International Journal of Science and Research (IJSR)
ISSN (Online): 2319-7064
Index Copernicus Value (2013): 6.14 | Impact Factor (2013): 4.438

Ultra Semi $^{\#}g\alpha$-Closed Graphs

M. Vivek Prabu[1], V. Kokilavani[2]

[1, 2]Department of Mathematics, Kongunadu Arts and Science College (Autonomous), Coimbatore-641029, Tamilnadu, India

Abstract: ***In this paper,we introduce the notion of ultra semi $^{\#}g\alpha$-closed graphs and strongly semi $^{\#}g\alpha$-closed graphs in topological spaces and investigate some of their properties via semi $^{\#}g\alpha$-open sets and semi $^{\#}g\alpha$-closure operator. We also introduce the notion of semi $^{\#}g\alpha$-Urysohn space and examine its properties.***

Keywords: semi $^{\#}g\alpha$-closed graphs, ultra semi $^{\#}g\alpha$-closed graphs, strongly semi $^{\#}g\alpha$-closed graphs, semi $^{\#}g\alpha$-Urysohn space, semi $^{\#}g\alpha$-T_1 space.

1. Introduction

In 2009, M.Caldas et.al [1] introduced and studied the concept of functions with strongly λ-closed graphs. V.Kokilavani and M.Vivek Prabu [4], introduced the notion of semi $^{\#}g\alpha$-closed sets in topological spaces and examined their relationship with the other existing sets. In this paper, we introduce the notion of ultra semi $^{\#}g\alpha$-closed graphs and strongly semi $^{\#}g\alpha$-closed graphs in topological spaces and investigate some of their properties via semi $^{\#}g\alpha$-open sets and semi $^{\#}g\alpha$-closure operator. We also introduce the notion of semi $^{\#}g\alpha$-Urysohn space and examine its properties.

2. Preliminaries

Definition 2.1 A subset A of X is called

1) g-closed [6] if $cl(A) \subseteq$ U, whenever A $\subseteq$ U and U is open in X. The complement of g-closed set is called g-open.
2) $g^{\#}\alpha$-closed [7] if $\alpha cl(A) \subseteq$ U, whenever A $\subseteq$ U and U is g-open in X. The complement of $g^{\#}\alpha$-closed set is called $g^{\#}\alpha$-open.
3) $^{\#}g\alpha$-closed [2] if $\alpha cl(A) \subseteq$ U, whenever A $\subseteq$ U and U is $g^{\#}\alpha$-open in X. The complement of $^{\#}g\alpha$-closed set is called $^{\#}g\alpha$-open.
4) semi $^{\#}g\alpha$-closed [4] if scl(A) $\subseteq$ U, whenever A $\subseteq$ U and U is $^{\#}g\alpha$-open in X. The complement of semi $^{\#}g\alpha$-closed set is called semi$^{\#}g\alpha$-open.

The union (resp. intersection) of all semi $^{\#}g\alpha$-open (resp. semi $^{\#}g\alpha$-closed) sets, each contained in (resp. containing) a set A of X is called the semi $^{\#}g\alpha$-interior (resp. semi $^{\#}g\alpha$-closure) of A, which is denoted by semi $^{\#}g\alpha$-int(A) (resp. semi $^{\#}g\alpha$-cl(A)).

Definition 2.2 A function f: $X \rightarrow Y$ is said to be

1) semi $^{\#}g\alpha$-continuous [4] if for every closed set in Y, its inverse image is semi $^{\#}g\alpha$-closed in X.
2) semi $^{\#}g\alpha$-irresolute [4] if for every semi $^{\#}g\alpha$-closed set in Y, its inverse image is semi $^{\#}g\alpha$-closed in X.

Definition 2.3

1) A space X is said to be semi $^{\#}g\alpha$-T_0 [5] if for each pair of distinct points x and y in X, there exists semi $^{\#}g\alpha$-open sets U and V containing x and y respectively, such that x ∈ U and y ∉ U or y ∈ V and x ∉ V.
2) A space X is said to be semi $^{\#}g\alpha$-T_1 [5] if for each pair of distinct points x and y in X, there exists semi $^{\#}g\alpha$-open sets U and V containing x and y respectively, such that y ∉ U and x ∉ V.
3) A space X is said to be semi $^{\#}g\alpha$-T_2 [5] if for each pair of distinct points x and y in X, there exists semi $^{\#}g\alpha$-open sets U and V containing x and y respectively, such that U ∩ V = ∅.

