Factorization of a Matrix by the Elements of its Row Space

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Abstract: ***Factorization of a matrix is a means by which the regularity and order in phenomena can be discerned .Factorization of a matrix can be used in applications relevant to various scientific and policy concerns, we can mention here some examples of different uses for matrix factorization, although many of these uses overlap, so we can mention ,interdependency and pattern delineation, parsimony or data reduction, discovering the basic structure of a domain. As a case in point, a scientist may want to uncover the primary independent lines or dimensions--such as size, leadership, and age--of variation in group characteristics and behavior, data collected on a large sample of groups and factorization can help disclose this structure. So factorization of a matrix is a fundamental theme in linear Algebra and applied statistics which has both scientific and engineering significance. In this paper we give a simple way for to factorize a matrix and we propose an algorithm with which we can factorize a matrix using some vectors of its row space.***

Keywords: Factorization of a matrix, row space . singular matrix ,invertible matrix –vectors of row space

1. Introduction

A simple way to factorize a matrix could be found ,since any $m\times n$ matrix A can be written as $A = I_m A$ or as $A = AI_n$ where I_m and I_n are the identity matrices of type $m\times m$ and $n\times n$ respectively, and we can write I_m as a product of two matrices any square matrix B and its inverse B^{-1} ,so we can use this idea to factorize any A matrix by using the associative law for matrix multiplication and writing it as $A = B(B^{-1}A)$ or $A = (AB)B^{-1}$

Also $A = I_m I_m ... I_m A = B_1 B_1^{-1} ... B_n B_n^{-1} A$ .So by this method we can find infinite number of different factorization for any matrix M since we can find an infinite number of invertible matrices to give us the identities matrices I_m or I_n .

This paper was organized as follows:

Section 2 presented a simple way to factorize an $m\times n$ matrix section 3 give us a proposition from which we can deduce that if we are able to factorize an square matrix M then must be a singular one .section 4 described a method for making use of the mentioned proposition with a numerical example .section 5 explain an algorithm by which we can be able to reduce the entities of a data set matrix with some numerical examples .section 5 conclusion of the paper.

2. Proposition

If an square $n\times n$ matrix C can be written as $C = AB$ such that , A of type $n\times k$, B of type $k\times n$ and $k < n$ then A is singular matrix

Proof

Let $A = \begin{bmatrix} a_{11} & . & . & . & a_{1k} \\ . & . & . & . & . \\ . & . & . & .. & . \\ . & . & . & . & . \\ a_{n1} & . & . & . & a_{nk} \end{bmatrix}$, $B = \begin{bmatrix} b_{11} & . & . & . & b_{1n} \\ . & . & . & . & . \\ . & . & . & .. & . \\ . & . & . & . & . \\ b_{k1} & . & . & . & b_{kn} \end{bmatrix}$

since $k < n$ we can add $n-k$ zero columns to the matrix A to get a new square $n\times n$ matrix L ,and we can also add $n-k$ zero rows or any $n-k$ rows to get another new square $n\times n$ matrix N

Now we have

$$L = \begin{bmatrix} a_{11} & . & . & . & a_{1k}\ 0...0 \\ . & . & . & . & . \\ . & . & . & .. & . & .. \\ . & . & . & . & . \\ a_{n1} & . & . & . & a_{nk}\ 0...0 \end{bmatrix}, N = \begin{bmatrix} b_{11} & ... & & . & b_{1n} \\ . & . & . & . & . \\ .b_{k1} & . & . & .. & .b_{kn} \\ 0 & . & . & . & .0 \\ 0 & . & . & . & o \end{bmatrix}$$

Already we have $C = AB$ and now we note that also $C = LN$, and we have $\det(L) = 0$, $\det(N) = 0$, So we get that $\det(C) = \det(L)\times\det(N) = 0\times 0 = 0$

Therefore C is singular.

