International Journal of Science and Research (IJSR)
ISSN (Online): 2319-7064
Index Copernicus Value (2013): 6.14 | Impact Factor (2013): 4.438

Modelling and Non-linear Control Design for Coupled Twin Tank Level Process

Jiffy Anna John[1], Riya Mary Francis[2]

[1]PG student, Department of Electrical & Electronics Engineering, TKM College of Engineering, Kollam, India

[2]Assistant Professor, Department of Electrical & Electronics Engineering, TKM College of Engineering, Kollam, India

Abstract: ***The liquid level control in Coupled Tank System is a basic problem in process industries. Many times the liquids will be processed by chemical or mixing treatment in the tanks, but always the level of fluid in the tanks must be controlled and the flow between tanks must be regulated in presence of non-linearity, disturbance and time varying system parameters. The control of liquid level in tanks presents a challenging problem due to its non-linear behavior. This paper deals with non-linear control designs using higher order Sliding Mode Control (SMC). For the consideration of non-linearity and to realize level position regulating and tracking control, a non-linear Integral Sliding Mode Control (ISMC) is used which guarantees the asymptotic stability of closed loop system and reduce the chattering problem associated with Standard Sliding Mode Control (SSMC). The goal of the control algorithm is to track the desired level of liquid in second tank by using flow rate of liquid into first tank as the manipulated variable.***

Keywords: Coupled tank, Integral Sliding Mode Control, Level Control, Non-linear model, Process control

1. Introduction

The level control is one of the control system variable which are more important in process industries. The liquid level control through regulation of flow rate is an important application in various engineering areas such as steam generators in power generating processes, reactors in many chemical plants, storage tank in oil and gas production industry etc.. The problem of level control in coupled tank process are its interacting characteristics and system dynamics. The process industries requires liquid to be pumped as well as stored in tanks and then re-pumped to another tank. It is essential for control system engineers to understand how tank control systems work and how the level control problem is solved. The liquid level system has time varying system parameters and non-linear characteristics in the complex industrial process, also number of disturbances acting on the system during operation.

Various attempts in controlling liquid level of coupled tank system were proposed. The mathematical modelling and designing of Sliding Mode Control for a liquid level control system when tanks are coupled was proposed by Hur Abbas et al [1]. Ahence Boubakir et al proposed a neurofuzzy-sliding mode control using non-linear sliding surface applied to coupled tank system[2]. A robust control of non-linear uncertain system via sliding mode and backstepping was proposed by R.Benayache et al [3]. Parvat. B. J et al proposed the design of sliding mode control using constant relay gain approach [4] and a Direct Model Reference Adaptive Control for Coupled Tank System was proposed by Muhammad Nasiruddin Mahyuddin et al [5].

This paper presents the mathematical modelling of non-linear coupled tank system and performance analysis of the system by using an Integral Sliding Mode Control (ISMC), also verify the ability of the proposed controller for disturbance rejection.

The structure of this paper is as follows. Section 2 deals with the system description. The non-linear modelling of the coupled tank system is explained in section 3. Section 4 highlights the Sliding Mode Control (SMC) designs. Simulation results with Standard Sliding Mode Control (SSMC) and Integral Sliding Mode Control (ISMC) is given in section 5. Conclusion is discussed in section 6.

2. System Description

The coupled tank system includes two tanks mounted above a reservoir, which function as a storage for liquid. It has an independent pump to pump liquid from reservoir to tanks. The two tanks are connected in an interactive manner. When two tanks are coupled, the liquid in two tanks interact and exhibit a non-linear behavior. The liquid meets resistance when flowing through a conduit such as a pipeline. If a liquid flow through the pipe is under turbulent flow condition, the outlet flow rate being function of the square root of the tank height. The discharge coefficient of liquid flowing out of tank can vary by using valves.

Both tanks are identical in cross section and is represented as A (cm^2). The inlet flow Q_{in} is given to the tank 1 and the outlet flow Q_{out} is taken from tank 2. A manual valve is available between tank1 and tank2 which can be used to change the interaction between the tanks.

