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The Schrodinger Equation of Linear Potential Solution Specializing to the Stark Effects

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Abstract: ***The Shifts in the energy level spectrum due to the external electric field is examine, for a one dimensional quantum mechanical system describe by the Schrodinger equation specializing to the linear potential which is perturbed by electric field. The 1D model of linear potential that is known as a quantum bouncer, which is define as ($v(y) = Fy\ for\ y > 0\ and\ v(y) = \infty\ for\ y < 0$) and the symmetric linear potential which is ($v(y) = Fy$). In this case the Airy function property is use to give a close result the shift in energy level under the electric field (Stark Effect), The approximate and exact result is compared.***

Keywords: Linear potential, stark effect, quantum bouncer, Schrodinger equation.

1. Introduction

The analysis of various quantum mechanical potentials is important. A system with a symmetrical linear potential $V(y) = F|y|$ is a quantum equivalent of a 'bouncing ball', the case of symmetric linear potential is seemingly small variation on the quantum bouncer potential. This symmetric linear potential is a simple model to analyze the quantum states of neutrons in the Earth's gravitational field [1-4]. The 1D quantum mechanical system in linear potential for quantum bouncer define as

$$V(y) = \begin{cases} mgy\ for\ 0 \le y \\ \infty\ for\ y < 0 \end{cases} \tag{1}$$

The shift in the energy spectrum that reacts to the quantized energy levels due to a constant external force. Here we consider the stark effect as a perturbation term of desire potential, the symmetric linear potential given by $V(y) = F|y|$.In this paper we show how direct use of the airy function properties gives a close result for the stark shift in quantum bouncer system [4].

2. Solving Schrodinger equation with linear potential

Since the gravitational interaction is weak, so the terrestrial objects are confined by the earth's gravitational field. The gravitational quantum effect are not observed in the macro-world, evidence for quantum states of the gravitational energy levels in the quantum bouncer [4].It has been a favorite example for solving of 1D problem in teaching of quantum physics. This academic problem has received renewed attention in the earth's gravitational field [1]. Now we begin by

Considering the Schrodinger equation of a quantum bouncer problem define by the linear potential in Eqn.(1).

Using the symmetric potential to solve Eqn.(1) for $y > 0$ Eqn.(1) reduce to:

$$\frac{-\hbar^2}{2m}\frac{d^2\psi_n(y)}{dy^2} + fz\psi_n(y) = E_n\psi_n(y) \tag{2}$$

For even $\left(\psi^{(+)}(y)\right)$ *and odd* $\left(\psi^{(-)}(y)\right)$ states and extend them to negative value of y by using

$$\psi^{(+)}(-y) = \psi^{(+)}(y)\ and\ \psi^{(-)}(y) = -\psi^{(-)}(y) \tag{3}$$

Using change of variable to $y = \rho z + \sigma$ To simplify Eqn.(2) and find the result for a particle confined by a (half-well) and for a particle in one-dimensional constant force field by the Airy function, the Air functions of figure 1 shows Ai(x) and Bi(x) [where Ai(x) and Bi(x) are the Airy functions].

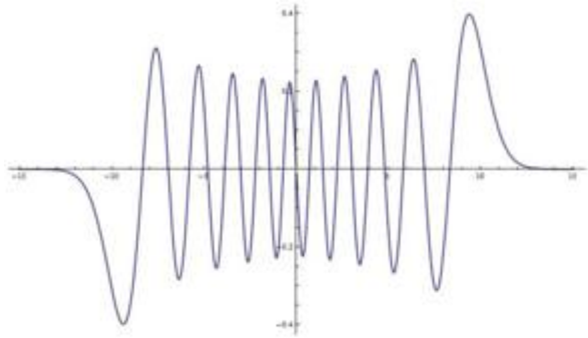

Figure 1: Plot of eigenfunctions Ai(x) and Bi(x)

Exact solution

The Schrodinger equation for quantum bouncer can be written as:

$\psi_n''(x) = (x - \sigma_n)\psi_n(x)$

Using the change of variable $z = \rho x$ and the definitions:

$$\rho = \left(\frac{\hbar^2}{2mf}\right)^{1/3}\ and\ \sigma = \frac{E_n}{F_n} = \frac{E_n}{\varepsilon_0} \tag{4}$$

The solution of Eqn.(3) are two linearly independent Airy functions $Ai(x - \sigma_n)$ and $Bi(x - \sigma_n)$ [5]. The Bi solution diverges for large positive argument and does not satisfy the boundary condition $\psi(z \to \infty) = 0$ and so excluded. The energy Eigen values are determined by the boundary condition imposed by the infinite wall at origin, specifically that $\psi(z = 0) = Ai(-\sigma_n)$.