3. Remark

Due to the above proposition we note that for any $m\times n$,$n \geq m$ matrix A with full row rank or not we can find an $n\times m$ matrix B such that $AB = C$ is an $m\times m$ invertible matrix , so we get

$$I_m = C^{-1}AB \qquad (1)$$

where I_m is the identity matrix of rank m

Multiplying both side of (1) from right by an $m\times r$ matrix M we get

$$M = C^{-1}ABM \qquad \text{or}$$

$$M = (C^{-1}A)\,(BM) \qquad (2)$$
$$M = LN \qquad (3)$$

Where $(C^{-1}A) = L$ is an $m \times n$, $n \geq m$ and $(BM) = N$ is an $n \times r$ one .

Now we can use equation (3) for factorizing any matrix into two matrices

Example

Suppose we want the factorization of $M = \begin{bmatrix} 1 & 0 & 2 & 4 \\ 2 & 3 & 1 & 1 \end{bmatrix}$

Since M is 2×4 matrix and in equation (2) we have $M = (C^{-1}A)\,(BM)$

Therefore B must be an $n \times 2$ matrix and A must be $2 \times n$ matrix $n \geq 2$, so let $n = 2$ and

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & 1 \end{bmatrix} , B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \Rightarrow C = AB = \begin{bmatrix} 5 & 3 \\ 2 & 1 \end{bmatrix}$$

$$\Rightarrow C^{-1} = \begin{bmatrix} -1 & 3 \\ 2 & -5 \end{bmatrix}$$

From (2)

$$M = \left(\begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{bmatrix} \right) \left(\begin{bmatrix} -1 & 3 \\ 2 & -5 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 & 4 \\ 2 & 3 & 1 & 1 \end{bmatrix} \right) \qquad (4)$$

$$M = \begin{bmatrix} 5 & 3 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 5 & 9 & 1 & -1 \\ -8 & -15 & -1 & 3 \end{bmatrix} \qquad (5)$$

Note in (4) M is equal to the product of four matrices , in (5) it is equal to product of two matrices ,also if we want one of the factor is an square matrix ,simply we multiply M from right or left with an invertible square matrix for example can factorize it as follows

$$\begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix} \times M = \begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 & 4 \\ 2 & 3 & 1 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} 4 & 3 & 5 & 9 \\ 15 & 12 & 18 & 32 \end{bmatrix} \Rightarrow$$

$$M = \begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}^{-1} \begin{bmatrix} 4 & 3 & 5 & 9 \\ 15 & 12 & 18 & 32 \end{bmatrix}$$
$$M = \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix} \begin{bmatrix} 4 & 3 & 5 & 9 \\ 15 & 12 & 18 & 32 \end{bmatrix}$$

Note that M before factorization has 8 entities and after factorization it has 12 entities , so there is no reduction . Therefore we will give another approach for the reduction of the entities of a matrix M by letting it equal to a product of two matrices , we can use rank factorization method but we will give a new method for this reduction

4. How we can Reduce the Entities of a Data Set Matrix

We know that matrix factorization has numerous algorithms and it has many benefits ,so we can make use of it to reduce the entities of any matrix for example if we have 100×20 matrix M ,we note that M has 2000 entities , by writing $M = LN$ where L is 100×2 and N is 2×20 matrix , L will has 200 entities and N will has 40 ones and the total entities of the two matrices will be 240 instead of 2000 entities so we can save our data in our computer or our note books or any other documentary source as two matrices L, N and if we need this data we recall the result of the product of the two L, N matrices by using our calculator matrix to give us our original data set matrix M . And if we know the matrix N we can proceed as follows:

Suppose we have the matrix

$$M = \begin{bmatrix} 7 & 3 & 4 & 4 & 2 & 7 & 4 \\ 11 & 5 & 6 & 7 & 3 & 11 & 7 \\ 9 & 5 & 4 & 8 & 2 & 9 & 8 \\ 4 & 2 & 2 & 3 & 1 & 4 & 3 \end{bmatrix}$$

and we want to write it as $M = LN$,and we know that

$$N = \begin{bmatrix} 1 & 1 & 0 & 2 & 0 & 1 & 2 \\ 3 & 1 & 2 & 1 & 1 & 3 & 1 \end{bmatrix}$$

