Propagation Modes in Multimode Graded-Index Fibers

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Abstract: ***In this research some important parameters of graded index fiber have been studied such as numerical aperture, the normalized frequency and their effects on the modal dispersion. The modal dispersion and the propagation delay have been calculated and plottedas functions of radial number and azimuthal number.***

Keywords: graded index fiber, Step index fibers,modal dispersion, radial number, Azimuthal number

1. Introduction

As its name implies, multimode fibers propagate more than one mode. Multimode fibers can propagate over 100 modes. The number of propagated modes depends on the core size and numerical aperture NA . However,when the core size and NA increase, the number of modes increases. Typical values of fiber core size, and NA are 50 to 100 μ m and 0.20 to 0.29, respectively.Launched into a multimode fiber with more ease. The higher NA and the larger core size make it easier to make fiber connections. During fiber splicing, core-to-core alignment becomes less critical. Another advantage is that multimode fibers permit the utilization of light-emitting diodes LEDs. Single mode fibers typically must use laser diodes. LEDs are cheaper, less complex, and last longer. LEDs are preferred for most applications. Multimode fibers also have some disadvantages. As the number of modes increases, the effect of modal dispersion increases **[1].** Modal dispersion (intermodal dispersion) means that modes arrive at the fiber end at slightly different times. This time difference causes the light pulse to spread. Modal dispersion affects system bandwidth. Fiber manufacturers adjust the core diameter, NA , and index profile properties of multimode fibers to maximize system bandwidth. Then multimode optical fibers are categorized into two types according to their structure, Step index fibers (SIF_S) and Graded index fiber (GIF_S)**[2].**

2. Parameters of Graded Index Fiber

The determination of NA for graded index fiber is more complex than that for step index fiber.In graded index fiber, NA is a function of position across the core end face. This is in contrast to the step index fiber, where NA is constant across the core. Geometrical optics consideration shows that the light incident on the fiber core at position R will propagate as a guided mode only if it is within the local numerical aperture $NA(R)$ at that point. The local numerical aperture is defined as [**3**]

$$NA(R)=\begin{cases} NA(0)\sqrt{1-R^{q}} & R\le 1 \\ 0 & R>1 \end{cases} \quad (1)$$

Where q the graded order, the axial local numerical aperture $NA(0)$ is defined as $NA(0)=n_1\sqrt{2\Delta}$ .Where Δ is the core-cladding index difference? It is clear that NA of a graded index fiber decreases from $NA(0)$ to zero as moves from the fiber axis to the core cladding boundary. As the use of modal distribution $\psi(R)$, the fundamental mode in the step index fiber is mostly approximated with Gaussian distribution of the form [**4**]

$$\psi(R)=e^{-R^2/w^2} \quad (2)$$

where the width parameter is determined by curve fitting or by following a variational procedure. The quality of fit is generally quite good for values of V (normalized frequency) the neighborhood of 2. The spot size can be determined from an analytical approximation accurate to within 1% for $1.2<V<2.405$ and given by **[5]**

$$\frac{w}{a}\approx 0.65+1.619\,V^{-3/2}+2.879\,V^{-6} \quad (3)$$

The spot size w is different for each q , where the smaller q is the larger spot size w .

From the definition $V=\beta_1 a\sqrt{2\Delta}$, V is proportional to β_1 if Δ is relatively independent of V and β_1 . In this case,

$$\frac{\partial}{\partial\beta_1}\left(\frac{\partial\beta_z}{\partial\beta_1}\right)=\left[\frac{\partial}{\partial\beta_1}\left(\frac{\partial}{\partial\beta_1}\frac{\beta_z}{\beta_1}\beta_1\right)\right]=\beta_1\frac{\partial^2}{\partial\beta_1^2}\left(\frac{\beta_z}{\beta_1}\right)+2\frac{\partial}{\partial\beta_1}\left(\frac{\beta_z}{\beta_1}\right) \quad (4)$$

Now, using the definition $V=\beta_1 a\sqrt{2\Delta}$ and chain rule, one may be found

$$\frac{\partial}{\partial\beta_1}=\frac{\partial V}{\partial\beta_1}\frac{\partial}{\partial V}=a\sqrt{2\Delta}\frac{\partial}{\partial V} \quad , \quad \frac{\partial^2}{\partial\beta_1^2}=\frac{\partial V}{\partial\beta_1}\frac{\partial}{\partial V}a\sqrt{2\Delta}\frac{\partial}{\partial V}=2a^2\Delta\frac{\partial^2}{\partial V^2}$$