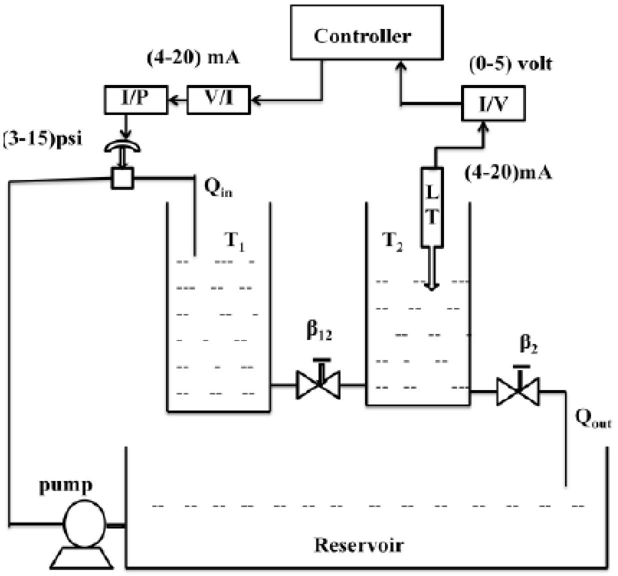

Figure 1: Block Diagram of Coupled Tank System

The change in water level h_1(cm) in tank1 affects the water level h_2 (cm) in tank 2. The water level variation in tank1 and tank2 depends on the inlet and outlet flows. The liquid level in second tank ie, h_2 (cm) is maintained at some desired value by using flow rate of the liquid into first tank $Q_{in}(cm^3/sec)$ as the manipulated variable. The control of liquid level in tanks presents a challenging problem due to its non-linear behavior which is due to the interacting characteristics. In interacting process, dynamics of tank 1 affects the dynamics of tank 2 and vice versa because flow rate depends on the difference between the liquid levels.

3. Mathematical Modelling of Coupled Tank System

Let,
h_1 and h_2 be the height of liquid in tank 1 and tank 2 respectively (cm)
A_1 and A_2 be the cross-sectional area of tank 1 and tank 2 respectively (cm^2)
Q_{in} be flow rate of liquid into tank 1(cm^3/sec)
Q_{out} be flow rate of liquid out of tank 2(cm^3/sec)
a_2 be the cross sectional area of outlet pipe in tank 2 (cm^2)
a_{12} be the cross sectional area of interaction pipe between tank 1 and tank 2 (cm^2)
β_{12}be the valve ratio of interaction pipe between tank 1 and tank 2
β_2 be the valve ratio of outlet pipe of tank 2
g be the acceleration due to gravity

It is assumed that the liquid used is non-viscous, incompressible. The nonlinear equation of the coupled tank system can be obtained by mass balance equation and it is given by,

Rate of change of mass in the tank = Mass flow in - Mass flow out

ie, $$\frac{Adh}{dt} = Q_{in} - Q_{out}$$

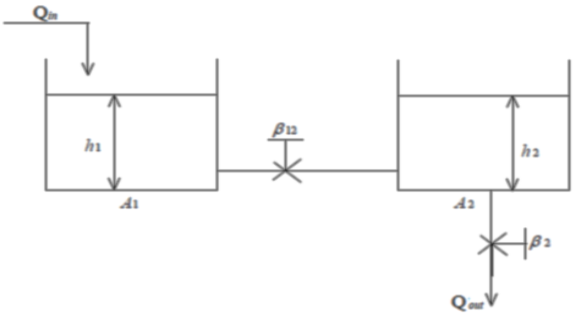

Figure 2: The Coupled tank SISO Process

The dynamic equations for tank 1:

$$A_1 \frac{dh_1(t)}{dt} = Q_{in} - \beta_{12}\, a_{12}\sqrt{2g\,[h_1(t) - h_2(t)]}$$