Definition 2.4 If f:$(X,\tau)\rightarrow(Y,\sigma)$ is any function, then the subset G(f) = {(x,f(x)) : x ∈ X} of the product space (X x Y, τ x σ) is called graph of f [3].

3. Ultra Semi $^{\#}g\alpha$-Closed Graphs

Definition 3.1 A function f: $(X,\tau)\rightarrow(Y,\sigma)$ is said to have a ultra semi $^{\#}g\alpha$-closed graph if for each (x,y) ∈ (X x Y) \ G(f), there exist U $\subseteq$ Semi$^{\#}G\alpha$O(X,x) and V $\subseteq$ Semi$^{\#}G\alpha$O(Y,y) such that f(U) ∩ semi $^{\#}g\alpha$-cl(V) = ∅.

Theorem 3.2 If f: $(X,\tau)\rightarrow(Y,\sigma)$ is a function with a ultra semi $^{\#}g\alpha$-closed graph, then for each x ∈ X, f(x) = ∩{semi $^{\#}g\alpha$-cl(f(U)) : U $\subseteq$ Semi$^{\#}G\alpha$O(X,x)}.

Proof. Suppose the theorem is false. Then there exists any y ≠ f(x) such that y ∈ ∩{semi $^{\#}g\alpha$-cl(f(U)) : U $\subseteq$ Semi $^{\#}G\alpha$O(X,x)}. Hence for every U $\subseteq$ Semi$^{\#}G\alpha$O(X,x), y ∈ semi $^{\#}g\alpha$-cl(f(U)). So V ∩ f(U) ≠ ∅ for every V $\subseteq$

Semi$^{\#}G\alpha$O(Y,y). This implies that semi $^{\#}g\alpha$-cl(V) ∩ f(U) ⊃ V ∩ f(U) ≠ ∅ which contradicts the hypothesis that f is a function with a ultra semi $^{\#}g\alpha$-closed graph. Hence the theorem holds.

Theorem 3.3 If f:$(X,\tau)\to(Y,\sigma)$ is semi $^{\#}g\alpha$-irresolute and Y is semi $^{\#}g\alpha$-T_2, then G(f) is ultra semi $^{\#}g\alpha$-closed.

Proof. Let (x,y) ∈ (X x Y) \ G(f) and V ⊆ Semi$^{\#}G\alpha$O(Y,y) such that f(x) ∉ semi $^{\#}g\alpha$-cl(V). It follows that there is U ⊆ Semi$^{\#}G\alpha$O(X,x) such that f(U)⊂ Y \ semi $^{\#}g\alpha$-cl(V). Hence, f(U) ∩ semi $^{\#}g\alpha$-cl(V) = ∅.
The converse of the above theorem need not be true which can be seen from the following example.

Example 3.4 Let X = {a,b,c}, τ = {∅,X,{a}} and f:$(X,\tau)\to(X,\tau)$ be an identity map. Then clearly f is semi $^{\#}g\alpha$-irresolute but X is not a semi $^{\#}g\alpha$-T_2 space. Therefore G(f) is not ultra semi $^{\#}g\alpha$-closed.

Theorem 3.5 If f:$(X,\tau)\to(Y,\sigma)$ is surjective and has a ultra semi $^{\#}g\alpha$-closed graph G(f), then Y is both semi $^{\#}g\alpha$-T_1 and semi $^{\#}g\alpha$-T_2.

Proof. Let $y_1 \neq y_2$ ∈ Y. Since f is surjective, there exists any x_1 ∈ X such that $f(x_1) = y_1$. Now (x_1,y_2) ∈ (X x Y) \ G(f). The ultra semi $^{\#}g\alpha$-closed graph G(f) of f implies U ⊆ Semi$^{\#}G\alpha$O(X,x_1) and V ⊆ Semi$^{\#}G\alpha$O(Y,y_2) such that f(U) ∩ semi $^{\#}g\alpha$-cl(V) = ∅, since y_1 ∉ semi $^{\#}g\alpha$-cl(V). Therefore there exists any W ⊆ Semi $^{\#}G\alpha$O(Y,y_1) such that W ∩ V = ∅. Thus Y is semi $^{\#}g\alpha$-T_2 and hence it is a semi $^{\#}g\alpha$-T_1 space.

