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Complement Properties on Normal Product of Strong Fuzzy Graphs

M. Vijaya

Department of Mathematics, Marudupandiyar College, Thanjavur-613403, Tamilnadu, India

Abstract: ***In this work we introduce the normal product of fuzzy graphs, normal product of strong fuzzy graphs and Complement Properties on normal product of strong fuzzy graphs are discussed.***

Keywords: Fuzzy graph, Complement of fuzzy graph, strong fuzzy graph, Normal product, Normal product of two strong fuzzy graphs

1. Introduction

Rosenfeld [3] introduced fuzzy graph in 1975. The operations of Cartesian product, compositions of fuzzy graphs were defined by Moderson.J.N and peng.C.S [6]. In this note, we discuss a sub class of fuzzy graphs called strong fuzzy graph which were introduced by Moderson.J.N and peng.C.S [6]. In this paper we introduced the normal product of fuzzy graphs. In the present study we have been introduced the normal product of fuzzy graphs. The operations on (crisp) graphs such as Cartesian product, composition, tensor, and normal products are extended to fuzzy graphs and some of their properties are incorporated to investigate. Properties found are related to complement of fuzzy graph and strong fuzzy graph.

2. Basic Definitions

Definition 2.1 Fuzzy graph with S as the underlying set is a pair G: (σ, μ) where σ: S $\rightarrow$ [0,1] is a fuzzy subset, μ:S x S$\rightarrow$ [0,1] is a fuzzy relation on the fuzzy subset σ, such that $\mu(x,y) \leq min\{\sigma(x),\sigma(y)\}$ for all for all $x,y \in S$.

Definition 2.2 Let (σ, μ) be fuzzy sub graph of G = (V, X). Then (σ, μ) is called a strong [6] fuzzy graph of G if $\mu(u, v) = \sigma(u) \wedge \sigma(v)$ for all $(u, v) \in X$.

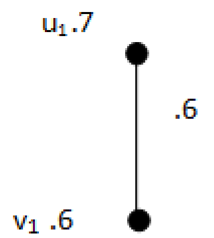

Figure 1: Strong fuzzy graph

Definition 2.3 The complement of a fuzzy graph $G^c : (\sigma^c,\mu^c)$ where $\sigma^c=\sigma$ and $\mu^c(uv)=0$ when $\mu(uv)=0$ and $\mu^c(uv)=\sigma(u) \wedge \sigma(v) - \mu(uv)$ when $\mu(uv) > 0$.
Note: $(G^c)^c$ = G if and only if G is a strong fuzzy graph.

Definition 2.4 The normal Product of two fuzzy graphs (σ_i, μ_i) on $G_i = (V_i , X_i)$, i=1,2 is said to be a fuzzy graph $G_1 \circ G_2 = (\sigma_1 \circ \sigma_2 , \mu_1 \circ \mu_2)$ on G = (V,X) where V= V_1 X V_2 and X = $\{(u,u_2)(u,v_2)$ / $u\in V_1$, $_1,u_2v_2 \in X_2\} \cup \{(u_1, w)(v_1,w)\backslash u_1v_1\in X_1,w\in V_2\} \cup \{(u_1, u_2)(v_1,v_2)\backslash u_1v_1\in X_1,u_2v_2\in X_2\}$. Let σ_i be a fuzzy subset of V_i and let μ_i be a fuzzy subset of X_i, i=1,2. Define the fuzzy subsets $\sigma_1\circ \sigma_2$ of V and $\mu_1\circ \mu_2$ of X as follows.

$(\sigma_1 \circ \sigma_2)$ $(u_1, u_2) = \{(\sigma_1 (u_1)\wedge \sigma_2 (u_2)\}$ for all $(u_1, u_2) \in V$
$(\mu_1 \circ \mu_2)$ $\{(u, u_2)(u, v_2)\} = \{ \sigma_1 (u_1) \wedge \mu_2 (u_2v_2)\}$ $u \in V_1$ and $u_2v_2 \in X_2$
$(\mu_1 \circ \mu_2)$ $\{(u_1,w)(v_1,w)\} = \{ \mu_1 (u_1v_1)\wedge \sigma_2 (w_2)\}$ $u_1v_1 \in X_1$ and $w \in V_2$
$(\mu_1 \circ \mu_2)$ $\{(u_1, u_2)(v_1, v_2)\} = \{ \mu_1 (u_1v_1)\wedge \mu_2 (u_2v_2)\}$ $u_1u_2 \in X_1$ and $v_1v_2 \in X_2$