U (t) = Q_{in}

$$\frac{dh_1(t)}{dt} = \frac{u(t)}{A_1} - \frac{\beta_{12}a_{12}}{A_1}\sqrt{2g[h_1(t) - h_2(t)]} \qquad (1)$$

The dynamic equations for tank 2:

$$A_2 \frac{dh_2(t)}{dt} = \beta_{12}a_{12}\sqrt{2g\,[h_1(t) - h_2(t)}$$

$$-\beta_2 a_2 \sqrt{2g\,h_2(t)}$$

$$\frac{dh_2(t)}{dt} = \frac{\beta_{12}a_{12}}{A_2}\sqrt{2g[h_1(t) - h_2(t)]}$$

$$-\frac{\beta_2\, a_2}{A_2}\sqrt{2g\,h_2} \qquad (2)$$

At equilibrium, for constant water level set point, the derivatives must be zero ie, $\dot{h}_1 = \dot{h}_2 = 0$. In addition, for the case when $h_1 = h_2$, the system model is decoupled. So h_1 must be greater than h_2.

Let,

$z_1 = h_2 > 0$ and $z_2 = h_1 - h_2 > 0$

$z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$, u = q (t)

and

b = $\frac{\beta_{12}a_{12}}{A_2}\sqrt{2g}$, c = $\frac{\beta_2\, a_2}{A_2}\sqrt{2g}$, a = $\frac{1}{A_1}$

The output of the coupled tank system is taken to be the level of the second tank. Therefore, the dynamic model of coupled tank in eqs. (1) and (2) can be written as:

$$\dot{z}_1 = b\sqrt{z_2} - c\sqrt{z_1} \qquad (3)$$

$$\dot{z}_2 = au - 2b\sqrt{z_2} + c\sqrt{z_1} \qquad (4)$$

y = z_1

The objective of the control scheme is to regulate the output y (t) = $z_1(t) = h_2(t)$ to a desired value$h_2(des)$. The dynamic model of the coupled tank system is highly non-linear. Therefore, we will define a transformation so that the dynamic model of the coupled tank system can be transformed into a form facilitates the control design.

Let,

$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ and define the transformation

x = T (z) such that

$$x_1 = z_1 \tag{5}$$

$$x_2 = b\sqrt{z_2} - c\sqrt{z_1} \tag{6}$$

The inverse transformation z = $T^{-1}(x)$ is such that

$$z_1 = x_1 \tag{7}$$

$$z_2 = \left(\frac{c\sqrt{x_1}+x_2}{b}\right)^2 \tag{8}$$

It can be checked that we can write the dynamic model of coupled tank system in eqn (5) and (6) can be written as:

$$x'_1 = x_2 \tag{9}$$

$$x'_2 = \frac{b z'_2}{2\sqrt{z_2}} - \frac{c z'_1}{2\sqrt{z_1}} \tag{10}$$

Substitute the values of z'_1 and z'_2 in eqn (10), we get

$$x'_2 = \frac{bc}{2}\left[\frac{\sqrt{z_1}}{\sqrt{z_2}} - \frac{\sqrt{z_2}}{\sqrt{z_1}}\right] + \frac{c^2}{2} - b^2 + \frac{ab}{2\sqrt{z_2}}u \tag{11}$$

Where the values of z_1 and z_2 in above equation are function of x_1 and x_2 as given in eqn (7) and (8).
Hence dynamic model of the coupled tank system can be written as:

$$x'_1 = x_2 \tag{12}$$

$$x'_2 = f + \phi u \tag{13}$$

$$y = x_1 \tag{14}$$

Where, $f = \frac{bc}{2}\left[\frac{\sqrt{z_1}}{\sqrt{z_2}} - \frac{\sqrt{z_2}}{\sqrt{z_1}}\right] + \frac{c^2}{2} - b^2$

$$\phi = \frac{ab}{2\sqrt{z_2}}$$

Table 1: Parameters of Coupled Tank System

Parameters	*Value*
$A_1, A_2\ (cm^2)$	154
$a_2, a_{12}\ (cm^2)$	0.5
β_{12}	1.5315195
β_2	0.6820043
g (cm^2/sec)	981

4. Sliding Mode Controller Design

The idea behind SMC is to choose a sliding surface along which the system can slide to its desired final value.