Then we can write $V^2\frac{\partial^2}{\partial V^2}\left(\frac{\beta_z}{\beta_1}\right)+2V\frac{\partial}{\partial V}\left(\frac{\beta_z}{\beta_1}\right)$ the terms within the brackets can be estimated from the curves of β_z/β_1 versus V. For small V and the fundamental mode, the bracketed term can be positive (or negative waveguide dispersion), so it is possible to use small V or a to cancel the material dispersion or to have zero chromatic dispersion. This is the basic principal of dispersion-shifted fibers. Note that, the curve of β_z/β_1 and V for the fundamental mode is different for different graded order. That is; the waveguide dispersion of the single mode fiber is affected by the graded order. In turn, the waveguide dispersion can be controlled by the fiber characteristics such as fiber radius and refractive indices as well as the index profile.

3. Graded-index Fibers

The modal dispersion in graded index fibers is computed by a complicated procedure that depends on many approximations. In the present work, we are attempted to deduce the modal dispersion of graded- index fibers. Multimode optical fibers described by $\psi_i(r,\varphi,z)=A_i(r,\varphi)e^{i(\omega t-\beta_{zi}z)}$ support a large but finite number of modes which are particular solutions of Maxwell's equations. Each mode propagates at its own velocity resulting from its particular propagation constant. From the WKB approximation, the modal propagation constant was approximated by **[6]**

$$\beta_z^{\ell m}=k_o n_1\sqrt{1-2\Delta\left(\frac{f}{F}\right)^g} \qquad (5)$$

where $g=q/(q+2)$, $f=2m+\ell-1$ stands for the principal mode number and F represents the total number of propagation modes given by Eq.(5), which may be rewritten as $F=g\Delta a^2n_1^2k_o^2$. Note that, the pair (ℓ,m) represent the radial and azimuthal numbers. Physically, (ℓ,m) have the meaning that they count the number of maximum intensities that may appear in the radial and azimuthal directions in the field intensities of a given mode. In a strict sense, the mode number f is a discrete integer parameter which takes values ranging from unity for LP_{01} mode to the total number of mode groups. However, very often f can be treated as a continuous variable. This approximation is of great interest because it allows one to replace the discrete mode spectrum by a model continuum. As a result, the WKB method can readily be used and modes sums can be converted to integrals that are easier to handle **[3,6]**.

From Eq. (5), the unit propagation delay is

$$\tau_g^{\ell m}=\frac{\partial\beta_z^{\ell m}}{\partial w}=\frac{n_1}{c}\frac{\partial}{\partial w}\left\{w\sqrt{1-2\Delta\left(\frac{f}{F}\right)^g}\right\} \qquad (6)$$

To perform the differentiation, note that F is a function of w. Let

$$S_f=\left(\frac{f}{F}\right)^g=\frac{A_f}{w^{2g}} \qquad (7)$$

where $A_f=(fc^2)/(g\Delta a^2n_1^2)$ that may be assumed does not depend on w, ignoring the effect of $\partial n_1/\partial w$ because it much smaller than the modal dispersion effect. As a consequence

$$\frac{\partial S_f}{\partial w}=-\frac{2gS_f}{w} \qquad (8)$$

Using Eq.(8) into (6), the unit propagation delay for f mode will be

$$\tau_g^{\ell m}=\frac{n_1}{c}\left[\sqrt{1-2\Delta S_f}+\frac{2g\Delta S_f}{\sqrt{1-2\Delta S_f}}\right] \qquad (9)$$

Eq.(9) may be rearranged to explain

$$\tau_g^{\ell m}=\frac{n_1}{c}\left[\frac{1+2(g-1)\Delta S_f}{\sqrt{1-2\Delta S_f}}\right] \qquad (10)$$

The binomial expansion for $\sqrt{1-2\Delta S_f}$ will give

$$\sqrt{1-2\Delta S_f}=1+\Delta S_f+\frac{3}{2}\Delta^2S_f^2+\ldots\ldots \qquad (11)$$

