International Journal of Science and Research (IJSR)
ISSN (Online): 2319-7064
Index Copernicus Value (2013): 6.14 | Impact Factor (2013): 4.438

A New method for Solving Fully Fuzzy Linear Programming Problem

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Abstract: ***In this paper, a new method is proposed for solving fully fuzzy linear programming (FFLP) problem, in which all the coefficients and decision variables are triangular fuzzy number and all the constraints are fuzzy equality or inequality. With the help of similarity measure and ranking function, FFLP problem is transformed into crisp nonlinear programming problem. In the end, the proposed methodology is illustrated by numerical example.***

Keywords: Similarity measure, Ranking function, optimal solution.

1. Introduction

LP is an widely applied optimization technique in day today life because of its effectiveness. In conventional LP problem it is assumed that decision maker (DM) is sure about the accurate values of the decision parameters. However, the observed values of the data in real-life problems are often unclear. Fuzzy sets theory has been applied to tackle imprecise data in LP.

Lotfi et al. [1] have solved FFLP problem by approximated the fuzzy parameters to the nearest symmetric triangular fuzzy numbers and find the fuzzy optimal approximate solution. Amit et al. [2] have used the ranking function method to transform fuzzy objective function into crisp one and obtain the fuzzy optimal solution of FFLP problem. Khan et al. [3] have given a modified version of simplex method for FFLP problem. Ezzati et al. [5] have introduced a new lexicographical ordering of triangular fuzzy numbers and transform the FFLP problem into crisp multi-objective linear programming problem and find the exact optimal solution of FFLP problem. Recently, Kaur and Kumar [4] have applied Mehar's method on FFLP problem in which parameters are L-R fuzzy number.

The paper is organized as follows. Some basic definitions of fuzzy set theory are presented in Section 2. In Section 3, FFLP problem is presented and a new method for solving FFLP problem is discussed. Section 4 presents a numerical illustration and finally the paper concludes in Section 5.

2. Preliminaries

In this section, some basic definitions and arithmetic operations related to triangular fuzzy numbers, which will be used in the rest of the paper, are given.

Definition 2.1[2]. A fuzzy number $\tilde{A} = (a_1, a_2, a_3)$ is said to be a triangular fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & a_1 \le x \le a_2, \\ \dfrac{a_3 - x}{a_3 - a_2}, & a_2 \le x \le a_3, \\ 0, & otherwise. \end{cases}$$

Let TF(R) denotes the set of all triangular fuzzy numbers.

Definition 2.2 [2]. A triangular fuzzy number $\tilde{A} = (a_1, a_2, a_3)$ is said to be non-negative fuzzy number if and only if $a_1 \ge 0$.
Let $TF(R)^+$ denotes the set of all non-negative triangular fuzzy numbers.

Definition 2.3 [2]. A ranking function is a function $\Re : F(R) \to R$, where F(R) is the set of fuzzy numbers defined on the set of real numbers, which maps each fuzzy number into the real line, where a natural order exists. Let $\tilde{A} = (a_1, a_2, a_3)$ be a triangular fuzzy number then $\Re(\tilde{A}) = \dfrac{a_1 + 2a_2 + a_3}{4}$.

Definition 2.4 [2]. Let $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ be two triangular fuzzy numbers then

1) $\tilde{A} \preceq \tilde{B}$ iff $\Re(\tilde{A}) \le \Re(\tilde{B})$.

2) $\tilde{A} \succeq \tilde{B}$ iff $\Re(\tilde{A}) \ge \Re(\tilde{B})$.

3) $\tilde{A} \simeq \tilde{B}$ iff $\Re(\tilde{A}) = \Re(\tilde{B})$.

Definition 2.5 [6]. Let $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ be two triangular fuzzy numbers then the similarity between two fuzzy numbers is defined as

$$d(\tilde{A}, \tilde{B}) = \sqrt{\frac{1}{3}\left[(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2\right]}$$

Definition 2.6[2]. The arithmetic operations on two triangular fuzzy numbers $\tilde{A}=(a_1, a_2, a_3)$ and $\tilde{B}=(b_1, b_2, b_3)$ are given by:

1) $\tilde{A}\oplus\tilde{B}=(a_1, a_2, a_3)\oplus(b_1, b_2, b_3)=(a_1+b_1, a_2+b_2, a_3+b_3)$

2) $-\tilde{A}=-(a_1, a_2, a_3)=(-a_3, -a_2, -a_1)$

3) $\tilde{A}\Theta\tilde{B}=(a_1, a_2, a_3)\Theta(b_1, b_2, b_3)=(a_1-b_3, a_2-b_2, a_3-b_1)$

4) Let $\tilde{A}=(a_1, a_2, a_3)$ be any triangular fuzzy number and $\tilde{X}=(x_1, x_2, x_3)$ be a non-negative triangular fuzzy numbers then

$$\tilde{A}\otimes\tilde{X}\cong\begin{cases}(a_1x_1, a_2x_2, a_3x_3), & a_1\geq 0,\\ (a_1x_3, a_2x_2, a_3x_3), & a_1<0,\ a_3\geq 0,\\ (a_1x_3, a_2x_2, a_3x_1), & a_3<0.\end{cases}$$