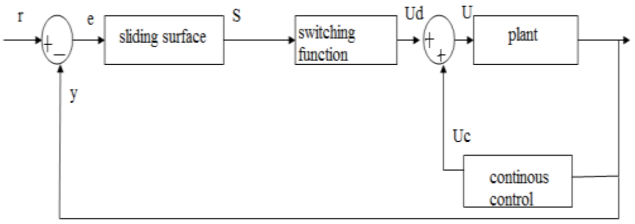

Figure 3: Closed loop schematic diagram of SMC

For designing SMC, firstly the sliding surface select and then design a suitable control law, so that the control variable is being driven to its reference value. The structure of SMC law U (t) is based on two main parts: a continuous part $U_C(t)$ and a discontinuous part $U_D(t)$.
ie, U(t) = $U_C(t) + U_D(t)$

Where, $U_C(t)$ is the dominated equivalent control, represents the continuous part of the controller that maintains the output of the system restricted to the sliding surface. It is a function of reference value and controlled variable. The discontinuous part $U_D(t)$ of SMC comprise a non-linear element that contains the switching element of the control law. This part of the controller is discontinuous across the sliding surface. The objective is to make the error and derivative of error equal to zero. As the system error is defined as the difference between actual height and desired height, mathematically
$e = x_1 - x_{1(des)}$

The expression for the n^{th} order sliding function is given by:
S (t) = $\left(\frac{d}{dt} + \lambda\right)^{n-1}$e

where n is the order of the system and λ > 0 is the slope of the sliding surface.
S (t) = $\dot{e}$+λe

In order to reduce the chattering problem associated with Standard Sliding Mode Controller (SSMC), an **Integral Sliding Mode Controller (ISMC)** is used. The sliding surface s (t) for integral sliding mode controller is presented by Slotine and Li [7],

$$S = \left(\frac{d}{dt} + \lambda\right)^n \int_0^t e\, dt$$

$$S(t) = \dot{e} + 2\lambda e + \lambda^2 \int_0^t e\, dt$$

$$S(t) = \dot{x}_1 + 2\lambda\left(x_1 - x_{1(des)}\right) + \lambda^2 \int_0^t \left(x_1 - x_{1(des)}\right) dt \tag{15}$$

Stability Condition:
Consider a candidate Lyapunov function,

$$V = \frac{1}{2}S^2$$

From Lyapunov theorem we know that if $\dot{V}$ is a negative definite, the system trajectory will be driven and attracted towards the sliding surface and remains sliding on it until the origin is reached asymptotically.

$$\dot{V} = S\dot{S}$$

A sufficient condition for the stability of the system is,
$\dot{S} \leq -|W| \leq 0$
Where W is a positive constant. This equation is called reaching condition or sliding condition.
On taking derivative of eqn (15) w.r.t time,
$\dot{s} = [f + \phi u] + 2\lambda[x_2] + \lambda^2[x_1 - x_{1(des)}]$
On putting $\dot{s} = 0$, we get the continuous part of control law as:
$U_C(t) = \frac{1}{\phi}\left[-f - 2\lambda x_{2} - \lambda^2[x_1 - x_{1(des)}]\right]$
The basic discontinuous control law of SMC is given by,
$U_D(t)$ = -Ksgn(S)
Where the parameter K is a constant manual tuning parameter.
Therefore the integral sliding mode controller is,
U (t) = $U_C(t) + U_D(t)$

U (t) =

$$\frac{-\left\{\frac{bc}{2}\left[\frac{b\sqrt{x_1}}{c\sqrt{x_1+x_2}}-\frac{c\sqrt{x_1+x_2}}{b\sqrt{x_1}}\right]+\frac{c^2}{2}-b^2\right\}-2\lambda x_2-\lambda^2\left[x_1-x_{1(des)}\right]}{\frac{ab^2}{2\left[c\sqrt{x_1+x_2}\right]}}$$

- Ksgn(S)

Where, λ and K are strictly positive constant. This asymptotically stabilize the output of the system y (t) = $x_1(t) = h_2(t)$ to its desired value.