In this case we can take a matrix as in equation (3) $M = LN$, then multiplying both sides of this equation from the right by N^T or any other 7×2 matrix we get

$$MN^T = LNN^T = LD \qquad (6)$$

Where $NN^t = D$ is invertible matrix or we can multiply both sides of equation (3) by any other matrix S to get an invertible one $NS = D$

So multiplying (6) from right with D^{-1} we deduce

$$MN^T D^{-1} = L \qquad (7)$$

Now we have

$$NN^t = D = \begin{bmatrix} 11 & 11 \\ 11 & 26 \end{bmatrix} \Rightarrow D^{-1} = \frac{1}{165} \begin{bmatrix} 26 & -11 \\ -11 & 11 \end{bmatrix}$$

And from (7) we get

$$L = MN^T D^{-1}$$
$$= \begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 3 & 2 \\ 1 & 1 \end{bmatrix}$$

So we get

$$\begin{bmatrix} 7 & 3 & 4 & 4 & 2 & 7 & 4 \\ 11 & 5 & 6 & 7 & 3 & 11 & 7 \\ 9 & 5 & 4 & 8 & 2 & 9 & 8 \\ 4 & 2 & 2 & 3 & 1 & 4 & 3 \end{bmatrix} =$$
$$\begin{bmatrix} 1 & 2 \\ 2 & 3 \\ 3 & 2 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 2 & 0 & 1 & 2 \\ 3 & 1 & 2 & 1 & 1 & 3 & 1 \end{bmatrix}$$

We note that the entities was reduced from 28 to 22. Now if we do not know the matrix N what can we do?

Since we can viewed multiplying a matrix L by a matrix N as a linear combination of L's columns using coefficients from N, or another way to look at it is that it's a linear combination of the rows of N using coefficients from L, so we can take the rows of N equal to the basis of the row space of matrix M or another matrix with its rows some elements (vectors) in the row space of M not necessary be the basis . If we want to reduce the entities of M and M is an square matrix then according to the proposition, M must be singular.

Example

Let $M = \begin{bmatrix} 5 & 7 & 8 & 3 & 4 \\ 7 & 10 & 11 & 4 & 5 \\ 8 & 11 & 13 & 5 & 7 \\ 3 & 4 & 5 & 2 & 3 \\ 4 & 5 & 7 & 3 & 5 \end{bmatrix}$

Then we can do row operation to M so as to get its basis as follows:

$R_2 - \frac{7}{5}R_1 \to R_2$

$$\begin{bmatrix} 5 & 7 & 8 & 3 & 4 \\ 0 & \frac{1}{5} & -\frac{1}{5} & -\frac{1}{5} & -\frac{3}{5} \\ 8 & 11 & 13 & 5 & 7 \\ 3 & 4 & 5 & 2 & 3 \\ 4 & 5 & 7 & 3 & 5 \end{bmatrix}$$

$R_3 - \frac{8}{5}R_1 \to R_3$

$$\begin{bmatrix} 5 & 7 & 8 & 3 & 4 \\ 0 & \frac{1}{5} & -\frac{1}{5} & -\frac{1}{5} & -\frac{3}{5} \\ 0 & -\frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{3}{5} \\ 3 & 4 & 5 & 2 & 3 \\ 4 & 5 & 7 & 3 & 5 \end{bmatrix}$$

$R_4 - \frac{3}{5}R_1 \to R_4$

$$\begin{bmatrix} 5 & 7 & 8 & 3 & 4 \\ 0 & \frac{1}{5} & -\frac{1}{5} & -\frac{1}{5} & -\frac{3}{5} \\ 0 & -\frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{3}{5} \\ 0 & -\frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{3}{5} \\ 4 & 5 & 7 & 3 & 5 \end{bmatrix}$$

$R_5 - \frac{4}{5}R_1 \to R_5$

$$\begin{bmatrix} 5 & 7 & 8 & 3 & 4 \\ 0 & \frac{1}{5} & -\frac{1}{5} & -\frac{1}{5} & -\frac{3}{5} \\ 0 & -\frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{3}{5} \\ 0 & -\frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{3}{5} \\ 0 & -\frac{3}{5} & \frac{3}{5} & \frac{3}{5} & \frac{9}{5} \end{bmatrix}$$