The quantized energies are then given in terms of zeros of the well-behaved Airy function $Ai(-\mathcal{E}_n)$ with $E_n = +\varepsilon_0\mathcal{E}_n$, the result of the Asymptotic function behavior of the $Ai(x)$ zeros [6] gives:

$E_n \approx \mathcal{E}_n\left[\frac{3\pi}{2}(4n-1)\right]^{2/3}$ This after expanding becomes:

$$E_n = \mathcal{E}_n\left[\frac{3\pi}{8}(4n-1)\right]^{2/3}\left[1 + \frac{5}{48}\left(\frac{3\pi}{8}(4n-1)\right)^{-2} - \right.$$
$$5\ 363\pi 8(4n-1)-4 \tag{5}$$

The wave functions and normalization constant are then given by

$$\psi_n(z) = N_n Ai\left(\frac{z}{\rho} - \sigma_n\right), where\ N_n = \frac{1}{\sqrt{\rho}A'i(-\mathcal{E}_n)} \quad (6)$$

$A'(-\mathcal{E}_n)$ is differential of $Ai(-\mathcal{E}_n)$ where $|\psi_n(z)|^2$ is the probability distribution for a particle of a quantum bouncer. In these types of problem the eigenfunctions are pieces of the same function, the Airy function Ai shifted in all the cases so that it has zero at $z = 0$ and by $z < 0$ part truncated. The first five Eigen values of linear potential are illustrated in table 1[6].

Table 1: Eigenvalues of the linear potential

n	$E(joule)$
1	2.33811
2	4.08795
3	5.52056
4	6.78671
5	7.94413

3. Linear Potential with Stark Effect

The change in the energy levels of an atom in the presence of electric field [7], considering an electron which is subjected to gravity (g) and an electric field $\in_0$. In this situation the result of the perturbing potential of the form $V(y) = \bar{F}y$. the solution of this potential remain the same with the global substitution $\bar{F} \to (F + \bar{F})$. The perturbed eigenvalues are obtained from Eqn. (4) as:

$$\tilde{E}_n = (\mathcal{E}_0\sigma_n)\left[1 + \frac{\bar{F}}{F}\right]^{2/3} \quad (7)$$

So the first, second and third order energy shifts are given by Eqn. (8) below.

$$E_n^{(1)} = \frac{2}{3}\left(\frac{\bar{F}}{F}\right)E_n^{(0)}, E_n^{(2)} = -\frac{1}{9}\left(\frac{\bar{F}}{F}\right)^2 E_n^{(0)}\ and\ E_n^{(3)} = \frac{4}{81}\left(\frac{\bar{F}}{F}\right)^3 E_n^{(0)} \quad (8)$$

The energy level shifts values are given in Table 2 below for the first five states [3].

Table 2: Shift of the first order energy (linear stark effect)

n	$\Delta E(joule)$
1	0.27
2	0.44
3	0.60
4	0.74
5	0.84

The Hamiltonian of particle in the gravity if earth is:

$$H_0 = \frac{p^2}{2m} + Fy \quad (9)$$

If we consider the first and second order stark effect:

$$H = H_0 + H_1 = \frac{p^2}{2m} + Fy + e \in_0 y + {}^1\!/_2\, e^2 \in_0^2 y^2 \quad (10)$$

By the basis of Eqn.10 the Schrodinger equation will be written as

$$\psi_n''(y) = \left(\frac{m}{\hbar^2}e^2 \in_0^2 y^2 + \frac{2m}{\hbar^2}(F + e \in_0)y - \frac{2m}{\hbar^2}E_n\right)\psi_n(y) \quad (11)$$

The Eqn.11 can be re- write in the form:

$$\psi_n''(y) - (py^2 + qy - r)\psi_n(y) \quad (12)$$

Where:

$$p = \frac{m}{\hbar^2}e^2 \in_0^2, q = \frac{2m}{\hbar^2}(F + e \in_0)\ and\ c = \frac{2m}{\hbar^2}E_n \quad (13)$$

To solve Eqn.13 we change the variables:

$$\sigma = y + \frac{q}{2p}, u = \sqrt{4}a\sigma \quad (14)$$

The Eqn.12 will become:

$$\frac{d^2\psi}{du^2} + \left(\frac{r - \frac{q^2}{4p}}{\sqrt{p}} - u\right)\psi(n) = 0 \quad (15)$$