5. Results and Discussion

The response of the system for different operating level and set point tracking performance of the system with designed Integral Sliding Mode Control (ISMC) are observed.

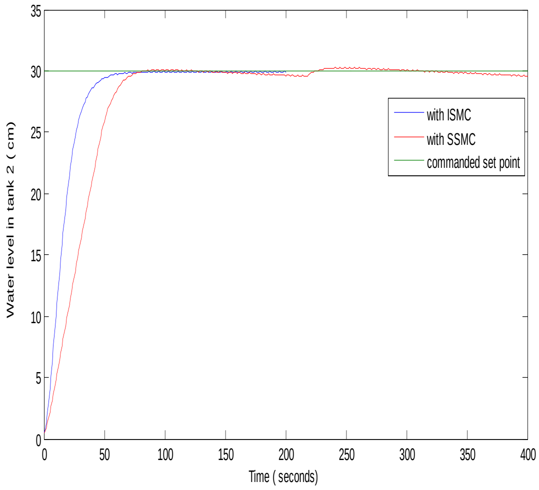

Figure 4: Response of the system with Integral Sliding Mode Control(ISMC) and Standard Sliding Mode Control(SSMC)

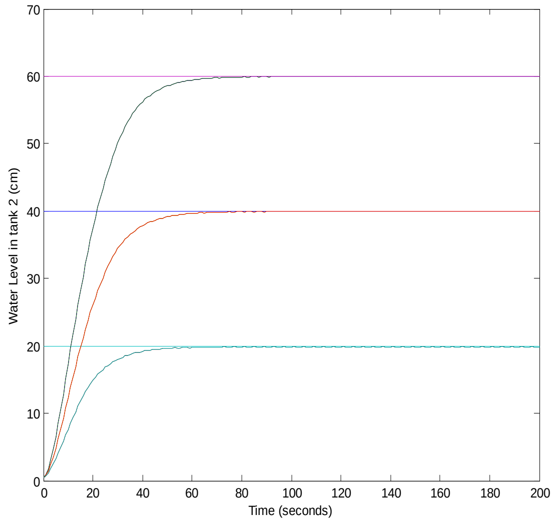

Figure 5: Response of the system with Integral Sliding Mode Control for different operating level

In figure 4, the system response with Standard Sliding Mode Control (SSMC) and Integral Sliding Mode Control (ISMC) is shown. The chattering problem associated with Standard Sliding Mode Control can be reduced by using Integral Sliding Mode Control. In figure 5, the response of the system with Integral Sliding Mode Control for different operating level is shown. The system achieves consistent performance and maintains the desired transient response characteristic throughout all operating points [at 20 cm, 40 cm, 60 cm] without overshoot.

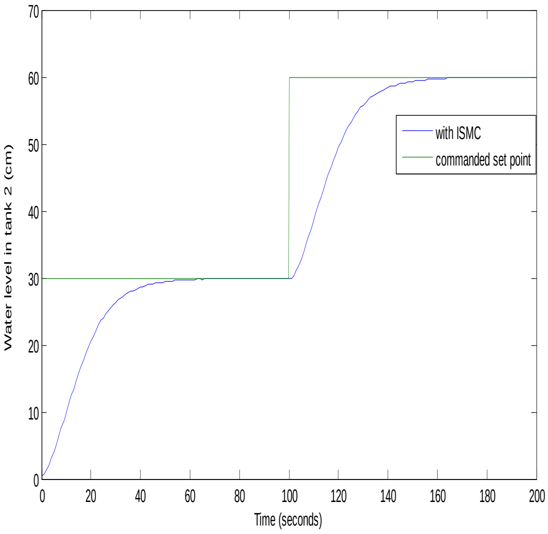

Figure 6: Set point tracking performance of the system with Integral Sliding Mode Control (ISMC)

The set point tracking test consist of changing the set point consecutively during the operation. The set point change is done at 100 second by a magnitude of 30 cm height in water level and the tracking performance of the system with Integral Sliding Mode Control is shown in figure 6. The response of the system shown in figure 6 clearly indicate how the controller takes the action for the given set point. The disturbance rejection of the controlled system shown in figure 7 and the response shows confirms the controller action even in the presence of disturbance (load disturbance) so as to reach the desired set point.