Substituting this expansion into Eq.(10) will obtain

$$\tau_g^{\ell m}=\frac{n_1}{c}\left[1+2(g-1)\Delta S_f\right]\left[1+\Delta S_f+\frac{3}{2}\Delta^2S_f^2+\ldots\right] \qquad (12)$$

Ignoring the terms with Δ^3 and higher, the last equation will be

$$\tau_g^{\ell m}=\frac{n_1}{c}\left[1+(2g-1)\Delta S_f+(2g-0.5)\Delta^2S_f^2\right] \qquad (13)$$

To calculate the modal dispersion, we find the propagation modes that give the maximum and minimum group delay. The minimum dispersion occurs when the delays at the two endpoints are $S_f=(1/F)^g$ and $S_f=1$. That is;

$$\tau_g^1=\frac{n_1}{c}\left[1+\varepsilon\right] \qquad (14\text{ a})$$

$$\tau_g^F=\frac{n_1}{c}\left[1+(2g-1)\Delta+(2g-0.5)\Delta^2\right] \qquad (14\text{ b})$$

are the same, where

$$\varepsilon=(2g-1)\Delta\frac{1}{F^g}+(2g-0.5)\Delta^2\frac{1}{F^{2g}} \qquad (15)$$

For multimode fibers $F>>1$, such that $\varepsilon\approx 0$. In this case, the optimum g may be found by equalizing the results in Eq.(14) to get

$$(2g_{opt}-1)=-(2g_{opt}-0.5)\Delta \qquad (16)$$

Eq.(16) may be reformed to obtain

$$g_{opt}=\frac{1+0.5\Delta}{2(1+\Delta)} \qquad (17)$$

Using the definition $g=q/(q+2)$ and Eq.(17), we will get

$$q_{opt}=\frac{2+\Delta}{1+1.5\Delta}=2(1-\Delta)+3\Delta^2+.... \qquad (18)$$

Under the above conditions, modal dispersion can be explained as the difference between τ_g^1 and τ_g^F. That is; the modal dispersion D_{mod} represents the difference

$$D_{\text{mod}}=\tau_g^1-\tau_g^F \qquad (19)$$

Substituting Eq.(14) into (19) and using the result in Eq.(18), the modal dispersion will be $D_{\text{mod}}=0$, while the approximation $q_{opt}=2(1-\Delta)$ will make

$$D_{\text{mod}}=\frac{n_1}{c}\frac{\Delta^2}{8} \qquad (20)$$

Like that D_{mod} for graded index fiber is proportional to Δ^2, which is much smaller than $D_{\text{mod}}=n_1\Delta/c$, for step index fiber. It is important to note that the above approximations are accurate only if the modal dispersion is considered and the chromatic dispersion is ignored. This may be attributed to the frequency dependence of Δ and n_1 that assumed constants in the above derivation.

In general, the modal dispersion is computed using Eq.(13), where for any mode and any graded order $\tau_g^{\ell m}$ may be determined. The dispersion between two modes will be the difference. For the step -index fiber that is a special case of graded -index fiber with $q\to\infty$ and $g\to 0$, we have $S_f=0$ for the minimum mode order and hence $\tau_g^1=n_1/c$, while the maximum mode order has $S_f=1$ that gives

$$\tau_g^F=\frac{n_1}{\text{c}}\left[\frac{1-2\Delta}{\sqrt{1-2\Delta}}\right]\approx\frac{n_1}{\text{c}}(1-\Delta) \qquad (21)$$

However, the modal dispersion for step index fiber will be $D_{\text{mod}}=\tau_g^1-\tau_g^F=n_1\Delta/c$.

4. Results and Discussion

Fig. (1) Illustrates the variation of the factor $V^2\frac{\partial^2}{\partial V^2}\left(\frac{\beta_z}{\beta_1}\right)+2V\frac{\partial}{\partial V}\left(\frac{\beta_z}{\beta_1}\right)$ as a function of V for all collection cases of q. All the descriptions pertains the LP_{01} mode because it is the alone mode that is be limited within a limited range of V, while the remainder modes will be non-limited. The figure shows that the mentioned factor be positive with a range of V. Therefore, $D_{wav}<0$ being within this range as a result it may be using the negative value to control in D_{ch} magnitude to having a λ_o shifted custom made. It is noted that from the figure the step index fiber case the intentional range is (0.2-2.2) while the other cases of graded index fiber causes this range to be shifted to the right. Therefore, D_{wav} depends on graded order basically as well as to its dependency on V. In other words, the controlling in chromatic dispersion will depends on q. From the other hand, as long as D_{wav} is important only in the single modefiber, so that there is not needing to plot the other modes.