3. Properties

Normal product of two strong fuzzy graphs 3.1 The normal product of two strong fuzzy graphs (σ_i, μ_i) on Gsi = (V_i , X_i), i=1,2 is said to as a strong fuzzy graph $\mathbf{G_{s_1}\circ G_{s_2}}$ = $(\sigma_1 \circ \sigma_2 , \mu_1 \circ \mu_2)$ on G = (V,X) where V= V_1 X V_2 and X $=\{((u,u_2)(u,v_2)) \setminus u\in V_1, (u_2,v_2)\in X_2\} \cup \{((u_1,w)(v_1,w))\backslash (u_1,v_1) \in X_1,w \in V_2\}\cup\{((u_1, u_2)(v_1, v_2))\backslash(u_1,v_1)\in X_1,(u_2,v_2) \in X_2\}$. Fuzzy sets $\sigma = \sigma_1 \circ \sigma_2$ and $\mu = \mu_1 \circ \mu_2$ are defined as $\sigma(u_1, u_2) = (\sigma_1 (u_1) \wedge \sigma_2(u_2))$ $\mu\{(u, u_2), (u, v_2)\} = \{\sigma (u, u_2)\wedge \sigma (u,v_2)\}$ $\mu\{(u_1, w), (v_1, w)\} = \{\sigma (u_1, w)\wedge \sigma(v_1,w)\}$ $\mu((u_1,v_1),(u_2,v_2)) = \{\sigma(u_1, v_1)\wedge \sigma(u_2,v_2)\}$

Theorem 3.2 Let $G_{s1}:(\sigma_1, \mu_1)$ and $G_{s2}: (\sigma_2, \mu_2)$ be two strong fuzzy graphs. Then $G_{s1}\circ G_{s_2}$ is a strong fuzzy graph. Let $G_{s_1} \circ G_{s_2}$ = G:(σ,μ) where $\sigma = \sigma_1 o\, \sigma_2$, $\mu= \mu_1 \circ\mu_2$ and G* =(V,E) where V=V_1 x V_2 , E=$\{(u,u_2)(u,v_2):u\in V_1,u_2v_2 \in E_2\}\cup (u_1,w)(v_1,w):w\in V_2 ,u_1v_1\in E_1\} \cup \{(u_1,u_2)(v_1,v_2):u_1v_1 \in E_1,u_2v_2 \in E_2\}$. case 1. $\mu((u, u_2) (u,v_2)) = \sigma_1(u) \wedge \mu_2 (u_2,v_2) = \sigma_1(u) \wedge (\sigma_2(u_2) \wedge \sigma_2(v_2))$ since G_{s2} being strong. $= (\sigma_1(u) \wedge \sigma_2(u_2)) \wedge (\sigma_1(u) \wedge \sigma_2(v_2)) = (\sigma_1 o\, \sigma_2)(u,u_2)\wedge (\sigma_1 o\, \sigma_2) (u,v_2) = \sigma(u, u_2) \wedge \sigma(u, v_2)$

case 2. $\mu (u_1, w) (v_1, w) = \sigma_2 (w) \wedge \mu_1 (u_1, v_1) = \sigma_1 (u_1) \wedge \sigma_1 (v_1) \wedge \sigma_2 (w)$, since G_{s1} being strong. $= (\sigma_1 (u_1) \wedge \sigma_2 (w)) \wedge (\sigma_1 (v_1) \wedge \sigma_2 (w)) = (\sigma_1\circ \sigma_2) (u_1, w) \wedge (\sigma_1 \circ \sigma_2) (v_1, w) = \sigma (u_1, w) \wedge \sigma (v_1, w)$ case 3. $\mu (u_1, u_2) (v_1, v_2) = \mu_1 (u_1, v_1) \wedge \mu_2 (u_2, v_2) = (\sigma_1 (u_1) \wedge \sigma_1 (v_1)) \wedge (\sigma_2 (u_2) \wedge \sigma_2 (v_2))$, since G_{s1} being strong. $= (\sigma_1 (u_1) \wedge \sigma_2 (u_2)) \wedge (\sigma_1 (v_1) \wedge \sigma_2 (v_2)) = (\sigma_1\circ \sigma_2) (u_1, u_2) \wedge (\sigma_1 \circ \sigma_2) (v_1, v_2) = \sigma(u_1, u_2) \wedge \sigma (v_1, v_2)$. Thus from above cases it follows that G= $G_{s1}\circ$ G_{s_2} is a strong fuzzy graph.