5) $$\lambda\tilde{A}=\begin{cases}(\lambda a_1, \lambda a_2, \lambda a_3), & \lambda\geq 0,\\ (\lambda a_3, \lambda a_2, \lambda a_1), & \lambda<0.\end{cases}$$

3. Fully Fuzzy Linear Programming Problem

The FFLP problem is written as:

(P1) $$Max\ Z(\tilde{X})=\sum_{j=1}^{n}\tilde{c}_j\otimes\tilde{x}_j$$

Subject to . $\sum_{j=1}^{n}\tilde{a}_{ij}\otimes\tilde{x}_j\ (\preceq, \approx, \succeq)\ \tilde{b}_i$ *i = 1, 2….. , m,*

$\tilde{x}_j$ is non -negative triangular fuzzy, *j = 1, 2….. , n,.*

where $\tilde{X}=[\tilde{x}_j]_{n\times 1}$, $\tilde{c}_j$, $\tilde{a}_{ij}$, $\tilde{b}_i, \tilde{x}_j \in$ TF(R) , j = 1, 2…, n, p = 1, 2…, k_i and i = 1, 2…., m.

3.1 Proposed Method

In this section, a new algorithm to find a fuzzy optimal solution of FFLP problem is proposed. The steps of the proposed algorithm are as follows:

Step 1: Using definition 2.4, the fuzzy constraints of the FFLP problem can be converted into the crisp constraints. The obtained fuzzy linear programming problem can be written as:

(P2) $$Max\ Z(\tilde{X})=\sum_{j=1}^{n}\tilde{c}_j\otimes\tilde{x}_j$$

Subject to $\Re\left(\sum_{j=1}^{n}\tilde{a}_{ij}\otimes\tilde{x}_j\right)(\preceq, \approx, \succeq)\Re(\tilde{b}_i)$ *i = 1, 2….. , m,*

$x_j,\ y_j-x_j, z_j-y_j\geq 0$ j = 1, 2…... n.

Step 2: With regard to Definition 2.5, the problem in step 1 is converted into the following crisp multi-objective non-linear programming problem.

(P3) $$Max\ d\left(Z_q(\tilde{X}),\ \tilde{0}\right)$$

Subject to $\Re\left(\sum_{j=1}^{n}\tilde{a}_{ij}\otimes\tilde{x}_j\right)(\preceq, \approx, \succeq)\Re(\tilde{b}_i)$ *i = 1, 2.. , m,*

$x_j,\ y_j-x_j, z_j-y_j\geq 0$ j = 1, 2…... n.

Step 3 Solve the crisp non-linear programming problem, obtained in Step 2, to find the optimal solution of x_j, y_j and z_j.

4. Numerical Example

In this section proposed algorithm is illustrated with the help of numerical example.

Example 6.1

$$MaxZ_1(\tilde{X})=(5,7,9)\tilde{x}_1+(4,5,6)\tilde{x}_2+(1,2,3)\tilde{x}_3$$

Subject to

$$(2,5,7)\tilde{x}_1+(2,3,4)\tilde{x}_2+(1,2,3)\tilde{x}_3\approx(8,16,24)$$
$$(1,2,3)\tilde{x}_1+(1,2,3)\tilde{x}_2+(1,3,4)\tilde{x}_3\preceq(7,17,22) \quad (1)$$
$$(2,3,4)\tilde{x}_1+(1,2,4)\tilde{x}_2+(2,3,4)\tilde{x}_3\succeq(12,18,25)$$

$\tilde{x}_1, \tilde{x}_2, \tilde{x}_3$ are non – negative triangular fuzzy numbers.

From step 1 and step 2, (1) can be written as

$$MaxZ_1(\tilde{X})=\sqrt{\frac{1}{3}\left((5x_1+4x_2+x_3)^2+(7y_1+5y_2+2y_3)^2+(9z_1+6z_2+3z_3)^2\right)}$$

Subject to $2x_1+2x_2+x_3+10y_1+6y_2+4y_3+7z_1+4z_2+3z_3=64$

$$x_1+x_2+x_3+4y_1+4y_2+6y_3+3z_1+3z_2+3z_3\leq 63 \quad (2)$$
$$2x_1+x_2+2x_3+6y_1+4y_2+6y_3+4z_1+4z_2+4z_3\geq 73$$
$$x_j, y_j-x_j, z_j-y_j\geq 0 \quad j=1,2,3$$

Now solving the problem (2) by LINGO 14.0, we get the optimal solution and optimal value of objective functions of problem (1) are given in the table 1

Table 1

$\tilde{x}_1^*$	$\tilde{x}_2^*$	$\tilde{x}_3^*$	$Z(\tilde{X}^*)$
(0,0,0)	(0,0,9.75)	(1.25, 1.25, 6.25)	(1.25, 2.5, 77.25)

5. Conclusions

In this paper, a new approach for solving the FFLP problem is proposed. In this method, ranking function is applied on the fuzzy constraints to convert it into crisps constraints and similarity measure on objective functions.

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