Using the harmonic oscillator Schrodinger equation as in[5].

$$\psi''(y) + (2n + 1 - y^2)\psi(y) = 0 \quad (16)$$

The energy value will be:

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega, n = 0.1, 2 \ldots \quad (17)$$

Comparing Eqn.15 and Eqn.16 and considering the boundary condition $\psi(y \to \infty) = 0$ implies:

$$r = (2n + 1)\sqrt{p} + \frac{q^2}{4p} \quad (18)$$

Substituting Eqn.13 in Eqn.18 the Exact solution will be:

$$E_n = \left(n + \frac{1}{2}\right)\frac{e\in_0\hbar}{\sqrt{m}} + \frac{(F + e\in_0)^2}{2e^2\in_0^2} \quad (19)$$

But $\in_0 = 10^{-11}\, {}^{v}\!/_{m}$, $\hbar = 1.054 \times 10^{-34} Js$, $m_e = 9.11 \times 10^{-31} kg$, $e = 1.6 \times 10^{-19} c$, $g = 9.8\, m/s^2$ while using $n = 1\ and\ F = mg$ The Exact value for First energy level is:

$$E = 8.1 \times 10^{-49} j \quad (20)$$

The shift in the energy level

The normalized Eigen functions of the unperturbed linear potential are in Eqn.6. By considering the linear stark effect, the perturbed Hamiltonian will become:

$$H = H_0 + H_1 = \frac{p^2}{2m} + Fy + e \in_0 y \quad (21)$$

The first-order energy shift is giving by:

$$\Delta E_n^{(1)} = \langle\psi_n|V(y)|\psi_n\rangle = \bar{F}\langle\psi_n|y|\psi_n\rangle = \frac{2}{3}\left(\frac{\bar{F}}{F}\right)\mathcal{E}_0\sigma_n \quad (22)$$

By replacing the perturbed potential we have:

$$\Delta E_n^{(1)} = \frac{2}{3}\left(\frac{e\in_0}{F}\right)E_n^{(0)} \quad (23)$$

Similarly the second order shift is [4]:

$$\Delta E_n^{(2)} = -\frac{1}{9}\left(\frac{e\in_0}{F}\right)^2 E_n^{(0)} \quad (24)$$

From Eqn.23 to find the energy shift with perturbed potential which is

$$V = \frac{1}{2}e^2 \in_0^2 y^2 \quad (25)$$

We have:

$$\Delta E_n^{(1)} = \left\langle\psi_n\left|\frac{1}{2}e^2 \in_0^2 y^2\right|\psi_n\right\rangle = \frac{1}{2}e^2 \in_0^2 \langle\psi_n|y^2|\psi_n\rangle \quad (26)$$

Eqn.26 reduces to:

$$\Delta E_n^{(1)} = -\frac{4}{15}\left(\frac{e\in_0}{F}\right)^2 E_n \quad (27)$$

Where $\in_0 = 10^{-11}\, V/m$, $m_e = 9.11 \times 10^{-31} kg$, $e = 1.6 \times 10^{-19} c\ and\ g = 9.8\, m/s^2$. Table 3 shows the first five shift's in energy by the perturbed potential.

Table 3: Shifts in energy by perturbed potential

n	$\Delta E(joule)$
1	0.02
2	0.03
3	0.05
4	0.06
5	0.07

4. Discussion and Conclusion

In this paper the perturbed linear potential due to constant external field (stark effect) is discuss, also the expression for the second order energy shift is studied using the solution of

Schrodinger equation for linear potential. The stark effect was investigated by exact and perturbed method; the Airy functions $Ai(x)$ and $Bi(x)$ are Eigen functions of the unperturbed linear potential, but the $Bi(x)$ solution diverges for large value and does not satisfy the condition $\psi(\infty) = 0$ the $Bi(x)$ is excluded.

The exact solution of stark effect leads to eigenvalue relation of:

$$E_n = \left(n + \frac{1}{2}\right)\frac{\bar{F}\hbar}{\sqrt{m}} + \frac{(F+\bar{F})^2}{2\bar{F}^2}, \bar{F} = e \in_0 .$$

The quantized energy is given in terms of the solution of the well-behaved zeros of the Airy function $Ai(-\sigma_n)$ with $E_n = \mathcal{E}_0\sigma_n$. The eigenvalues obtain by the perturbation is so near to the Exact result but distinction of exactness due to perturbation.

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