Some New Estimators of Population Mean Using PPS Sampling

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Abstract: ***In this study we consider some ratio and product type estimators we calculate bias and mean square error of these estimators. These estimators are compared with usual ratio and product estimators & with mean- per unit. An empirical illustration is also included to justify the practical utility of the proposed estimators.***

Keywords: Ratio estimators, product estimators, probability proportional to size sampling Bias, Mean square error

1. Introduction

The method of PPS selection was familiar to Mahalanobis even as early as 1937. About seventy years ago, Hansen and Hurwitz (1943) introduced the method of selecting units of a finite population with probability proportional to a given size measure (PPS) and demonstrated its efficiency over simple random sampling method. **Unequal probability sampling** There are situations in which it is desirable to have unequal probability of selecting elements into the sample. The estimators based on unequal probability sampling and the ratio estimators are generally more efficient than the simple estimator based on equal probability sampling when the regression line of y on x passes through the origin.

Notations

Let y_i ,x_{1i} and x_{2i} be two values of characters y under study and values of the two auxiliary characters x_1 and x_2 for the i^{th} unit in the population (i =1,2….N) of the population of size N. Let a sample of size n be drawn with ppswr sampling (based on x_1) and Y, X_1 ,X_2 be population total of y ,x_1 x_2 respectively

$u_i = \frac{y_i}{NP_{1i}}$ $v_i = \frac{x_{2i}}{NP_{1i}}$ $p_{1i} = \frac{x_{1i}}{X_1}$

$\bar{u}_n = \frac{1}{n}\sum_{i=1}^{n} u_i = \bar{y}_{1pps}$

$\bar{v}_n = \frac{1}{n}\sum_{i=1}^{n} v_i = \bar{x}_{2pps}$

$\sigma_u{}^2 = \sum_{i=1}^{N} p_{1i}\,(u_i - \bar{Y})^2$, $\frac{\sigma_u{}^2}{\bar{y}^2} = C_u{}^2$

$\sigma_v{}^2 = \sum_{i=1}^{N} p_{1i}\,(v_i - \bar{X}_2)^2$, $\frac{\sigma_v{}^2}{\bar{X}_2{}^2} = C_v{}^2$

$\rho_{uv} = \frac{\sum_{i=1}^{N} p_{1i}(u_i - \bar{Y}(v_i - \bar{X}_2)}{\sigma_u \sigma_v}$

$\bar{y}_R = \frac{\bar{u}_n}{\bar{v}_n}\bar{X}_2$

$\bar{y}_P = \frac{\bar{u}_n \bar{v}_n}{\bar{X}_2}$

2. Proposed Estimators

$$\bar{y}_{RT} = \bar{y}_{pps} - \frac{\bar{x}_{pps}}{\bar{X}} + 1 \qquad --- (1)$$

$$\bar{y}_{PT} = \bar{y}_{pps} - \frac{\bar{X}}{\bar{x}_{pps}} + 1 \qquad --- (2)$$

$$\bar{y}_{RPT_1} = \left(\bar{y}_{pps} - \frac{\bar{x}_{pps}}{\bar{X}} + 1\right) \frac{\bar{X}}{\bar{x}_{pps}} \qquad --- (3)$$

$$\left(\bar{y}_{pps} - \frac{\bar{X}}{\bar{x}_{pps}} + 1\right) \frac{\bar{x}_{pps}}{\bar{X}} \qquad ---- (4)$$

$$\bar{y}_{RPT_3} = \left(\bar{y}_{pps} - \frac{\bar{x}_{pps}}{\bar{X}} + 1\right) \frac{\bar{x}_{pps}}{\bar{X}} \qquad ------- (5)$$

$$\bar{y}_{RPT_4} = \left(\bar{y}_{pps} - \frac{\bar{X}}{\bar{x}_{pps}} + 1\right) \frac{\bar{X}}{\bar{x}_{pps}} \qquad -------- (6)$$

Let $\bar{y} = (1+e_0)\bar{Y}$	$E(e_0{}^2) = C_u{}^2$
$\bar{x} = (1+e_1)\bar{X}$	$E(e_1{}^2) = C_v{}^2$
$E(e_0) = E(e_1) = 0$	$E(e_0 e_1) = \rho_{uv} C_u C_v$