Fig. (2) shows the behavior of D_{mod} with the radial number for many cases of q where it is using the disparity between both the maximum and minimum values which revert to the same azimuthal number.However, from the figure, one can see that when $q=0$ the parameter D_{mod} equates to zero, while for the cases in which q less than 2, the D_{mod} has negative values. So, the D_{mod} increases with increasing q until arriving to its maximum value at $q=4$ after which, it is turnabout to decreasing because of the nature of the minimum and maximum values that is revert to a specific azimuthal number. The physical causation behind this behavior reverts to the modes nature, where there is a commutation takes place among them when $q<2$ and this significant is because there is an antecedence or slowness for the mode relative to the other mode. The continuation in q increasing return in D_{mod} value to a stationary case deference on zero which is so large in comparison with the case $q=2$ or in comparison with the chromatic dispersion.

Fig. (3) Shows the behavior of D_{mod} with the azimuthal number, where the difference gave between the minimum and maximum values relating to a specific radial number. From the two figures comparison we notice a variation behavior symmetrical with D_{mod} values varying.

Fig. (4) Illustrates τ_g as a function of radial numberfor a number of q cases and for azimuthal number divers values. From figure it isclarified that τ_g has a slight start point and a high end point. The perceptive difference in antecedent figures in the disparity between these two points except the case $q=1$ in which one can see the reverse occurrence. We note that the increasing of L mean the curve elevation except the case $q=1$ occurrences the reverse too. The existence of $q<2$ conversed caused existence $D_{\text{mod}}<0$ this is apparent in the two antecedent figures.

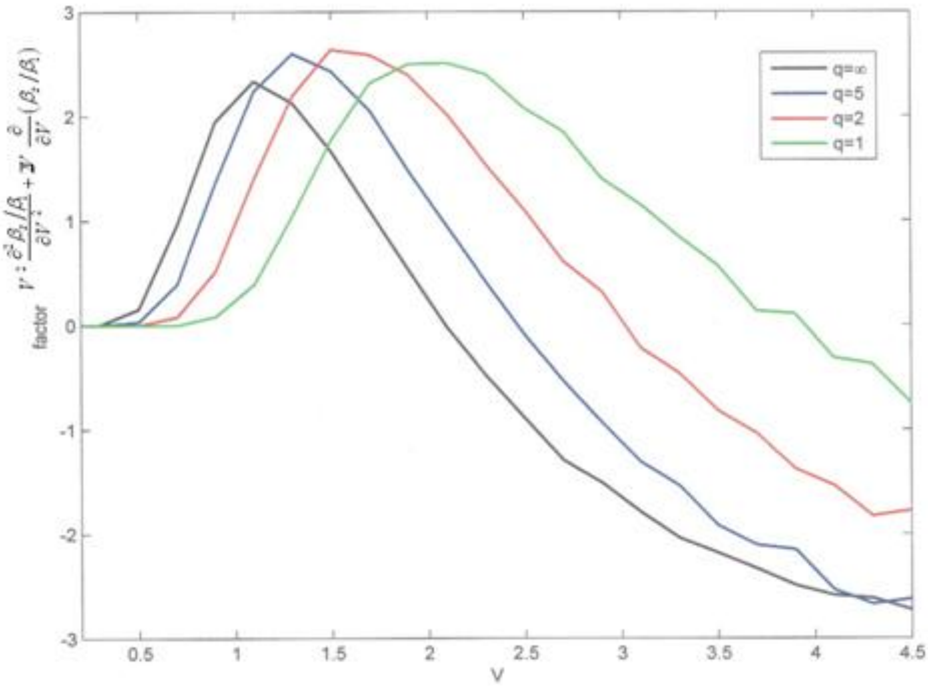

Figure 1: The factor $V^{2}\dfrac{\partial^{2}\beta_z/\beta_1}{\partial V^{2}}+2V\dfrac{\partial}{\partial V}\left(\beta_z/\beta_1\right)$ as a function of V for different graded orders.