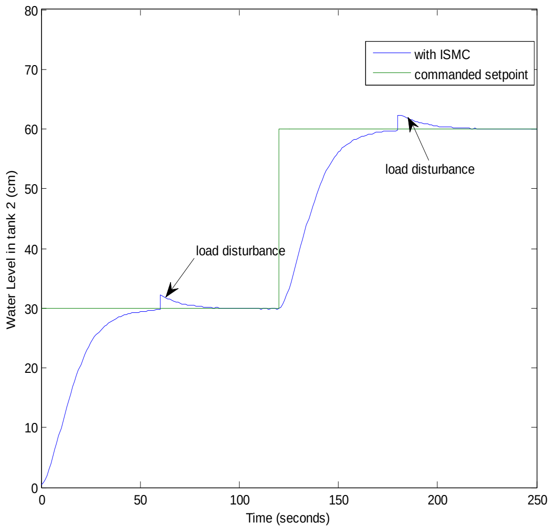

Figure 7: Disturbance rejection of the controlled system

6. Conclusion

The non-linear control designs using Standard Sliding Mode Control (SSMC) for the application of level control of coupled tank system is used. In order to reduce the chattering problem associated with Standard Sliding Mode Control (SSMC), an Integral Sliding Mode Control (ISMC) is used. It can be shown that the Integral Sliding Mode Control (ISMC) can cope with the coupled tank non-linear characteristics at all operating points. The designed non-linear controllers are able to sustain the desired transient response throughout the set point changes without significant overshoot, maximally fast and with high degree of accuracy and also the designed controller allows good disturbance rejection, thereby maintaining best dynamic performance.

References

[1] Hur Abbas, Sajjad and Shahid Qamar, "Sliding Mode Control of coupled tank liquid level control system", IEEE 10th International Conference on Frontires of Information Technology, 2012.
[2] Ahence Boubakir, Fares Boudjema and Salim Labiod, "A neurofuzzy- sliding mode control using non-linear sliding surface applied to coupled tank system", International journal of Automation and Computing, February 2009.
[3] R. Benayache, L.Chirifi Alaoui, A. Benamor, X. Dovifaaz and P.Bussy "Robust controlof non-linear uncertain system via sliding mode with backstepping design". American Control Conference, USA , July 2010.
[4] Parvat B. J, Jadhav V. K, Lokhande. N. N "Design and implementation of sliding mode controller for level control". International Journal of Electronics and Communication Engineering, ISSN 2278-8735.
[5] Ahmed RHIF, Zohra Kardous, Naceur, BenHadj Braiek "A PID- Sliding Mode Control design for a coupled tank". International Conferences CRATT, Rades Tunisia 2012.
[6] Boubakir, F. Boudjema and S. Laboid "A Fuzzy Sliding Mode Controller using non-linear sliding surface applied to the coupled tank system". International Journal of Fuzzy System, Vol. 10, No. 2, June 2008.
[7] J. J. Slotine and S.S. Sastry "Tracking control of non-linear system using sliding surface with application to robot manipulators". International journal control, Vol.38, pp. 465-492, 1983.

Author Profile

Jiffy Anna John was born in Kerala, India in 21/01/1991. She received B.Tech degree in Electrical and Electronics Engineering from Baselios Mathews II College of Engineering, Kollam, Kerala in 2013.Currently, she is pursuing her M Tech degree in Industrial Instrumentation & Control from TKM College of Engineering, Kollam, India. Her research interests includes Process Control and Control Systems. (jiffyannajohn@gmail.com)