$R_3 + R_2 \to R_3$, $R_4 + R_2 \to R_4$, $R_5 + 3R_2 \to R_5$

$$\begin{bmatrix} 5 & 7 & 8 & 3 & 4 \\ 0 & \frac{1}{5} & -\frac{1}{5} & -\frac{1}{5} & -\frac{3}{5} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Now we can take

$AB = M$ where

$$B = \begin{bmatrix} 5 & 7 & 8 & 3 & 4 \\ 0 & 1 & -1 & -1 & -3 \end{bmatrix}$$

$$A \times \begin{bmatrix} 5 & 7 & 8 & 3 & 4 \\ 0 & 1 & -1 & -1 & -3 \end{bmatrix} = M$$

Multiplying both sides with B^t we get

$$A \times \begin{bmatrix} 5 & 7 & 8 & 3 & 4 \\ 0 & 1 & -1 & -1 & -3 \end{bmatrix} B^t = MB^t$$

$$A \times \begin{bmatrix} 5 & 7 & 8 & 3 & 4 \\ 0 & 1 & -1 & -1 & -3 \end{bmatrix} \begin{bmatrix} 5 & 0 \\ 7 & 1 \\ 8 & -1 \\ 3 & -1 \\ 4 & -3 \end{bmatrix} = \begin{bmatrix} 5 & 7 & 8 & 3 & 4 \\ 7 & 10 & 11 & 4 & 5 \\ 8 & 11 & 13 & 5 & 7 \\ 3 & 4 & 5 & 2 & 3 \\ 4 & 5 & 7 & 3 & 5 \end{bmatrix} \begin{bmatrix} 5 & 0 \\ 7 & 1 \\ 8 & -1 \\ 3 & -1 \\ 4 & -3 \end{bmatrix} \Rightarrow$$

$$A \times \begin{bmatrix} 163 & -16 \\ -16 & 12 \end{bmatrix} = \begin{bmatrix} 163 & -16 \\ 225 & -20 \\ 264 & -28 \\ 101 & -12 \\ 140 & -20 \end{bmatrix}$$

Multiplying both sides with

$$D^{-1} = \begin{bmatrix} 163 & -16 \\ -16 & 12 \end{bmatrix}^{-1} = \begin{bmatrix} \frac{3}{425} & \frac{4}{425} \\ \frac{4}{425} & \frac{163}{1700} \end{bmatrix}$$

We get

$$A = \begin{bmatrix} 163 & -16 \\ 225 & -20 \\ 264 & -28 \\ 101 & -12 \\ 140 & -20 \end{bmatrix} \begin{bmatrix} \frac{3}{425} & \frac{4}{425} \\ \frac{4}{425} & \frac{163}{1700} \end{bmatrix} \Rightarrow A = \begin{bmatrix} 1 & 0 \\ \frac{7}{5} & \frac{1}{5} \\ \frac{8}{5} & -\frac{1}{5} \\ \frac{3}{5} & -\frac{1}{5} \\ \frac{4}{5} & -\frac{3}{5} \end{bmatrix}$$

Now let $r_1 = (5,7,8,3,4)$, $r_2 = (0,1,-1,-1,-3)$ and let $w_1 = r_1 + 2r_2 = (5,9,6,1,-2)$, $w_2 = r_1 + 3r_2 = (5,10,5,0,-5)$,so we can form another factorization for M by taking the matrix

$$B = \begin{bmatrix} 5 & 9 & 6 & 1 & -2 \\ 5 & 10 & 5 & 0 & -5 \end{bmatrix}$$

So we will get

$AB = M$

$$\Rightarrow A \times \begin{bmatrix} 5 & 9 & 6 & 1 & -2 \\ 5 & 10 & 5 & 0 & -5 \end{bmatrix} = M$$