3. Bias and MSE of the proposed Estimators

Bias	MSE
B ($\bar{y}_{RT}$) 0	M ($\bar{Y}^2 C_u{}^2 + C_v{}^2 - 2\bar{Y}\rho_{uv} C_u C_v$)
$B(\bar{y}_{PT})$ $-C_v{}^2$	M ($\bar{Y}^2 C_u{}^2 + C_v{}^2 + 2\bar{Y}\rho_{uv} C_u C_v$)
$B(\bar{y}_{RPT_1})$ $(1+\bar{Y})\,C_v{}^2 - \bar{Y}\rho_{uv} C_u C_v$	M ($\bar{Y}^2 C_u{}^2 + (1+\bar{Y})^2 C_v{}^2 - 2\bar{Y}(1+\bar{Y})\rho_{uv} C_u C_V$)
B $(\bar{y}_{RPT_2})$ $\bar{Y}\rho_{uv} C_u C_v$	M ($\bar{Y}^2 C_u{}^2 + (1+\bar{Y})^2 C_v{}^2 + 2\bar{Y}(1+\bar{Y})\rho_{uv} C_u C_v$)
$B(\bar{y}_{RPT_3})$ $\bar{Y}\rho_{uv} C_u C_v - C_v{}^2$	M ($\bar{Y}^2 C_u{}^2 + (\bar{Y}-1)^2 C_v{}^2 + 2\bar{Y}(\bar{Y}-1)\rho_{uv} C_u C_v$)
$B(\bar{y}_{RPT_4})$ $(\bar{Y}-2)\,C_v{}^2 - \bar{Y}\rho_{uv} C_u C_v$	M ($\bar{Y}^2 C_u{}^2 + (1-\bar{Y})^2 C_v{}^2 - 2\bar{Y}(\bar{Y}-1)\rho_{uv} C_u C_v$

4. Comparison of the Proposed Estimators

4.11 MSE ($\bar{y}_{RT}$) < MSE ($\bar{y}_R$)

$\bar{Y}^2 C_u{}^2 + C_v{}^2 - 2\bar{Y}\rho_{uv} C_u C_v < \bar{Y}^2[C_u{}^2 + C_v{}^2 - 2\rho_{uv} C_u C_v]$

$(1-\bar{Y}^2)\,C_v{}^2 - 2\bar{Y}(1-\bar{Y})\rho_{uv} C_u C_v < 0$

$(1+\bar{Y})C_v{}^2 - 2\bar{Y}\rho_{uv} C_u C_v < 0$

$\frac{1+\bar{Y}}{2\bar{Y}} < \rho\frac{C_u}{C_v}$

4.12 With Mean per unit

MSE ($\bar{y}_{RT}$) < MSE ($\bar{y}$)

$\bar{Y}^2 C_u{}^2 + C_v{}^2 - 2\bar{Y}\rho_{uv} C_u C_v < C_u{}^2$

$(\bar{Y}^2 - 1)C_u{}^2 + C_v{}^2 - 2\bar{Y}\rho_{uv} C_u C_v < 0$

$(\bar{Y}^2 - 1)C_u{}^2 + C_v{}^2 < 2\bar{Y}\rho_{uv} C_u C_v$

4.21 MSE ($\bar{y}_{PT}$) < MSE ($\bar{y}_P$)

$\bar{Y}^2 C_u{}^2 + C_v{}^2 + 2\bar{Y}\rho_{uv}C_uC_v < \bar{Y}^2[C_u{}^2 + C_v{}^2 + 2\rho_{uv}C_uC_v]$

$(1-\bar{Y}^2)\,C_v{}^2 + 2\bar{Y}(1-\bar{Y})\rho_{uv}C_uC_v < 0$

$(1+\bar{Y})C_v{}^2 < -2\bar{Y}\rho_{uv}C_uC_v$

$\frac{1+\bar{Y}}{2\bar{Y}} < -\rho\frac{C_u}{C_v}$

4.22 With Mean per unit

MSE ($\bar{y}_{PT}$) < MSE ($\bar{y}$)