Theorem 3.6 If f: $(X,\tau)\to(Y,\sigma)$ is injective and has a ultra semi $^{\#}g\alpha$-closed graph G(f), then X is a semi $^{\#}g\alpha$-T_1 space.

Proof. Since f is injective, for any pair of distinct points x_1,x_2 ∈ X, $f(x_1) \neq f(x_2)$. Here $(x_1,f(x_2))$ ∈ (X x Y) \ G(f). Since G(f) is ultra semi $^{\#}g\alpha$-closed graph, there exist U ⊆ Semi$^{\#}G\alpha$O(X,x_1) and V ⊆ Semi$^{\#}G\alpha$O$(Y,f(x_2))$ such that f(U) ∩ semi $^{\#}g\alpha$-cl(V) = ∅. Therefore we have x_2 ∉ U. So there exist any W ⊆ Semi$^{\#}G\alpha$O(X,x_2) such that x_1 ∉ W. Hence, X is a semi $^{\#}g\alpha$-T_1 space.

Remark 3.7 If f:$(X,\tau)\to(Y,\sigma)$ is bijective and has a ultra semi $^{\#}g\alpha$-closed graph G(f), then X and Y are semi $^{\#}g\alpha$-T_1 spaces.

Theorem 3.8 A space X is semi $^{\#}g\alpha$-T_2 if and only if the identity function f: $(X,\tau)\to(X,\tau)$ has a ultra semi $^{\#}g\alpha$-closed graph G(f).

Proof. Let X be a semi $^{\#}g\alpha$-T_2 space. Since the identity function f: $(X,\tau)\to(X,\tau)$ is semi $^{\#}g\alpha$-irresolute, from Theorem 3.3 we conclude that it has a ultra semi $^{\#}g\alpha$-closed graph G(f).
Conversely suppose that f has a ultra semi $^{\#}g\alpha$-closed graph G(f). Here clearly f is surjective and hence by Theorem 3.5, X is a semi $^{\#}g\alpha$-T_2 space.

Definition 3.9 A function f: $(X,\tau)\to(Y,\sigma)$ is called quasi semi $^{\#}g\alpha$-irresolute, if for each x ∈ X and each V ⊆ Semi$^{\#}G\alpha$O(Y,f(x)), there exist U ⊆ Semi$^{\#}G\alpha$O(X,x) such that f(U) ⊂ semi $^{\#}g\alpha$-cl(V).

Theorem 3.10 If a function f:$(X,\tau)\to(Y,\sigma)$ is quasi semi $^{\#}g\alpha$-irresolute, injective and has a ultra semi $^{\#}g\alpha$-closed graph G(f), then X is semi $^{\#}g\alpha$-T_2.
Proof. Since f is injective, for any pair of distinct points x_1, x_2 ∈ X, $f(x_1) \neq f(x_2)$. Here $(x_1,f(x_2))$ ∈ (X x Y) \ G(f). Since G(f) is ultra semi $^{\#}g\alpha$-closed graph, there exist U ⊆ Semi$^{\#}G\alpha$O(X,x_1) and V ⊆ Semi$^{\#}G\alpha$O$(Y,f(x_2))$ such that f(U) ∩ semi $^{\#}g\alpha$-cl(V) = ∅, which implies U ∩ f^{-1}(semi $^{\#}g\alpha$-cl(V)) = ∅. Consequently f^{-1}(semi $^{\#}g\alpha$-cl(V)) ⊂ X \ U. Moreover since f is quasi semi $^{\#}g\alpha$-irresolute, there exists any W ⊆ Semi$^{\#}G\alpha$O(X,x_2) such that f(W) ⊂ semi $^{\#}g\alpha$-cl(V). i.e., W ⊂ f^{-1}(semi $^{\#}g\alpha$-cl(V)) ⊂ X \ U. Thus W ∩ U = ∅. Hence X is semi $^{\#}g\alpha$-T_2.