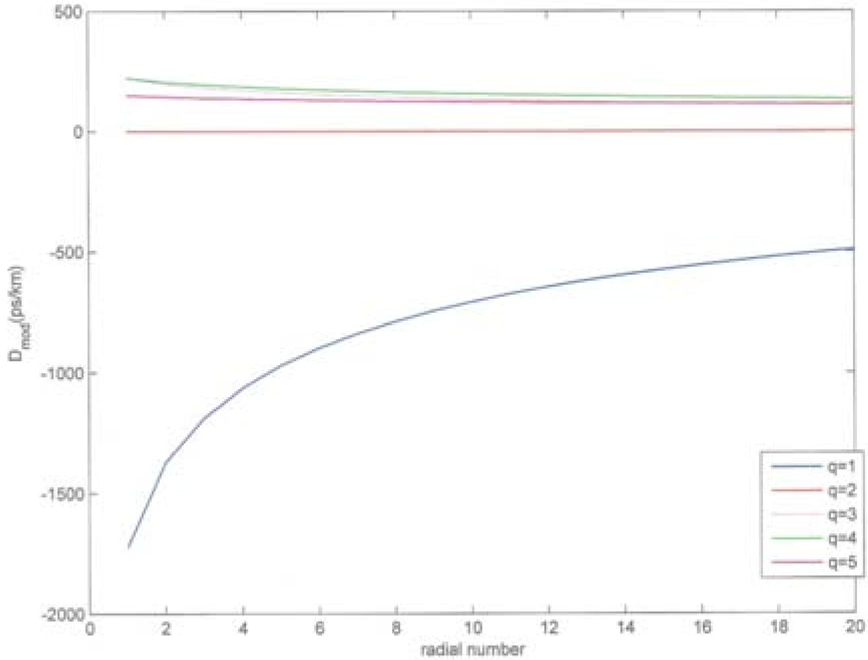

Figure 2: D_{mod} as a function of radial number for different graded orders

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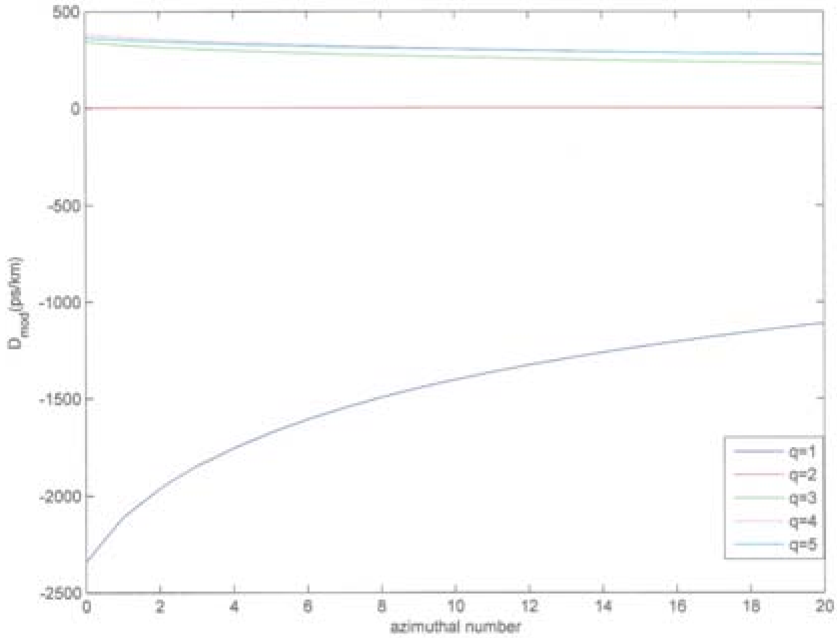