$\bar{Y}^2 C_u{}^2 + C_v{}^2 + 2\bar{Y}\rho_{uv}C_uC_v < C_u{}^2$

$(\bar{Y}^2-1)C_u{}^2 + C_v{}^2 + 2\bar{Y}\rho_{uv}C_uC_v < 0$

$(\bar{Y}^2-1)C_u{}^2 + C_v{}^2 < -2\bar{Y}\rho_{uv}C_uC_v$

4.31 (MSE) $\bar{y}_{RPT_1}$ < MSE ($\bar{y}_R$)

$\bar{Y}^2 C_u{}^2 + (1+\bar{Y})^2 C_v{}^2 - 2\bar{Y}(1+\bar{Y})\rho_{uv}C_uC_v < \bar{Y}^2[C_u{}^2 + C_v{}^2 - 2\rho_{uv}C_uC_v]$

$(1+2\bar{Y})\,C_v{}^2 - 2\bar{Y}\rho_{uv}C_uC_v < 0$

$(1+2\bar{Y})\,C_v{}^2 < 2\bar{Y}\rho_{uv}C_uC_v$

$\frac{(1+2\bar{Y})}{2\bar{Y}} < \rho\frac{C_u}{C_v}$

4.32 With Mean per unit

MSE $\bar{y}_{RPT_1}$ < MSE ($\bar{y}$)

$\bar{Y}^2 C_u{}^2 + (1+\bar{Y})^2 C_v{}^2 - 2\bar{Y}(1+\bar{Y})\rho_{uv}C_uC_v < C_u{}^2$

$(\bar{Y}^2-1)C_u{}^2 + (1+\bar{Y})^2 C_v{}^2 < 2\bar{Y}(1+\bar{Y})\rho_{uv}C_uC_v$

$(\bar{Y}-1)\,C_u{}^2 + (1+\bar{Y})\,C_v{}^2 < 2\bar{Y}\rho_{uv}C_uC_v$

4.41 MSE($\bar{y}_{RPT_2}$) < MSE ($\bar{y}_P$)

$\bar{Y}^2 C_u{}^2 + (1+\bar{Y})^2 C_v{}^2 + 2\bar{Y}(1+\bar{Y})\rho_{uv}C_uC_v < \bar{Y}^2[C_u{}^2 + C_v{}^2 + 2\rho_{uv}C_uC_v]$

$(1+2\bar{Y})\,C_v{}^2 + 2\bar{Y}\rho_{uv}C_uC_v < 0$

$(1+2\bar{Y})\,C_v{}^2 < -2\bar{Y}\rho_{uv}C_uC_v$

$\frac{(1+2\bar{Y})}{2\bar{Y}} < -\rho\frac{C_u}{C_v}$

4.42 With Mean per unit

MSE($\bar{y}_{RPT_2}$) < MSE ($\bar{y}$)

$\bar{Y}^2 C_u{}^2 + (1+\bar{Y})^2 C_v{}^2 + 2\bar{Y}(1+\bar{Y})\rho_{uv}C_uC_v < C_u{}^2$

$(\bar{Y}^2-1)C_u{}^2 + (1+\bar{Y})^2 C_v{}^2 < -2\bar{Y}(1+\bar{Y})\rho_{uv}C_uC_v$

$(\bar{Y}-1)\,C_u{}^2 + (1+\bar{Y})\,C_v{}^2 < -2\bar{Y}\rho_{uv}C_uC_v$

4.51 MSE($\bar{y}_{RPT_3}$) < MSE ($\bar{y}_P$)

$\bar{Y}^2 C_u{}^2 + (\bar{Y}-1)^2 C_v{}^2 + 2\bar{Y}(\bar{Y}-1)\rho_{uv}C_uC_v < \bar{Y}^2[C_u{}^2 + C_v{}^2 + 2\rho_{uv}C_uC_v]$

$(1-2\bar{Y})\,C_v{}^2 - 2\bar{Y}\rho_{uv}C_uC_v < 0$

$(1-2\bar{Y})\,C_v{}^2 < 2\bar{Y}\rho_{uv}C_uC_v$

$\frac{(1-2\bar{Y})}{2\bar{Y}} < \rho\frac{C_u}{C_v}$

4.52 With Mean per unit

MSE($\bar{y}_{RPT_3}$) < MSE ($\bar{y}$)