Theorem 3.11 If a function f:$(X,\tau)\to(Y,\sigma)$ is semi $^{\#}g\alpha$-irresolute, injective and has a ultra semi $^{\#}g\alpha$-closed graph G(f), then X is semi $^{\#}g\alpha$-T_2.

Proof. Since every semi $^{\#}g\alpha$-irresolute function is quasi semi $^{\#}g\alpha$-irresolute, the proof follows from Theorem 3.10.

Theorem3.12 If a function f:$(X,\tau)\to(Y,\sigma)$ is quasi semi $^{\#}g\alpha$-irresolute, bijective and has a ultra semi $^{\#}g\alpha$-closed graph G(f), then X and Y are semi $^{\#}g\alpha$-T_2.
Proof. It is obvious from Theorem 3.10 and Theorem 3.5.

4. Strongly Semi $^{\#}g\alpha$-Closed Graphs

Definition 4.1 A function f: $(X,\tau)\to(Y,\sigma)$ is said to have a strongly semi $^{\#}g\alpha$-closed graph if for each (x,y) ∈ (X x Y) \ G(f), there exist U ⊆ Semi $^{\#}G\alpha$O(X,x) and an open set V of Y containing y such that f(U) ∩ V = ∅.

Theorem 4.2 Every ultra semi $^{\#}g\alpha$-closed graph is strongly semi $^{\#}g\alpha$-closed graph.

Proof. It follows from the definitions 3.1 and 4.1.

Theorem 4.3 If f: $(X,\tau)\rightarrow(Y,\sigma)$ is semi $^{\#}g\alpha$-continuous and Y is Hausdroff, then G(f) is strongly semi $^{\#}g\alpha$-closed in X x Y.

Proof. Let (x,y) ∈ (X x Y) \ G(f). Then f(x) ≠ y. Since Y is Hausdroff, there exist open sets V and W in containing f(x) and y respectively such that V ∩ W = ∅. Also since f is semi $^{\#}g\alpha$-continuous, there exists U ⊆ Semi$^{\#}G\alpha$O(X,x) such that f(U) ⊂ V. Hence f(U) ∩ W = ∅, G(f) is strongly semi $^{\#}g\alpha$-closed.

Theorem 4.4 If f: $(X,\tau)\rightarrow(Y,\sigma)$ is surjective and has a strongly semi $^{\#}g\alpha$-closed graph G(f), then Y is T_1.

Proof. Let $y_1 \neq y_2$ ∈ Y. Since f is surjective, there exists a x ∈ X such that f(x) = y_2. Hence (x,y_1) ∉ G(f). Then by the definition 4.1, there exist semi $^{\#}g\alpha$-open set U and an open set V containing x and y_1 respectively, such that f(U) ∩ V = ∅. Hence y_2 ∉ V. Thus Y is T_1.

Theorem 4.5 If f: $(X,\tau)\rightarrow(Y,\sigma)$ is a function with a strongly semi $^{\#}g\alpha$-closed graph, then for each x ∈ X, f(x) = ∩{semi $^{\#}g\alpha$cl(f(U)) : U ⊆ Semi$^{\#}G\alpha$O(X,x)}.

Proof. It follows from the Theorem 3.2 and Theorem 4.2.

Theorem 4.6 If f: $(X,\tau)\rightarrow(Y,\sigma)$ is surjective and has a strongly semi $^{\#}g\alpha$-closed graph G(f), then Y is both semi$^{\#}g\alpha$-T_2 and semi $^{\#}g\alpha$-T_1.

Proof. It follows from Theorem 3.5 and Theorem 4.2.

Theorem 4.7 If f: $(X,\tau)\rightarrow(Y,\sigma)$ is an injection and G(f) is strongly semi $^{\#}g\alpha$-closed, then X is semi $^{\#}g\alpha$-T_1.

Proof. It follows from the Theorem 3.6 and Theorem 4.2.

Theorem4.8 If f: $(X,\tau)\rightarrow(Y,\sigma)$ is a bijective function with strongly semi $^{\#}g\alpha$-closed graph G(f), then (X,τ) and (Y,σ) are semi $^{\#}g\alpha$-T_1 space.