Example 3.3

Let G_{s_1} and G_{s_2} be two strong fuzzy graphs

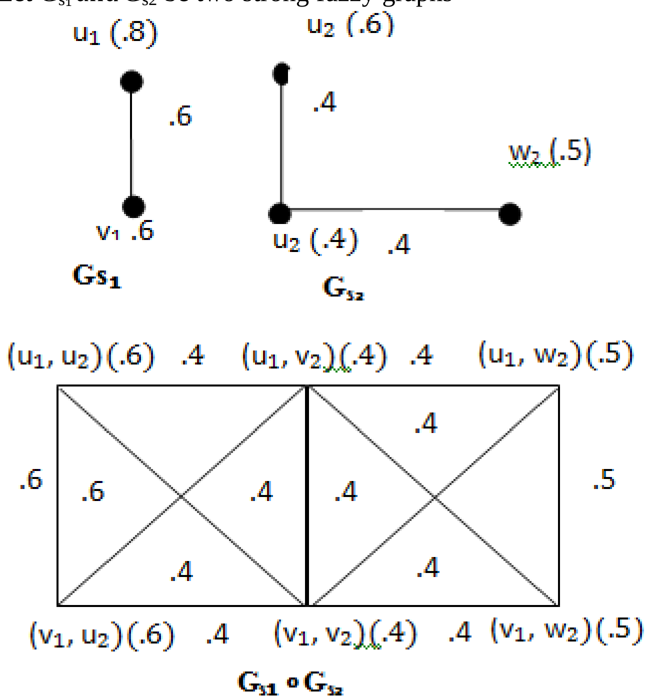

Figure 2

From fig 2 implies $G_{s1} \circ G_{s_2}$ is the normal product of two strong fuzzy graphs is also a strong fuzzy graph.

From the above figs

$\mu((u_1,u_2)(v_1,u_2)) = \sigma(u_1,u_2) \wedge \sigma(v_1, u_2) = .6 \wedge .6 = .6$

$\mu((u_1,v_2)(u_1,w_2)) = \sigma(u_1,v_2) \wedge \sigma(u_1,w_2) = .4 \wedge .5 = .4$

Similarly finding the membership value of all the edges, we get a strong fuzzy graph. Hence the normal product $G_{s1} \circ G_{s_2}$ of two strong fuzzy graphs G_{s1} and G_{s_2} is also a strong fuzzy graph.

Theorem 3.4 If $G_{s1} :(\sigma_1, \mu_1)$ and $G_{s_2}: (\sigma_2, \mu_2)$ be two strong fuzzy graphs then $\overline{G_{s_1} \circ G_{s_2}} = \overline{G_{s_1}} \circ \overline{G_{s_2}}$

Proof: Let $G_{s1} :(\sigma_1, \mu_1)$ and $G_{s_2}: (\sigma_2, \mu_2)$ are strong fuzzy graphs. $\overline{G} :(\sigma,\overline{\mu})= \overline{G_{s_1} \circ G_{s_2}}$ $\overline{\mu}=\overline{\mu_1 \circ \mu_2}$, $\overline{G}$: $(V,\overline{E})$ $\overline{G_{s_1}}$ $(\sigma_1,\overline{\mu_1})= \overline{G_1}$ $(V_1 ,\overline{E_1})$ $\overline{G_{s_2}}(\sigma_2, \overline{\mu_2})= \overline{G_2}$ $(V_2 ,\overline{E_2})$ $\overline{G_{s_1}} \circ \overline{G_{s_2}}$: $(\sigma_1 \circ \sigma_2, \overline{\mu_1} \circ \overline{\mu_2})$ Now , the various types of edges say e , joining the vertices of V are the following and it suffices to prove that $\overline{\mu_1 \circ \mu_2} = \overline{\mu_1} \circ \overline{\mu_2}$ in each case. Case(i)

$e=(u,u_2)(u,v_2)$ $u_2v_2 \in E_2$ Then $e \in E$ and G being strong hence $\overline{\mu}$ (e)=0 Also $(\overline{\mu_1} \circ \overline{\mu_2})$(e)=0 $u_2v_2 \notin E_2 = \overline{\mu}$ (e). Case (ii)

$e=(u,u_2)(u,v_2)$ $u_2 \neq v_2$ and $u_2v_2 \notin E_2$ Then $e \notin E$, so $\mu(e)=0$

Now $\overline{\mu}(e) = \sigma(u, u_2) \wedge \sigma (u, v_2)$

$= [\sigma_1(u) \wedge (\sigma_2(u_2)] \wedge [(\sigma_1(u) \wedge \sigma_2(v_2)]$

$= \sigma_1(u) \wedge [\sigma_2(u_2) \wedge \sigma_2(v_2)]$ $u_2v_2 \notin E_2$ $(\overline{\mu_1} \circ \overline{\mu_2})(e) = \sigma_1(u) \wedge \overline{\mu_2}(u_2v_2)$