$\bar{Y}^2 C_u{}^2 + (\bar{Y}-1)^2 C_v{}^2 + 2\bar{Y}(\bar{Y}-1)\rho_{uv}C_uC_v < C_u{}^2$

$(\bar{Y}^2-1)C_u{}^2 + (\bar{Y}-1)^2 C_v{}^2 < -2\bar{Y}(\bar{Y}-1)\rho_{uv}C_uC_v$

$(\bar{Y}+1)\,C_u{}^2 + (1-\bar{Y})\,C_v{}^2 < -2\bar{Y}\rho_{uv}C_uC_v$

4.61 MSE$\left(\bar{y}_{RPT_4}\right)$ < MSE ($\bar{y}_R$)

$(\bar{Y}^2 C_u{}^2 + (1-\bar{Y})^2 C_v{}^2 - 2\bar{Y}(\bar{Y}-1)\rho_{uv}C_uC_v < \bar{Y}^2[C_u{}^2 + Cv2-2\rho uvCuCv$

$(1-2\bar{Y})\,C_v{}^2 + 2\bar{Y}\rho_{uv}C_uC_v < 0$

$(1-2\bar{Y})\,C_v{}^2 < -2\bar{Y}\rho_{uv}C_uC_v$

$\frac{(1-2\bar{Y})}{2\bar{Y}} < -\rho\frac{C_u}{C_v}$

4.62 With Mean per unit

MSE$\left(\bar{y}_{RPT_4}\right)$ < MSE ($\bar{y}$)

$(\bar{Y}^2 C_u{}^2 + (1-\bar{Y})^2 C_v{}^2 - 2\bar{Y}(\bar{Y}-1)\rho_{uv}C_uC_v < C_u{}^2$

$(\bar{Y}^2-1)C_u{}^2 + (1-\bar{Y})^2 C_v{}^2 < 2\bar{Y}(\bar{Y}-1)\rho_{uv}C_uC_v$

$(\bar{Y}+1)C_u{}^2 + (1-\bar{Y})C_v{}^2 < 2\bar{Y}\rho_{uv}C_uC_v$

5. Empirical Study

To see the performance of the proposed estimators in comparison to other estimators, description of population data are given below:

Population: [Source: Steel and Torrie (1960, p.282)]

Y: Long of leaf burn in sec.,

X_1 :Potassiam Percentage

X_2: Clorine Percentage.

The required population parameters are:

$\bar{Y}$=0.6860, C_y=0.4803, N=30, n =6

ρ_{yx_1} =0.1794, C_{x_1}=0.2295, C_{x_2}=0.7493, $\rho_{x_1x_2}$ =0.4074

$\bar{X}_1$ =4.6537, ρ_{yx_2}=-0.4996 $\bar{X}_2$ =0.8077, g=2

MSE of proposed estimators

1. M ($\bar{y}_{RT}$) =0.13409 , 2.M ($\bar{y}_{PT}$) =0.18836 , 3. M ($\bar{y}_{RPT_1}$) =0.21252

4. M ($\bar{y}_{RPT_2}$) =0.30401, 5.M ($\bar{y}_{RPT_3}$) = 0.10523 , 6. M ($\bar{y}_{RPT_4}$) =0.12227

7. M **($\bar{y}_R$)=0.23478, 8.M ($\bar{y}_P$) =0.51077, 9.M (** $\bar{y}$) =0.23068

Percentage Relative Efficiencies (PRE)

Estimators	R.E over $\bar{y}$	R.E over $\bar{y}_R$	R.E over $\bar{y}_P$
$\bar{y}$	___	101.77	221.41
$\bar{y}_R$	98.25	___	217.55
$\bar{y}_P$	45.16	45.96	___
$\bar{y}_{RT}$	171.97	175.09	___
$\bar{Y}_{PT}$	122.46	___	271.16
$\bar{Y}_{RPT1}$	108.54	110.47	___
$\bar{Y}_{RPT2}$	75.87	___	168.01
$\bar{Y}_{RPT3}$	219.21	___	485.38
$\bar{Y}_{RPT4}$	188.66	192.01	___

6. Conclusion

The proposed estimators are found to be better usual Mean per unit , ratio and product estimators.

References

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