Longitudinally Rough Short Bearing

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Abstract: Here we have studied the performance of short bearing under the presence of magnetic fluid as a lubricant. Bearing surfaces are considered to be longitudinally rough. The roughness is characterized by a stochastic random variable with non-zero mean, variance and skewness. The modified Reynolds equation is solved with suitable boundary conditions to obtain the pressure distribution. This expression for pressure distribution is used to calculate the load carrying capacity. The results are presented graphically. It is seen that due to magnetization the performance of bearing system gets improvement. It is also observed that the effect of roughness is negative on the performance of the bearing. The investigation suggests that the negative effect of roughness can be reduced by positive effect of magnetization parameter. It is also observed that the performance gets improve in the case of suitable combination of roughness parameters.

Keywords: Short bearing, Longitudinal roughness, Magnetic fluid, Reynolds’s equation, Load carrying capacity

1. Introduction

The slider bearing is the simplest and continuously encountered among the hydrodynamic bearings. In slider bearing, the film is continuous and non-diverging. These kinds of bearings are designed to support the axial loads. Particular solutions of Reynold’s equation for slider bearing with various simple film geometries are described in several books and research papers (Lord Rayleigh [1], Archibald [2]). Prakash and Vij [3] analysed the hydrodynamic lubrication of a plane inclined slider bearing taking various geometries into consideration and given that the quality of being porous decreased the friction and load carrying capacity. Patel and Gupta [4] extended the above analysis of Prakash and Vij [3] by integrating slide velocity and proved that in order to increase the performance of the bearing system the value of the slide parameter deserved to be minimized.

However, bearing surfaces could be roughened through manufacturing process, the wear and the impulsive damage. In order to understanding for the effect of surface roughness Christensen [5-6] developed a stochastic concept and introduced an averaging film model to lubricated surfaces with striated roughness. Many investigators has applied a stochastic method to model the random roughness (Tzeng and Seibel [7], Christensen and Tonder [8-10]). Christensen and Tonder [8-10] presented all inclusive general analysis for surface roughness based on a general probability density function by modifying and developing the method of Tzeng and Seibel [7]. Accordingly many investigators have been carried out to study the effect of surface roughness such as the works in the hydrodynamic journal bearing by Taranga et.al. [11], the in the hydrodynamic slider bearings by Christensen and Tonder [12] and the squeeze film spherical bearing by Andharia et al. [13]. In all these studies straight lubricant were used. The use of magnetic fluid as a lubricant modifying the performance of the bearing has splendidly recognized. Agrawal [14] considered the configuration of Prakash and Vij [3] in the presence of a magnetic fluid lubricant and establish its performance better than the one with straight lubricant. Bhat and Deheri [15] extended the analysis of Agrawal [14] by studying a magnetic fluid based porous composite slider bearing. Bhat and Deheri [16] discussed a general porous slider bearing with squeeze film formed by a magnetic fluid. Patel and Deheri [17] presented behavior of transversely rough magnetic fluid based porous short bearing. Also Andharia et al. [18] has discussed performance of a magnetic fluid based longitudinally rough short bearing. Recently Andharia et al. [22] presented surface roughness effect of transverse patterns on the performance of short bearing.

Here it has been proposed to investigate and examine the performance of longitudinally rough short bearing in the presence of a magnetic fluid as a lubricant.

2. Analysis

The geometry and configuration of bearing is shown in Fig. 1, which is infinite in Z-direction.

The slider moves with the uniform velocity U in X-direction. The length of bearing L and breadth B is in Z-direction, where B<<L. The pressure gradient ∂p/∂z is very larger than pressure gradient ∂p/∂x. The maximum and minimum film thicknesses are h₁ and h₂ respectively. The assumptions of usual hydrodynamic lubrication theory are taken into consideration in the development of the analysis.

The lubricant film is supposed to be isoviscous and incompressible and the flow is laminar. The magnetic field is oblique to the stator as in Agrawal [14]. Following discussions carried out by Prajapati [19] regarding the effect
of various forms of magnitude of magnetic field is expressed as
\[ M^2 = KB^2 \left( \frac{1 + e^b}{2b} \right) \sin^2 \left( \frac{z}{2b} \right) \left( \frac{1 + e^b}{2b} \right) \left( \frac{1 + e^b}{2b} \right) \] (1)
Where \( B \) is the breadth of bearing and \( K \) is a suitably chosen constant from dimensionless point of view (Bhat and Deheri [16]).