Proof. It follows from the Theorem 3.7 and Theorem 4.2.

Theorem 4.9 If f: $(X,\tau)\rightarrow(Y,\sigma)$ is semi $^{\#}g\alpha$-irresolute and Y is semi $^{\#}g\alpha$-T_2, then G(f) is strongly semi $^{\#}g\alpha$-closed.

Proof. It follows from the Theorem 3.3 and Theorem 4.2.

Example 4.10 Let X = {a,b,c}, τ = {∅,X,{a}} and f: $(X,\tau)\rightarrow(X,\tau)$ be an identity map. Then clearly f is semi $^{\#}g\alpha$-irresolute but X is not a semi$^{\#}g\alpha$-T_2 space. Therefore G(f) is not strongly semi $^{\#}g\alpha$-closed.

Theorem 4.11 A space X is semi $^{\#}g\alpha$-T_2 if and only if the identity function f: $(X,\tau)\rightarrow(X,\tau)$ has a strongly semi $^{\#}g\alpha$-closed graph G(f).

Proof. It follows from the Theorem 3.8 and Theorem 4.2.

Theorem 4.12 If a function f: $(X,\tau)\rightarrow(Y,\sigma)$ is a quasi semi $^{\#}g\alpha$-irresolute injection with a strongly semi $^{\#}g\alpha$-closed graph G(f), then X is semi $^{\#}g\alpha$-T_2.

Proof. It follows from the Theorem 3.10 and Theorem 4.2.

Theorem 4.13 If a function f: $(X,\tau)\rightarrow(Y,\sigma)$ is semi $^{\#}g\alpha$-irresolute, injective and has a strongly semi $^{\#}g\alpha$-closed graph G(f), then X is semi $^{\#}g\alpha$-T_2.

Proof. Since every semi $^{\#}g\alpha$-irresolute function is quasi semi $^{\#}g\alpha$-irresolute, the proof follows from Theorem 3.11.

Theorem 4.14 If a function f: $(X,\tau)\rightarrow(Y,\sigma)$ is quasi semi $^{\#}g\alpha$-irresolute, bijective and has a strongly semi $^{\#}g\alpha$-closed graph G(f), then X and Y are semi $^{\#}g\alpha$-T_2.

Proof. It is obvious from Theorem3.12 and Theorem4.2.

5. Semi $^{\#}g\alpha$-Urysohn Space

Definition 5.1 A topological space X is called semi $^{\#}g\alpha$-Urysohn if every pair of distinct points x, y ∈ X, there exist U ⊆ Semi$^{\#}G\alpha$O(X,x) and V ⊆ Semi$^{\#}G\alpha$O(X,y) such that semi $^{\#}g\alpha$-cl(U) ∩ semi $^{\#}g\alpha$-cl(V) = ∅.

Theorem 5.2 Every semi $^{\#}g\alpha$-Urysohn space is a semi $^{\#}g\alpha$-T_2 space.

Proof. Let x and y be two distinct points of X. Since X is semi $^{\#}g\alpha$-Urysohn, there exist U ⊆ Semi$^{\#}G\alpha$O(X,x) and V ⊆ Semi$^{\#}G\alpha$O(X,y) such that semi $^{\#}g\alpha$-cl(U) ∩ semi $^{\#}g\alpha$-cl(V) = ∅. Hence U ∩ V = ∅. Thus X is semi $^{\#}g\alpha$-T_2.

Theorem 5.3 If Y is semi $^{\#}g\alpha$-Urysohn and f: $(X,\tau)\rightarrow(Y,\sigma)$ is quasi semi $^{\#}g\alpha$-irresolute injection, then X is semi $^{\#}g\alpha$-T_2.