$= \sigma_1(u) \wedge [\sigma_2(u_2) \wedge \sigma_2(v_2)]$

$= \overline{\mu_1 \circ \mu_2}$ (e)

Case(iii)

$e = (u_1,w)(v_1,w)$ $u_1v_1 \in E_1$ Here $e \in E$ and $\overline{\mu}$ (e)=0 Also $u_1v_1 \notin \overline{E_1}$ Hence $(\overline{\mu_1} \circ \overline{\mu_2})(e) = 0$

Case(iv)

$e = (u_1,w)(v_1,w)$ $u_1v_1 \notin E_1$ Here $e \notin E$, so $\mu(e)=0$ and

$\overline{\mu}(e) = \sigma(u_1, w_2) \wedge \sigma (v_1,w)$

$= [\sigma_1(u_1) \wedge (\sigma_2(w)] \wedge [(\sigma_1(v_1) \wedge \sigma_2(w)]$

$= [\sigma_1(u_1) \wedge \sigma_1(v_1)] \wedge \sigma_2(w)$

Also $u_1v_1 \notin \overline{E_1}$

$(\overline{\mu_1} \circ \overline{\mu_2})(e) = \overline{\mu_1}(u_2v_2) \wedge \sigma_2(w)$

$= [\sigma_1(u_1) \wedge \sigma_1(v_1)] \wedge \sigma_2(w)$

$= \overline{\mu}$ (e)

Case(v)

$e=(u_1,u_2)(v_1,v_2)$ $u_1v_1 \in E_1$, and $u_2v_2 \in E_2$ Here $e \in E$ and $\overline{\mu}$ (e)=0 Also since $u_1v_1 \notin \overline{E_1}$ and $u_2v_2 \notin \overline{E_2}$ We have $(\overline{\mu_1} \circ \overline{\mu_2})(e) = 0$

Case(vi)

$e=(u_1,u_2)(v_1,v_2)$ $u_1v_1 \in E_1$, and $u_2v_2 \notin \overline{E_2}$ Then $e \notin E$, so $\mu(e)=0$ Also $\overline{\mu}$ (e) = $\overline{\mu_1 \circ \mu_2}$ (e) = 0 $u_1v_1 \notin \overline{E_1}$ and $u_2v_2 \in E_2$ Then $(\overline{\mu_1} \circ \overline{\mu_2})(e) = 0$

Case(vii)

$e=(u_1,u_2)(v_1,v_2)$ $u_1v_1 \notin E_1$, and $u_2v_2 \in E_2$ Then $e \notin E$, so $\mu(e)=0$ Also $\overline{\mu}$ (e)=0 Then $(\overline{\mu_1} \circ \overline{\mu_2})(e) = 0$

Case(viii)

$e=(u_1,u_2)(v_1,v_2)$ $u_1v_1 \notin E_1$, and $u_2v_2 \notin \overline{E_2}$ Then $e \notin E$, so $\mu(e)=0$

$\overline{\mu}(e) = \sigma(u_1, u_2) \wedge \sigma (v_1,v_2)$

$= [\sigma_1(u_1) \wedge \sigma_2(u_2)] \wedge [\sigma_1(v_1) \wedge \sigma_2(v_2)]$

$= [\sigma_1(u_1) \wedge \sigma_1(v_1)] \wedge [\sigma_2(u_2) \wedge \sigma_2(v_2)]$

Since $u_1v_1 \notin E_1, \Rightarrow u_1v_1 \in \overline{E_1}$

$u_2v_2 \notin E_2 \Rightarrow u_2v_2 \in \overline{E_2}$

Hence $(\overline{\mu_1} \circ \overline{\mu_2})(e) = \overline{\mu_1}(u_1v_1) \wedge \overline{\mu_2}(u_2v_2)$

$=[\sigma_1(u_1) \wedge \sigma_1(v_1)] \wedge [\sigma_2(u_2) \wedge \sigma_2(v_2)]$

$= \overline{\mu}$ (e)

Thus from case (i) to (viii) it follows that $\overline{G_{s_1} \circ G_{s_2}} = \overline{G_{s_1}} \circ \overline{G_{s_2}}$.

4. Conclusion

In this paper we have proposed, complement of strong fuzzy graphs, normal products of strong fuzzy graphs and the complement properties for tensor product of strong fuzzy graphs. In the fuzzy environment it is reasonable to discuss complement of strong fuzzy graphs and its properties.

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Author Profile

Dr. M. Vijaya is Head, Department of Mathematics Marudupandiyar College, Thanjavur-613403, Tamilnadu, India