set of vertices such that $N(u_i) \cap N(u_j) = \phi$, where $1 \leq i \leq n$, $1 \leq j \leq n$ .Suppose there exists a minimal set $K_1 = \{u_1, u_2. u_3, \ldots u_n\} \in N(K)$, such that the sub graph $\langle K \cup K_1 \rangle$ has no isolated vertex. Further, if $K \cup K1$ covers all the vertices in G , then $K \cup K_1$ forms a minimal total dominating set of G . Since $V(G) = V(G^2)$, there exists a vertex set $D = \{v_1, v_2. v_3, \ldots v_m\} \subseteq K \cup K_1$ in G^2 , which covers all the vertices in G^2 and for every vertex $v \in V(G^2)$, there exists at least two vertices $\{u,w\} \in D$. Clearly, D forms a minimal double dominating set of G^2. Therefore, $\gamma_d(G^2) \leq \gamma_t(G)$

Theorem 4.7: For any connected fuzzy graph G , $\gamma_d(G^2) \leq \gamma_c(G)$

Proof: Suppose $C = \{v_1, v_2. v_3, \ldots v_n\} \subseteq V(G)$ be the set of all cut vertices in G . Further, if $C \cup I$, where $I \in N(C)$ with, and vertex v_i is adjacent to one or two vertices $\forall \{v_i\} \in I$ be the minimal set of vertices which covers all the vertices in G and if the sub graph $C \cup I$ is connected. Then $C \cup I$ forms a minimal connected dominating set of G . Let $D = \{v_1, v_2. v_3, \ldots v_k\} \subseteq V(G)$ be the minimal set of vertices which covers all the vertices in G^2 . Suppose for every vertex $v \in V(G^2)$, there exists at least two vertices $\{u,w\} \in D$. Then D itself forms a minimal double dominating set of G^2 . Therefore, it follows that, $\gamma_d(G^2) \leq \gamma_c(G)$

5. Conclusion

The Inverse and Double domination number in fuzzy square graph is defined. Theorems related to this concept are derived and the relation between fuzzy square graph of domination number and Inverse, Double domination number in fuzzy square graphs are established.

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