The bearing surfaces are assumed to be transversely rough. The thickness \( h \) of the lubricant film is given by
\[ h = \bar{h} + h_s \] (2)
Where \( \bar{h} \) is the mean film thickness and \( h_s \) is the deviation from the mean film thickness characterizing the random roughness of the bearing surfaces. \( h_s \) is considered to be stochastic in nature and governed by probability density function \( f(h_s) \), \( -c \leq h_s \leq c \) where \( c \) is the maximum deviation from the mean film thickness.

The random variable \( h_s \) are defined by the relationship:
\[ \alpha = E(h_s) \] (3)
\[ \sigma^2 = E[(h_s - \alpha)^2] \] (4)
and
\[ \mathcal{E} = E[(h_s - \alpha)^2] \] (5)
Where \( E \) is the expectancy operator defined by
\[ E(R) = \int_{-c}^{c} f(h_s) dh_s \] (6)
Wherein (Tzeng and Saibel [7])
\[ f(h_s) = \frac{35}{32c^3} (c^2 - h_s^2) \left( \frac{c^2 - h_s^2}{c^2} \right), -c \leq h_s \leq c = 0, \text{ elsewhere} \] (7)
It is easily observed that \( \alpha, \sigma \) and \( \mathcal{E} \) are independent of \( x \).

The standard deviation, the mean and the measure of symmetry \( \mathcal{E} \) the random variable \( h_s \) are defined by the relationship:
\[ \alpha = E(h_s) \] (3)
\[ \sigma^2 = E[(h_s - \alpha)^2] \] (4)
and
\[ \mathcal{E} = E[(h_s - \alpha)^2] \] (5)
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The standard deviation, the mean and the measure of symmetry play important role. Therefore with the usual notations of hydrodynamic lubrication, the modified Reynolds equation for film pressure (Prajapati [19], Bhat [20], Deheri, Andharia and Patel [21]) is given by
\[ \frac{d^2}{dz^2} \left( p - \frac{\mu_0 EM^2}{2} \right) = 6\mu U \left( \frac{1}{n(h)} \right) \] (8)
Where
\[ h = h_s \left[ 1 + m \left( \frac{1 - X}{1} \right) \right] \]
\[ (h) = h_s^2 \left[ 1 - 3m^2 + 6m^2(\alpha^2 + \sigma^2) - 10m^2(\mathcal{E} + 3\sigma^2(\alpha + \alpha^2)) \right] \]
\[ n(h) = h_s^2 \left[ 1 - 3m^2 + h_s^2(\alpha^2 + \sigma^2) - h_s^2(\mathcal{E} + 3\sigma^2(\alpha + \alpha^2)) \right] \]
Where in \( m = \frac{h_s - h_2}{h_2} \)
while \( \mu_0 \) is the magnetic susceptibility, \( \bar{\mu} \) is the free space permeability and \( \mu \) is the lubricant viscosity.