Multiplying both sides with B^t , we get

$$A\times\begin{bmatrix}5&9&6&1&-2\\5&10&5&0&-5\end{bmatrix}\times\begin{bmatrix}5&5\\9&10\\6&5\\1&0\\-2&-5\end{bmatrix}=M\times\begin{bmatrix}5&5\\9&10\\6&5\\1&0\\-2&-5\end{bmatrix}$$

$$A\times\begin{bmatrix}5&9&6&1&-2\\5&10&5&0&-5\end{bmatrix}\times\begin{bmatrix}5&5\\9&10\\6&5\\1&0\\-2&-5\end{bmatrix}=\begin{bmatrix}5&7&8&3&4\\7&10&11&4&5\\8&11&13&5&7\\3&4&5&2&3\\4&5&7&3&5\end{bmatrix}\times\begin{bmatrix}5&5\\9&10\\6&5\\1&0\\-2&-5\end{bmatrix}\Rightarrow$$

$$A\times\begin{bmatrix}147&155\\155&175\end{bmatrix}=\begin{bmatrix}131&115\\185&165\\208&180\\77&65\\100&80\end{bmatrix}$$

Multiplying both sides with

$$D^{-1}=\begin{bmatrix}147&155\\155&175\end{bmatrix}^{-1}=\begin{bmatrix}\frac{7}{68}&-\frac{31}{340}\\-\frac{31}{340}&\frac{147}{1700}\end{bmatrix}$$

We get

$$A=\begin{bmatrix}131&115\\185&165\\208&180\\77&65\\100&80\end{bmatrix}\begin{bmatrix}\frac{7}{68}&-\frac{31}{340}\\-\frac{31}{340}&\frac{147}{1700}\end{bmatrix}\Rightarrow A=\begin{bmatrix}3&-2\\3&-\frac{13}{5}\\5&-\frac{17}{5}\\2&-\frac{7}{5}\\3&-\frac{11}{5}\end{bmatrix}$$

Now we note that some entities of A are not integers ,and D is 2×2 matrix so if we choose D such that its determinant equal ± 1 all the entities of A will be integers because D^{-1} will not contains any fraction and this will be done as follows :

Let $r_1=(5,7,8,3,4)$, $r_2=(0,1,-1,-1,-3)$ and let

$w_1=r_1+2r_2=(5,9,6,1,-2)$, $w_2=\frac{1}{5}(r_1+3r_2)=(1,2,1,0,-1)$

,so we can form another factorization for M by taking the matrix

$$B=\begin{bmatrix}5&9&6&1&-2\\1&2&1&0&-1\end{bmatrix}$$

Then we will get

$AB=M$

$$\Rightarrow A\times\begin{bmatrix}5&9&6&1&-2\\1&2&1&0&-1\end{bmatrix}=M$$

Multiplying both sides with $\begin{bmatrix}1&0\\0&0\\0&0\\0&1\\0&0\end{bmatrix}$ instead of B^t

$$A\times\begin{bmatrix}5&9&6&1&-2\\1&2&1&0&-1\end{bmatrix}\times\begin{bmatrix}1&0\\0&0\\0&0\\0&1\\0&0\end{bmatrix}=M\begin{bmatrix}1&0\\0&0\\0&0\\0&1\\0&0\end{bmatrix}$$

$$A\times\begin{bmatrix}5&9&6&1&-2\\1&2&1&0&-1\end{bmatrix}\times\begin{bmatrix}1&0\\0&0\\0&0\\0&1\\0&0\end{bmatrix}=\begin{bmatrix}5&7&8&3&4\\7&10&11&4&5\\8&11&13&5&7\\3&4&5&2&3\\4&5&7&3&5\end{bmatrix}\begin{bmatrix}1&0\\0&0\\0&0\\0&1\\0&0\end{bmatrix}\Rightarrow$$

$$A\times\begin{bmatrix}5&1\\1&0\end{bmatrix}=\begin{bmatrix}5&3\\7&4\\8&5\\3&2\\4&3\end{bmatrix}$$

Multiplying both sides with

$$D^{-1}=\begin{bmatrix}5&1\\1&0\end{bmatrix}^{-1}=\begin{bmatrix}0&1\\1&-5\end{bmatrix}$$