Proof. Since f is injective, for any pair of distinct points $x_1, x_2 \in X$, $f(x_1) \neq f(x_2)$. Also since Y is semi $^{\#}g\alpha$-Urysohn, there exist $V_i \subseteq$ Semi$^{\#}G\alpha O(Y, f(x_i))$, i = 1,2 such that semi $^{\#}g\alpha$-cl$(V_1) \cap$ semi $^{\#}g\alpha$-cl$(V_2) = \emptyset$. Hence f^{-1}(semi $^{\#}g\alpha$-cl$(V_1)) \cap f^{-1}$(semi$^{\#}g\alpha$-cl$(V_2))=\emptyset$. Since f is quasi semi $^{\#}g\alpha$-irresolute, there exist $U_i \subseteq$ Semi$^{\#}G\alpha O(X, x_i)$, such that $f(U_i) \subseteq$ semi $^{\#}g\alpha$-cl(V_i), i = 1,2. Hence $U_i \subseteq f^{-1}$(semi $^{\#}g\alpha$-cl$(V_i))$, i = 1,2. Therefore $U_1 \cap U_2 \subseteq f^{-1}$(semi $^{\#}g\alpha$-cl$(V_1)) \cap f^{-1}$(semi $^{\#}g\alpha$-cl$(V_2)) = \emptyset$. Thus X is semi $^{\#}g\alpha$-T_2.

Definition 5.4 A function f: $(X,\tau)\rightarrow(Y,\sigma)$ is pre semi $^{\#}g\alpha$-open if $f(A) \subseteq$ Semi$^{\#}G\alpha O(Y)$ for all $A \subseteq$ Semi$^{\#}G\alpha O(X)$.

Lemma 5.5 Let f: $(X,\tau)\rightarrow(Y,\sigma)$ be pre semi $^{\#}g\alpha$-open, bijective. Then for any $B \subseteq$ Semi$^{\#}G\alpha C(X)$, $f(B) \subseteq$ Semi$^{\#}G\alpha C(Y)$.

Theorem 5.6 If f: $(X,\tau)\rightarrow(Y,\sigma)$ is pre semi $^{\#}g\alpha$-open, bijective and X is semi $^{\#}g\alpha$-Urysohn, then Y is semi $^{\#}g\alpha$-Urysohn.

Proof. Let $y_1 \neq y_2 \in Y$. Since f is bijective, $f^{-1}(y_1) \neq f^{-1}(y_2) \in X$. Also since X is semi $^{\#}g\alpha$-Urysohn, there exist $U \subseteq$ Semi$^{\#}G\alpha O(X, f^{-1}(y_1))$ and $V \subseteq$ Semi $^{\#}G\alpha O(X, f^{-1}(y_2))$ such that semi $^{\#}g\alpha$-cl(U) $\cap$ semi $^{\#}g\alpha$-cl(V) $= \emptyset$. Since semi $^{\#}g\alpha$-cl(U) is a semi $^{\#}g\alpha$-closed set in X, by Lemma 5.5 we have f(semi $^{\#}g\alpha$-cl(U)) $\subseteq$ Semi$^{\#}G\alpha C(Y)$. Also $U \subseteq$ semi $^{\#}g\alpha$-cl(U) implies $f(U) \subseteq$ f(semi $^{\#}g\alpha$-cl(U)) and hence semi $^{\#}g\alpha$-cl(f(U)) $\subseteq$ semi $^{\#}g\alpha$-cl(f(semi $^{\#}g\alpha$-cl(U))) = f(semi $^{\#}g\alpha$-cl(U)). Similarly we have semi $^{\#}g\alpha$-cl(f(V)) $\subseteq$ f(semi $^{\#}g\alpha$-cl(V)). Since f is injective, semi $^{\#}g\alpha$-cl(f(U)) $\cap$ semi $^{\#}g\alpha$-cl(f(V)) $\subseteq$ f(semi $^{\#}g\alpha$-cl(U)) $\cap$ f(semi $^{\#}g\alpha$-cl(V)) = f(semi $^{\#}g\alpha$-cl(U) $\cap$ semi $^{\#}g\alpha$-cl(V)) $= \emptyset$. Also since f is pre semi $^{\#}g\alpha$-open, there exist $f(U) \subseteq$ Semi$^{\#}G\alpha O(Y, y_1)$ and $f(V) \subseteq$ Semi $^{\#}g\alpha O(Y, y_2)$ such that semi $^{\#}g\alpha$-cl(f(U)) $\cap$ semi $^{\#}g\alpha$-cl(f(V)) $= \emptyset$. Thus Y is semi $^{\#}g\alpha$-Urysohn.