The associated boundary conditions are
\[ p = 0; \quad z = \pm \frac{e^b}{2} \quad \text{and} \quad \frac{dp}{dz} = 0; \quad z = 0 \] (9)
By integrating Eq. (8) with respect to \( z \)
\[ \frac{d}{dz} \left( p - \frac{\mu_0 EM^2}{2} \right) = 6\mu U \left( \frac{1}{n(h)} \right) \] (10)
Where \( C_1 \) is a constant.
\[ z = 0; \quad \frac{dp}{dz} = 0; \quad \frac{d}{dz} \left( M^2 \right) = 0 \quad \text{and} \quad C_1 = 0 \]
Again by integrating Eq. (10) with respect to \( z \)
\[ p - \frac{\mu_0 EM^2}{2} = 6\mu U \left( \frac{1}{n(h)} \right) \] (11)
Where \( C_2 \) is a constant.
\[ z = \pm \frac{e^b}{2}; \quad p = 0; \quad M^2 = 0 \quad \text{and} \quad C_2 = -3\mu_0 U \left( \frac{1}{n(h)} \right) \frac{\bar{h}_s}{b^2} \]
By Eq. (11), the pressure distribution is
\[ p = \frac{\mu_0 EM^2}{2} - 3\mu_0 U \left( \frac{1}{n(h)} \right) \frac{\bar{h}_s}{b^2} \left( \frac{1 - z^2}{B^2} \right) \] (12)
Introducing the following dimensionless quantities
\[ z = \frac{z}{B}; \quad x = \frac{X}{L}; \quad m = \frac{h_s - h_2}{h_2}; \quad \mu_s = \frac{h_s K h_b \bar{h}}{\mu \bar{U}} \]
\[ p = \frac{\mu_0 EM^2}{2} - 3\mu_0 U \left( \frac{1}{n(h)} \right) \frac{\bar{h}_s}{b^2} \left( \frac{1 - \bar{z}^2}{B^2} \right) \] (13)
The pressure distribution in dimensionless form as
\[ P = \frac{1}{2} \left[ \left( \frac{1}{2} + Z \right) \sin \left( \frac{\pi}{2} - Z \right) + \left( \frac{1}{2} - Z \right) \sin \left( \frac{\pi}{2} + Z \right) \right] \]
\[ + \left( \frac{3\mu_0 EM^2}{2} \right) \left( \frac{1}{n(h)} \right) \frac{\bar{h}_s}{b^2} \left( \frac{1 - \bar{z}^2}{B^2} \right) \] (14)
Where \( G(H) = \frac{A_1^2 - 2A_2^2 + (15\sigma^2 + 9\sigma^2)A_2}{A_1^2 - (3\sigma^2 + 3\sigma^2)A_2} \)
\[ + \left( 50\sigma^2 + 13\sigma^2 \right)^2 \left( 18\sigma^2 + 32\sigma^2 \right)^2 \]
\[ - \left( 34\sigma^2 + 31\sigma^2 + 54(\sigma^2 + \sigma^2) \right) + 182\sigma^2 \]
\[ + 40(15\sigma^2 + 32\sigma^2) \] (15)
Where \( A_1 = 1 + m(1 - X) \)
The load carrying capacity of the bearing is given by
\[ w = \frac{J}{B} \int_{0}^{1} p(x, z) \, dx \, dz \] (16)
Dimensionless load carrying capacity is obtained as
\[ W = \frac{1}{B} \frac{J}{z} \int_{0}^{1} P \, dx \, dZ \] (17)
Where \( Q_1 = \frac{J}{z} \int_{0}^{1} \left( \frac{1}{2} - \bar{z}^2 \right) G(H) \, dx \, dz \)

3. Result and Discussion

It is observed that Eq. (14) and Eq. (17) presents the dimensionless pressure distribution and dimensionless load carrying capacity respectively. These performance characteristics depend on various parameters such as magnetization parameter \( \mu^* \), length ratio \( L/h_2 \), breadth ratio \( b \), and the mean film thickness \( \bar{h} \).
B/h2, aspect ratio m, roughness parameters σ, α and Ε etc. Eq. (17) is numerically integrated using Simpson’s 1/3 rule for different values of μ*, σ, α and Ε. The results are presented graphically in Figures (2) – (23).

Figures (2) - (7) represent the variation of load carrying capacity with respect to magnetization parameter μ* for various values of L/h2, B/h2, σ/h2, α/h2, E/h2 and m respectively. These figures suggest that the load carrying capacity increases sharply due to magnetization parameter μ*.

Figures (8) - (12) represent the variation of load carrying capacity with respect to σ/h2 for various values of L/h2, B/h2, α/h2, E/h2 and m respectively. From these figures it is seen that the load carrying capacity increases significantly with σ/h2, but this increase is very less. However, the effect of σ/h2 was reported negative in the case of transverse surface roughness [22]. Figure (9) shows that load carrying capacity decreases marginally for increasing value of B/h2. Furthermore, the aspect ratio m has a strong positive effect in the sense of load carrying capacity increases sharply.
Figures (13) - (16) represent the variation of load carrying capacity with respect to $\alpha/h_2$ for various values of $L/h_2$, $B/h_2$, $\epsilon/h_2$ and $m$ respectively. From these figures it is clear that the load carrying capacity decreases due to $\alpha/h_2$. Figure (14) shows that load carrying capacity decreases marginally for increasing value of $B/h_2$.

Figures (17) - (19) present the variation of load carrying capacity with respect to $\epsilon/h_2$ for various values of $L/h_2$, $B/h_2$ and $m$ respectively. From this figures it is clear that the load carrying capacity decrease due to $\epsilon/h_2$. Figure (18) shows that load carrying capacity decreases marginally for increasing value of $B/h_2$.
4. Conclusion

The present study indicates that the effect of roughness parameters is trivial. This conditional effect increases with the larger values of $\sigma/h_2$ and $L/h_2$. The results show that the adverse effect of $B/h_2$, $\alpha/h_2$ and $E/h_2$ can be reduced to a larger extent by the positive effect of magnetization parameter $\mu^*$ and $L/h_2$, taking an appropriate values of aspect ratio $m$.

References


Author Profile

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