So we get

$$A=\begin{bmatrix}5&3\\7&4\\8&5\\3&2\\4&3\end{bmatrix}\begin{bmatrix}0&1\\1&-5\end{bmatrix}\Rightarrow A=\begin{bmatrix}3&-10\\4&-13\\5&-17\\2&-7\\3&-11\end{bmatrix}$$

We can use also the vectors of the row space for factorizing a rectangular matrix, so if we have a rectangular matrix

$$M=\begin{bmatrix}7&3&4&4&2&7&4\\11&5&6&7&3&11&7\\9&5&4&8&2&9&8\\4&2&2&3&1&4&3\end{bmatrix}$$

The basis of its row is

$r_1=(2,0,2,-1,1,2,-1)$, $r_2=(0,2,-2,5,-1,0,5)$,if we choose $w_1=\frac{r_1}{2}$, $w_2=\frac{r_2}{2}$ we note that the matrix D will be the identity matrix $D=\begin{bmatrix}1&0\\0&1\end{bmatrix}$ and that makes the calculation easy as we see in the following

Then

$$B=\begin{bmatrix}w_1\\w_2\end{bmatrix}=\begin{bmatrix}1&0&1&\frac{-1}{2}&\frac{1}{2}&1&\frac{-1}{2}\\0&1&-1&\frac{5}{2}&\frac{-1}{2}&0&\frac{5}{2}\end{bmatrix}$$

Now from

$$AB=M\Rightarrow A\times\begin{bmatrix}1&0&1&\frac{-1}{2}&\frac{1}{2}&1&\frac{-1}{2}\\0&1&-1&\frac{5}{2}&\frac{-1}{2}&0&\frac{5}{2}\end{bmatrix}=\begin{bmatrix}7&3&4&4&2&7&4\\11&5&6&7&3&11&7\\9&5&4&8&2&9&8\\4&2&2&3&1&4&3\end{bmatrix}$$

Multiplying both sides with $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$ instead of B^t this will give us in the left side

Then we get

$$D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A \times \begin{bmatrix} 1 & 0 & 1 & \frac{-1}{2} & \frac{1}{2} & 1 & \frac{-1}{2} \\ 0 & 1 & -1 & \frac{5}{2} & \frac{-1}{2} & 0 & \frac{5}{2} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 7 & 3 & 4 & 4 & 2 & 7 & 4 \\ 11 & 5 & 6 & 7 & 3 & 11 & 7 \\ 9 & 5 & 4 & 8 & 2 & 9 & 8 \\ 4 & 2 & 2 & 3 & 1 & 4 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow A \times \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 3 \\ 11 & 5 \\ 9 & 5 \\ 4 & 2 \end{bmatrix} \Rightarrow A = \begin{bmatrix} 7 & 3 \\ 11 & 5 \\ 9 & 5 \\ 4 & 2 \end{bmatrix}$$

So

$$\begin{bmatrix} 7 & 3 & 4 & 4 & 2 & 7 & 4 \\ 11 & 5 & 6 & 7 & 3 & 11 & 7 \\ 9 & 5 & 4 & 8 & 2 & 9 & 8 \\ 4 & 2 & 2 & 3 & 1 & 4 & 3 \end{bmatrix} = \begin{bmatrix} 7 & 3 \\ 11 & 5 \\ 9 & 5 \\ 4 & 2 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 1 & \frac{-1}{2} & \frac{1}{2} & 1 & \frac{-1}{2} \\ 0 & 1 & -1 & \frac{5}{2} & \frac{-1}{2} & 0 & \frac{5}{2} \end{bmatrix}$$

Note that the sum of the entities on the right hand side is 22 instead of 28 ones in the left side

5. Conclusion

The method of using vectors of the row space in factorizing a matrix is an efficient method for reducing the entities of a data set matrix ,although there are many algorithm for matrix factorization ,this a logarithm will be a good method because it is very simple, easy and accurate so it is more use full in reducing the entities of data set matrix.

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