Theorem5.7 If f: $(X,\tau)\rightarrow(Y,\sigma)$ is pre semi $^{\#}g\alpha$-open, bijective and X is semi $^{\#}g\alpha$-T_2, then G(f) is ultra semi $^{\#}g\alpha$-closed.

Proof. Let $(x,y) \in (X \times Y) \setminus G(f)$. Then $y \neq f(x)$. Since f is bijective, $x \neq f^{-1}(y)$. Also since X is semi $^{\#}g\alpha$-T_2, there exist $U_x, U_y \subseteq$ Semi$^{\#}G\alpha O(X)$ such that $x \in U_x$, $f^{-1}(y) \in U_y$ and $U_x \cap U_y = \emptyset$. Moreover as f is pre semi $^{\#}g\alpha$-open and bijective, we have $f(x) \in f(U_x) \subseteq$ Semi$^{\#}G\alpha O(Y)$, $y \in f(U_y) \subseteq$ Semi$^{\#}G\alpha O(Y)$ and $f(U_x) \cap f(U_y) = \emptyset$. Hence $f(U_x) \cap$ semi $^{\#}g\alpha$-cl$(f(U_y)) = \emptyset$. Therefore G(f) is ultra semi $^{\#}g\alpha$-closed.

Theorem5.8 If f:$(X,\tau)\rightarrow(Y,\sigma)$ is quasi semi $^{\#}g\alpha$-irresolute and Y is semi $^{\#}g\alpha$-Urysohn, then G(f) is ultra semi $^{\#}g\alpha$-closed.

Proof. Let $(x,y) \in (X \times Y) \setminus G(f)$. Then $y \neq f(x)$. Since Y is semi $^{\#}g\alpha$-Urysohn, there exist $V \subseteq$ Semi$^{\#}G\alpha O(Y,y)$ and $W \subseteq$ Semi$^{\#}G\alpha O(Y,f(x))$ such that semi $^{\#}g\alpha$-cl(V) $\cap$ semi $^{\#}g\alpha$-cl(W) $= \emptyset$. Since f is quasi semi $^{\#}g\alpha$-irresolute, there exists $U \subseteq$ Semi$^{\#}G\alpha O(X,x)$ such that $f(U) \subseteq$ semi $^{\#}g\alpha$-cl(W). Hence we have $f(U) \cap$ semi $^{\#}g\alpha$-cl(V) $= \emptyset$. Thus G(f) is ultra semi $^{\#}g\alpha$-closed.

References

[1] M.Caldas, S.Jafari and T.Noiri, *On functions with strongly λ-closed graphs, Southeast Asian Bulletin of Mathematics,33(2009),229-236.*

[2] R.Devi, H.Maki and V.Kokilavani, *The Group Structure of $^{\#}g\alpha$-Closed Sets in Topological Spaces, International Journal of general topology, 2(1), (2009), 21-30.*

[3] T.Husain, *Topology and Maps, Plenum Press,Newyork,(1977).*

[4] V.Kokilavani and M.Vivek Prabu, *Semi $^{\#}$Generalized α-Closed Sets and Semi $^{\#}$Generalized α-Homeomorphisms in Topological Spaces , Proceedings of National Conference on Recent Advances in Mathematical Analysis and Applications NCRAMAA-2013, pp. 153-161.*

[5] V. Kokilavani and M. VivekPrabu, *New Separation Axioms of Semi $^{\#}$Generalized α-Closed Sets in Topological Spaces,(Communicated).*

[6] N.Levine, *Generalized Closed Sets in Topology, Rend.Circ.Math.Palermo,19,(1970),89-96.*

[7] K.Nono, R.Devi, M.Devipriya, K.Muthukumaraswamy and H.Maki, *On $g^{\#}\alpha$-closed sets and the Digital Plane, Bull.FukuokaUniv.Ed.PartIII,53,(2004),15-24.*

Author Profile

Dr. V.Kokilavani is working as Assistant Professor of Mathematics in Kongunadu Arts and Science College (Autonomous), Coimbatore. She has more than 12 years of research experience and 10 years of teaching experience. She has published more than 50 research papers in reputed international and national journals.

Mr. M.Vivek Prabu is persuing his doctorate in Mathematics in Kongunadu Arts and Science College (Autonomous),Coimbatore. He has published 10 research papers in various international journals and proceedings.