Figure 3: D_{mod} as a function of azimuthal number for different graded orders

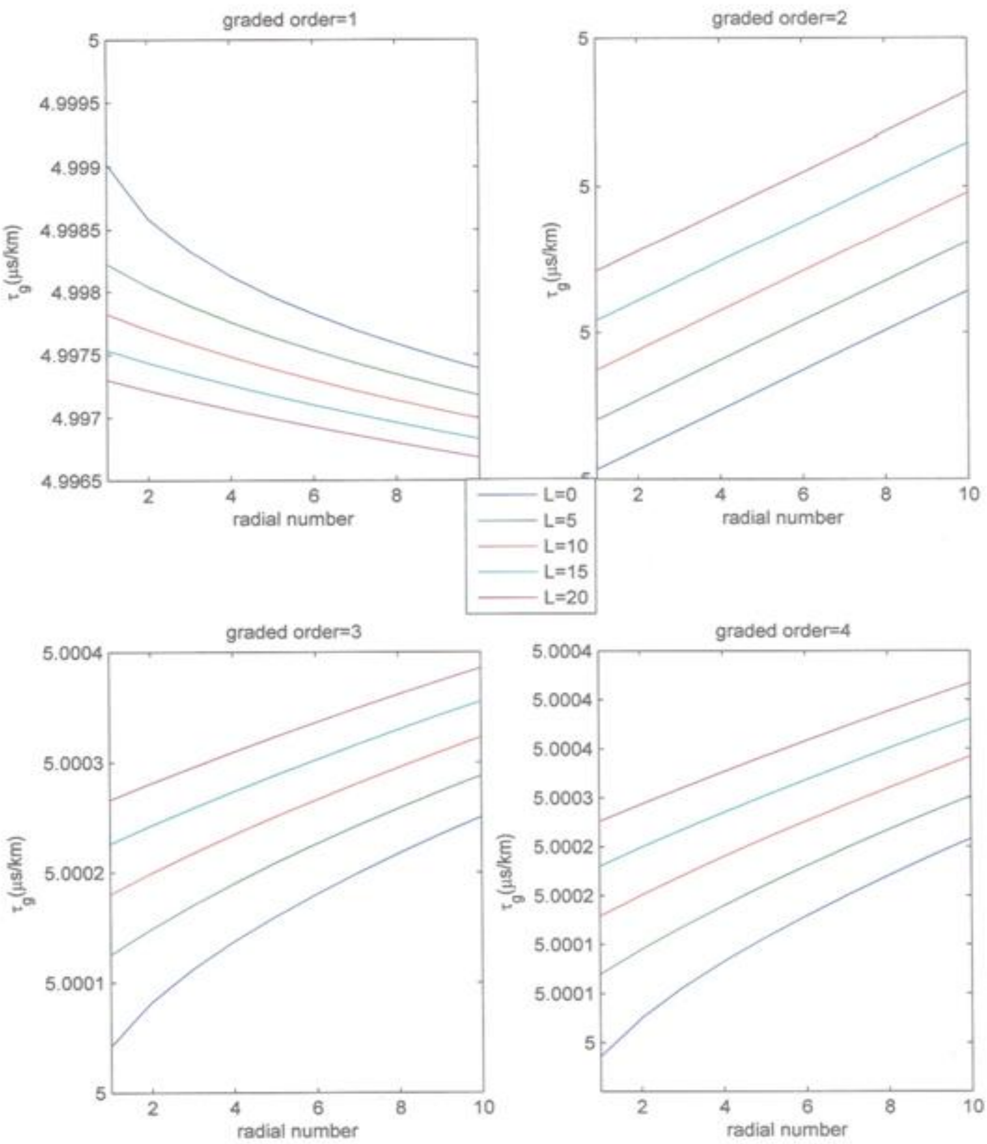

Figure 4: τ_g as a function of radial number for different azimuthal numbers and different graded orders.

5. Conclusion

In graded index fiber some parameters have the following behaviors

1) LP_{01} mode is the alone mode that is be limited within a limited range of *V*. So, the mentioned factor be positive with a range of *V*. Therefore, $D_{wav} < 0$ being within this range
2) D_{wav} dependon graded order and *V*.
3) When $q = 0$ the parameter D_{mod} equates to zero, while for the cases in which q less than 2, the D_{mod} has negative values.
4) D_{mod} increases with increasing q until arriving to its maximum value at $q = 4$ after which, it is turnabout to decreasing.
5) The continuation in q increasing return in D_{mod} value to a stationary case deference on zero which is so large in comparison with the case $q = 2$ or in comparison with the chromatic dispersion.
6) The varying of D_{mod} with the radial number is symmetrical with D_{mod} values varying with the azimuthal number.
7) τ_g have a slight start point and high end point.

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